Mr Robt Anderson

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FERGUSON'S LECTURES

ON

SELECT SUBJECTS,

IN

MECHANICS,	§	OPTICS,
HYDROSTATICS,	§	GEOGRAPHY,
HYDRAULICS,	§	ASTRONOMY, AND
PNEUMATICS,	§	DIALLING.

A NEW EDITION,

CORRECTED AND ENLARGED.

WITH

NOTES AND AN APPENDIX,

ADAPTED TO THE PRESENT STATE OF THE ARTS AND SCIENCES.

BY DAVID BREWSTER, A. M.

IN TWO VOLUMES,

WITH A VOLUME OF PLATES.

SECOND AMERICAN EDITION,

CAREFULLY REVISED AND CORRECTED,

BY ROBERT PATTERSON,

PROFESSOR OF MATHEMATICS, AND TEACHER OF NATURAL PHILOSOPHY, IN THE UNIVERSITY OF PENNSYLVANIA.

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CONTENTS OF THE SECOND VOLUME.

LECTURE X.

On the principles and art of Dialling—Dialling by the globe—Dialling lines—Tables of the sun's place and declination—Tables of the equation of time—Rules for finding the latitude of places 1

LECTURE XI.

Of Dialling—Dialling by trigonometry—Babylonian and Italian dials—On the placing of dials, and the regulation of time-keepers - 43

LECTURE XII.

Showing how to calculate the mean time of any new or full moon, or eclipse, from the creation of the world to the year of Christ 5800—Table of lunations—Tables of the moon's mean motion from the sun, &c. - - - - 72

SUPPLEMENT TO THE PRECEDING LECTURES, BY THE AUTHOR.

MECHANICS.

Description of a new and safe crane with different powers - - - - - 95
On Barker's water-mill without wheel or trundle 103

HYDROSTATICS.

A machine for demonstrating that, on equal bottoms, the pressure of fluids is in proportion to their perpendicular height, without any regard to their quantities - - - - - 106
A machine to be substituted in place of the common hydrostatic bellows - - - 111
The cause of reciprocating springs, and of ebbing and flowing wells, explained - - 113

HYDRAULICS.

Account of Blakey's fire engine - - 115
Archimedes's screw engine - - - 119
Quadruple pump-mill for raising water 120

DIALLING.

Universal dialling cylinder - - - 123
Cylindrical dial - - - - - - 127
To make three sun dials on three different planes 131
A universal dial on a plain cross - - 132
A universal dial by a terrestrial globe - 134

APPENDIX, BY THE EDITOR.

MECHANICS.

On the construction of undershot water-wheels for turning machinery - - - - - 141
On the construction of the mill-course 142
On the water-wheel and its float-boards 149
On the spur-wheel and trundle - 156
On the formation, size, and velocity, of the mill-stone - - - - 160
On the performance of undershot mills 165
On a new mill-wright's table - 169
On horizontal mills - - - 176
On double corn-mills - - - 181
On breast-mills - - - - 184
Practical remarks on the performance and construction of overshot water-wheels - 188
On the method of computing the effective power of overshot wheels in turning machinery - - - - - 188
On the performance of overshot and undershot mills - - - - 190
On the formation of the buckets, and the proper velocity, of overshot wheels 192
Account of an improvement in flour-mills 200
On the formation of the teeth of wheels and the leaves of pinions - - - - 202
On the formation of epicycloids, mechanically, and on the disposition of the teeth on the wheel's circumference - - 221

On the formation of cycloids, and epicycloids, geometrically, and the method of drawing lines parallel to them - - 223
On bevelled wheels, and the method of giving an epicycloidal form to their teeth 228
On the formation of the teeth of rack-work, the wipers of stampers, &c. - - - 231
On the nature and construction of wind-mills 248
Description of a wind-mill - - 248
On the form and position of wind-mill sails 256
To find the momentum of friction - 266
To find the velocity of the wind - 267
On the effect of wind-mill sails - 274
On horizontal wind-mills - - 277
On the nature of friction, and the method of diminishing its effects in machinery - 283
On the nature and operation of fly-wheels 302
On wheel carriages - - - - 310
On the formation of carriage wheels 311
On the position of the wheels - 327
On the line of traction, and the method by which horses exert their strength 328
On the position of the centre of gravity, and the manner of disposing the load 332
On the thrashing machine - - - 336
On thrashing machines driven by water 338
On thrashing machines driven by horses 342
On the power of thrashing machines 347
On the construction and effect of machines 349

Description of a simple and powerful capstane 369
A mechanical method of finding the centre of gravity - - - - - - - 374

HYDRAULICS.

On the steam-engine - - - - - 376
On the power of steam-engines, and the method of computing it - - 395
Description of a water-blowing machine - 400

OPTICS.

On achromatic telescopes - - - - 403
On achromatic object-glasses - 403
On achromatic eye-pieces - - 425
On the construction of optical instruments, with tables of their apertures, &c. and the method of grinding the lenses and mirrors of which they are composed - - - - - - 433
On the method of grinding and polishing lenses - - - - - - - 433
On the method of grinding and polishing the mirrors of reflecting telescopes - 438
On the single microscope - - 442
On the double microscope - - 446
On the refracting telescope - - 447
On the Gregorian telescope - - - 450
On the Cassegrainian telescope - 452
On the Newtonian telescope - - 454
Description of a new fluid microscope, invented by the editor - - - - - - 461

DIALLING.

Description of an analemmatic dial, which sets itself - - - - 464
Description of a new dial, invented by Lambert 471

ASTRONOMY.

On the cause of the tides - - 474

MECHANICS.

Extract from Marrat's Mechanics, on wheel-carriages - - - - 477

LECTURES

ON SELECT SUBJECTS.

LECTURE X.

The Principles and Art of Dialing.

A DIAL is a plane, upon which lines are described in such a manner, that the shadow of a wire, or of the upper edge of a plate-stile, erected perpendicularly on the plane of the dial, may, when the sun shines, shew the true time of the day. Preliminaries.

The edge of the plate by which the time of the day is found, is called the *gnomon* or *stile* of the dial, which must be parallel to the earth's axis; and the line on which the said plate is erected, is called the *substile.*

The angle included between the substile and stile, is called the *elevation*, or *height*, of the stile.

Those dials whose planes are parallel to the plane of the horizon, are called *horizontal* dials; and those dials whose planes are perpen-

dicular to the plane of the horizon, are called *vertical*, or *erect* dials.

Those erect dials, whose planes directly front the north or south, are called *direct* north or south dials; and all other erect dials are called *decliners*, because their planes are turned away from the north or south.

Those dials, whose planes are neither parallel nor perpendicular to the plane of the horizon, are called *inclining*, or *reclining* dials, according as their planes make acute or obtuse angles with the horizon; and if their planes are also turned aside from facing the south or north, they are called *declining-inclining*, or *declining-reclining* dials.

The intersection of the plane of the dial, with that of the meridian, passing through the stile, is called the *meridian of the dial*, or the hour-line of XII.

Those meridians, whose planes pass through the stile, and make angles of 15, 30, 45, 60, 75, and 90 degrees, respectively, with the meridian of the place, (which marks the hour-line of XII) are called *hour-circles;* and their intersections with the plane of the dial, are called *hour-lines.*

In all declining dials, the substile makes an angle with the hour-line of XII; and this angle is called the *distance of the substile from the meridian.*

The declining *plane's difference of longitude*, is the angle formed at the intersection of the stile and plane of the dial, by two meridians; one of which passes through the hour-line of XII, and the other through the substile.

This much being premised concerning dials in general, we shall now proceed to explain the different methods of their construction. PLATE XX.

If the whole earth *aPcp* were transparent, and hollow, like a sphere of glass, and had its equator divided into 24 equal parts by so many meridian semicircles, *a, b, c, d, e, f, g,* &c. one of which being the geographical meridian of any given place, as London, which is supposed to be at the point *a*; and if the hours of XII were marked at the equator, both upon that meridian and the opposite one, and all the rest of the hours in order on the rest of the meridians, those meridians would be the hour-circles of London; then, if the sphere had an opaque axis, as *PEp*, terminating in the poles *P* and *p*, the shadow of the axis would fall upon every particular meridian and hour, when the sun came to the plane of the opposite meridian, and would consequently shew the time at London, and at all other places on the meridian of London. Fig. 2. The universal principle on which *dialing* depends.

If this sphere were cut through the middle by a solid plane *ABCD*, in the rational horizon of London, one half of the axis *EP* would be above the plane, and the other half below it; and if straight lines were drawn from the centre of the plane, to those points where the circumference is cut by the hour-circles of the sphere, those lines would be the hour-lines of a horizontal dial for London: for the shadow of the axis would fall upon each particular hour-line of the dial, when it fell upon the like hour-circle of the sphere. *Horizontal dial.*

If the plane which cuts the sphere be upright, at *AFCG*, touching the given place (London) at *F*, and directly facing the meridian of Lon- Fig. 3.

don, it will then become the plane of an erect-direct south dial; and if right lines be drawn from its centre *E*, to those points of its circumference where the hour-circles of the sphere cut it, these will be the hour-lines of a vertical or direct south dial for London, to which the hours are to be set as in the figure, (contrary to those on a horizontal dial) and the lower half *Ep* of the axis will cast a shadow on the hour of the day in this dial, at the same time that it would fall upon the like hour-circle of the sphere, if the dial-plane were not in the way.

Vertical dial.

Inclining and *reclining dials.*

If the plane (still facing the meridian) be made to incline, or recline, by any given number of degrees, the hour-circles of the sphere will still cut the edge of the plane in those points to which the hour-lines must be drawn straight from the centre; and the axis of the sphere will cast a shadow on these lines at the respective hours. The like will still hold, if the plane be made to decline by any given number of degrees from the meridian, toward the east or west; provided the declination be less than 90 degrees, or the reclination be less than the co-latitude of the place: and the axis of the sphere will be a gnomon, or stile, for the dial. But it cannot be a gnomon, when the declination is quite 90 degrees, nor when the reclination is equal to the co-latitude; because, in these two cases, the axis has no elevation above the plane of the dial.

Declining dials.

And thus it appears, that the plane of every dial represents the plane of some great circle upon the earth; and the gnomon the earth's axis, whether it be a small wire, as in the above figures, or the edge of a thin plate, as in the common horizontal dials.

The whole earth, as to its bulk, is but a point, if compared to its distance from the sun; and, therefore, if a small sphere of glass be placed upon any part of the earth's surface, so that its axis be parallel to the axis of the earth, and the sphere have such lines upon it, and such planes within it, as above described, it will shew the hours of the day as truly as if it were placed at the earth's centre, and the shell of the earth were as transparent as glass. PLATE XX.

But because it is impossible to have a hollow sphere of glass perfectly true, blown round a solid plane: or if it were, we could not get at the plane within the glass to set it in any given position; we make use of a wire-sphere to explain the principles of dialing, by joining 24 semicircles together at the poles, and putting a thin flat plate of brass within it. Fig. 2, 3.

Dialing by the common Terrestrial Globe.

A common globe, of 12 inches diameter, has generally 24 meridian semicircles drawn upon it. If such a globe be elevated to the latitude of any given place, and turned about until any one of these meridians cuts the horizon in the north point, where the hour of XII is supposed to be marked, the rest of the meridians will cut the horizon at the respective distances of all the other hours from XII. Then, if these points of distance be marked on the horizon, and the globe be taken out of the horizon, and a flat board or plate be put into its place, even with the surface of the horizon, and if straight lines be drawn from the centre of the board to those points of distance on the horizon which were cut by the 24 meridian semicircles, these lines

will be the hour-lines of a horizontal dial for that latitude, the edge of whose gnomon must be in the very same situation with that of the axis of the globe before it was taken out of the horizon: that is, the gnomon must make an angle with the plane of the dial, equal to the latitude of the place for which the dial is made.

If the pole of the globe be elevated to the co-latitude* of the given place, and any meridian be brought to the north point of the horizon, the rest of the meridians will cut the horizon in the respective distances of all the hours from XII, for a direct south dial, whose gnomon must make an angle with the plane of the dial, equal to the co-latitude of the place; and the hours must be set the contrary way on this dial, to what they are on the horizontal.

But if your globe have more than 24 meridian semicircles upon it, you must take the following method for making *horizontal and south dials by it.*

To construct a *horizontal dial.*

Elevate the pole to the latitude of your place, and turn the globe until any particular meridian (suppose the first) comes to the north point of the horizon, and the opposite meridian will cut the horizon in the south. Then, set the hour-index to the uppermost XII on its circle; which done, turn the globe westward until 15 degrees of the equator pass under the brazen meridian, and then the hour-index will be at I, (for the sun moves 15 degrees every hour) and the first meridian will cut the horizon in the number of degrees from the north point, that I is distant from XII. Turn on until other 15 degrees of the equator pass under the brazen meridian,

* If the latitude be subtracted from 90 degrees, the remainder is called the co-latitude, or complement of the latitude.

and the hour-index will then be at II, and the first meridian will cut the horizon in the number of degrees that II is distant from XII: and thus, by making 15 degrees of the equator pass under the brazen meridian for every hour, the first meridian of the globe will cut the horizon in the distances of all the hours from XII to VI, which is just 90 degrees; and then you need go no farther, for the distances of XI, X, IX, VIII, VII, and VI, in the forenoon, are the same from XII, as the distances of I, II, III, IV, V, and VI, respectively, in the afternoon: and these hour-lines continued through the centre, will give the opposite hour-lines on the other half of the dial: but no more of these lines need be drawn than what answer to the sun's continuance above the horizon of your place on the longest day, which may be easily found by the 26th problem of the foregoing lecture.

Thus, to make a horizontal dial for the latitude of London, which is $51\frac{1}{2}$ degrees north, elevate the north pole of the globe $51\frac{1}{2}$ degrees above the north point of the horizon, and then turn the globe, until the first meridian (which is that of London on the English terrestrial globe) cuts the north point of the horizon, and set the hour-index to XII at noon.

Then, turning the globe westward until the index points successively to I, II, III, IV, V, and VI, in the afternoon; or until 15, 30, 45, 60, 75, and 90 degrees of the equator pass under the brazen meridian, you will find that the first meridian of the globe cuts the horizon in the following number of degrees from the north towards the east, viz. $11\frac{2}{3}$, $24\frac{1}{4}$, $38\frac{1}{12}$, $53\frac{1}{2}$, $71\frac{1}{15}$, and 90; which are the respective distances of

PLATE XXI. the above hours from XII upon the plane of the horizon.

Fig. 1. To transfer these, and the rest of the hours, to a horizontal plane—draw the parallel right lines *ac* and *bd* upon that plane, as far from each other as is equal to the intended thickness of the gnomon or stile of the dial, and the space included between them will be the meridian or XII o'clock line on the dial. Cross this meridian at right angles with the VI o'clock line *gh*, and setting one foot of your compasses in the intersection *a*, as a centre, describe the quadrant *ge* with any convenient radius or opening of the compasses: then, setting one foot in the intersection *b*, as a centre, with the same radius describe the quadrant *fh*, and divide each quadrant into 90 equal parts or degrees, as in the figure.

Because the hour-lines are less distant from each other about noon, than in any other part of the dial, it is best to have the centres of these quadrants at a little distance from the centre of the dial-plane, on the side opposite to XII, in order to enlarge the hour-distances thereabouts under the same angles on the plane.—Thus, the centre of the plane is at *C*, but the centres of the quadrants at *a* and *b*.

Lay a rule over the point *b*, (and keeping it there for the centre of all the afternoon-hours in the quadrant *fh*) draw the hour-line of I, through $11\frac{2}{3}$ degrees in the quadrant; the hour-line of II, through $24\frac{1}{4}$ degrees; of III, through $38\frac{1}{12}$ degrees; IV, through $53\frac{1}{2}$, and V, through $71\frac{1}{12}$: and because the sun rises about four in the morning, on the longest days at London, continue the hour-lines of IV and V, in the afternoon, through the centre *b* to the opposite

side of the dial.—This done, lay the rule to the centre *a*, of the quadrant *eg*, and through the like divisions or degrees of that quadrant, viz. $11\frac{2}{3}$, $24\frac{1}{4}$, $38\frac{1}{12}$, $53\frac{1}{2}$, and $71\frac{1}{12}$, draw the forenoon-hour-lines of XI, X, IX, VIII, and VII; and because the sun sets not before eight in the evening on the longest days, continue the hour-lines of VII and VIII in the afternoon, through the centre *a*, to VII and VIII in the afternoon; and all the hour-lines will be finished on this dial; to which the hours may then be set, as in the figure.

Lastly, through $51\frac{1}{2}$ degrees of either quadrant, and from its centre, draw the right line *ag* for the hypothenuse or axis of the gnomon *agi*; and from *g*, let fall the perpendicular *gi*, upon the meridian-line *ai*, and there will be a triangle made, whose sides are *ag*, *gi*, and *ai*. If a plate, similar to this triangle, be made as thick as the distance between the lines *ac* and *bd*, and set upright between them, touching at *a* and *b*, its hypothenuse *ag* will be parallel to the axis of the world, when the dial is truly set; and will cast a shadow on the hour of the day.

N. B. The trouble of dividing the two quadrants may be saved, if you have a scale with a line of chords upon it, such as that on the right hand of the plate; for if you extend the compasses from 0 to 60 degrees on the line of chords, and with that extent, as a radius, describe the two quadrants from their respective centres, the above distances may be taken with the compasses upon the line, and set off upon the quadrants.*

* The hour-angles on a dial, or any other plane angle, may, with great accuracy, be laid off by means of a line of *equal parts*,

PLATE XXI. Fig. 2. To construct an erect-direct south dial.

To make an erect-direct south dial.—Elevate the pole to the co-latitude of your place, and proceed in all respects as above taught for the horizontal dial, from VI in the morning to VI in the afternoon, only the hours must be reversed, as in the figure; and the hypothenuse *ag*, of the gnomon *agf*, must make an angle with the dial-plane, equal to the co-latitude of the place. As the sun can never shine longer on this dial than from six in the morning till six in the evening, there is no occasion for having any more than 12 hours upon it.*

To construct an erect declining dial.

To make an erect-dial, declining from the south toward the east or west.—Elevate the pole to the latitude of your place, and screw the quadrant of altitude to the zenith. Then, if your dial decline toward the east, (which we shall suppose it to do at present) count in the horizon the degrees of declination, from the east point toward the north, and bring the lower end of the quadrant to that degree of declination at which the reckoning ends. This done, bring any particular meridian of your globe (as suppose the first meridian) directly

thus:—From the angular point as a centre, with the radius $57\frac{3}{10}$ taken from any convenient line of equal parts, describe an arch, on which apply the chord of the given angle, if not exceeding 30°, taken from the same line of equal parts, calling each part or division a degree; but if the given angle exceed 30° or 60°, you are first to apply the radius (which is always equal to the chord of 60°,) and then apply the chord of the difference between 60° and the given angle, forwards or backwards, according as the given angle is greater or less than 60°, and then the two radii drawn through the extremities of the arch thus measured off will contain the given angle, *very nearly*. See Trans. Am. Phil. Soc. vol. 6, part 1.—A. Ed.

* A new and very simple geometrical method of constructing sun-dials may be seen in our author's Mechanical Exercises, p. 94.—E. Ed.

under the graduated edge of the upper part of the brazen meridian, and set the hour-index to XII at noon. Then, keeping the quadrant of altitude at the degree of declination in the horizon, turn the globe eastward on its axis, and observe the degrees cut by the first meridian in the quadrant of altitude, (counted from the zenith) as the hour-index comes to XI, X, IX, &c. in the forenoon, or as 15, 30, 45, &c. degrees of the equator pass under the brazen meridian at these hours respectively; and the degrees then cut in the quadrant by the first meridian, are the respective distances of the forenoon-hours from XII on the plane of the dial. Then, for the afternoon-hours, turn the quadrant of altitude round the zenith until it comes to the degree in the horizon opposite to that where it was placed before; namely, as far from the west point of the horizon toward the south, as it was set at first from the east point toward the north; and turn the globe westward on its axis, until the first meridian comes to the brazen meridian again, and the hour-index to XII: then, continue to turn the globe westward, and as the index points to the afternoon-hours I, II, III, &c. as 15, 30, 45, &c. degrees of the equator pass under the brazen meridian, the first meridian will cut the quadrant of altitude in the respective number of degrees from the zenith, that each of these hours is from XII on the dial.—And note, that when the first meridian goes off the quadrant at the horizon, in the afternoon, the hour-index shews the time when the sun will come upon this dial; and when it goes off the quadrant in the afternoon, the index will point to the time when the sun goes off the dial.

Having thus found all the hour-distances from XII, lay them down upon your dial-plate, either by dividing a semicircle into two quadrants of 90 degrees each (beginning at the hour-line of XII), or by the line of chords (or equal parts), as above directed.

In all declining dials, the line on which the stile or gnomon stands, (commonly called the *substile-line)* makes an angle with the twelve o'clock line, and falls among the forenoon-hour-lines, if the dial declines toward the east; and among the afternoon-hour-lines, when the dial declines toward the west; that is, to the left hand from the twelve o'clock line in the former case, and to the right hand from it in the latter.

To find the distance of the substile from the twelve o'clock line; when your dial declines from the south toward the east, count the degrees of that declination in the horizon from the east point toward the north, and bring the lower end of the quadrant of altitude to that degree of declination where the reckoning ends: then turn the globe until the first meridian cuts the horizon in the like number of degrees, counted from the south point toward the east; and the quadrant and first meridian will then cross one another at right angles, and the number of degrees of the quadrant, which are intercepted between the first meridian and the zenith, is equal to the distance of the substile-line from the twelve o'clock line; and the number of degrees of the first meridian, which are intercepted between the quadrant and the north pole, is equal to the elevation of the stile above the plane of the dial.

When the dial declines westward from the south, count that declination from the east point of the horizon towards the south, and bring the quadrant of altitude to the degree in the horizon at which the reckoning ends; both for finding the forenoon-hours, and the distance of the substile from the meridian: and for the afternoon-hours, bring the quadrant to the opposite degree in the horizon, namely, as far from the west toward the north, and then proceed in all respects as above.

Thus, we have finished our declining dial; and in so doing, have made four dials, viz.

1. A north dial, declining eastward by the same number of degrees. 2. A north dial, declining the same number west. 3. A south dial, declining east. And, 4. A south dial, declining west. Only, placing the proper number of hours, and the stile or gnomon respectively, upon each plane. For, (as above-mentioned) in the south-west plane, the substile-line falls among the afternoon-hours; and in the south-east, of the same declination, among the forenoon-hours, at equal distances from XII. And thus, all the morning-hours on the west decliner will be like the afternoon-hours on the east decliner; the south-east decliner will produce the north-west decliner; and the south-west decliner, the north-east decliner, by only extending the hour-lines, stile and substile, quite through the centre: the axis of the stile, (or edge that casts the shadow on the hour of the day) being, in all dials whatever, parallel to the axis of the world, and consequently pointing toward the north pole of the heavens in north latitudes, and toward the

PLATE XXI. south pole, in south latitudes. *See more of this in the following lecture.*

An easy method of constructing dials, by means of a dialing-scale.

But because every one who would like to make a dial, may not perhaps be provided with a globe to assist him, and may not understand the method of doing it by logarithmic calculation; we shall shew how to perform it by the plain dialing-lines, or scale of latitudes and hours; such as those on the right hand of Fig. 4, in Plate XXI, or at the top of Plate XXII, and which may be had on scales commonly sold by the mathematical instrument makers.

This is the easiest of all mechanical methods, and by much the best, when the lines are truly divided: not only the half hours and quarters may be laid down by all of them, but every fifth minute by most, and every single minute by those where the line of hours is a foot in length.

Fig. 3. Having drawn your double meridian-line *ab, cd,* on the plane intended for a horizontal dial, and crossed it at right angles by the six o'clock line *fe,* (as in Fig. 1.) take the latitude of your place with the compasses, on the scale of latitudes, and set that extent from *c* to *e,* and from *a* to *f,* on the six o'clock line: then, taking the whole six hours between the points of the compasses on the scale of hours, with that extent set one foot in the point *e,* and let the other foot fall where it will upon the meridian-line *cd,* as at *d.* Do the same from *f* to *b,* and draw the right lines *e d* and *f b,* each of

which will be equal in length to the whole scale of hours. This done, setting one foot of the compasses in the beginning of the scale at XII, and extending the other to each hour on the scale, lay off these extents from *d* to *e* for the afternoon-hours, and from *b* to *f* for those of the forenoon: this will divide the lines *de* and *bf* in the same manner as the hour-scale is divided, as 1, 2, 3, 4, 5, and 6, on which the quarters may also be laid down, if required.—Then, laying a rule on the point *c*, draw the first five hours in the afternoon, from that point, through the dots at the numeral figures 1, 2, 3, 4, 5, on the line *de*; and continue the lines of IV and V through the centre *c* to the other side of the dial, for the like hours of the morning; which done, lay the rule on the point *a*, and draw the last five hours in the forenoon through the dots 5, 4, 3, 2, 1, on the line *fb*; continuing the hour-lines of VII and VIII through the centre *a* to the other side of the dial, for the like hours of the evening; and set the hours to their respective lines as in the figure. Lastly, make the gnomon the same way as taught above for the horizontal dial, and the whole will be finished. PLATE XXI.

To make an erect south dial, take the co-latitude of your place from the scale of latitudes, and then proceed in all respects for the hour-lines, as in the horizontal dial; only reversing the hours, as in Fig. 2.; and making the angle of the stile's height equal to the co-latitude.

I have drawn out a set of dialing-lines upon the top of Plate XXII large enough for making a dial of nine inches diameter, or more, if required; and have drawn them tolerably exact for common practice, to every quarter of an hour.

PLATE XXI. This scale may be cut off from the plate, and pasted upon wood, or upon the inside of one of the boards of this book; and then it will be somewhat more exact than it is on the plate, for being rightly divided upon the copper-plate, and printed off on wet paper, it shrinks as the paper dries; but when it is wetted again, it stretches to the same size as when newly printed; and if pasted on while wet, it will remain of that size afterward.

But lest the young dialist should have neither globe, nor wooden scale, and should tear or otherwise spoil the paper one in pasting, we shall now shew him how he may make a dial without any of these helps. Only, if he have not a line of chords, (or equal parts) he must divide a quadrant into 90 equal parts or degrees for taking the proper angle of the stile's elevation, which is easily done.

Fig. 4. With an opening of the compasses at *ZL*, describe the two semicircles *LFk* and *LQk*, upon the centres *Z* and *z*, where the six o'clock line crosses the double meridian-line, and divide each semicircle into 12 equal parts, beginning at *L*; though, strictly speaking, only the quadrants from *L* to the six o'clock line need be divided: then connect the divisions which are equi-distant from *L*, by the parallel lines *KM*, *IN*, *HO*, *GP*, and *FQ*. Draw *VZ* for the hypothenuse of the stile, making the angle *VZE* equal to the latitude of your place; and continue the line *VZ* to *R*. Draw the line *Rr* parallel to the six o'clock line, and set off the distance *aK* from *Z* to *Y*, the distance *bI* from *Z* to *X*, *cH*, from *Z* to *W*, *dG* from *Z* to *T*, and *eF* from *Z* to *S*. Then draw the lines *Ss*, *Tt*, *Ww*, *Xx*, and *Yy*, each parallel to *Rr*. Set off the distance *yY*

Horizontal dial.

from *a* to 11, and from *f* to 1: the distance *xX* from *b* to 10, and from *g* to 2; *wW* from *c* to 9, and from *h* to 3; *tT* from *d* to 8, and from *i* to 4; *sS* from *e* to 7, and from *n* to 5. Then, laying a rule to the centre *Z*, draw the forenoon-hour-lines through the points 11, 10, 9, 8, 7; and laying it to the centre *z*, draw the afternoon-lines through the points 1, 2, 3, 4, 5; continuing the forenoon-lines of VII and VIII through the centre *Z*, to the opposite side of the dial, for the like afternoon-hours; and the afternoon-lines IV and V through the centre *z*, to the opposite side, for the like morning-hours. Set the hours to these lines as in the figure, and then erect the stile or gnomon, and the horizontal dial will be finished. PLATE XXI.

South dial.

To construct a south dial, draw the lines *VZ*, making an angle with the meridian *ZL*, equal to the co-latitude of your place, and proceed in all respects as in the above horizontal dial for the same latitude, reversing the hours as in Fig. 2, and making the elevation of the gnomon equal to the co-latitude.

Perhaps it may not be unacceptable to explain the method of constructing the dialing-lines, and some others; which is as follows:

PLATE XXII. Fig. 1. Dialing-lines, how constructed.

With any opening of the compasses, as *EA*, according to the intended length of the scale, describe the circle *ADCB*, and cross it at right angles by the diameters *CEA* and *DEB*. Divide the quadrant *AB* first into 9 equal parts, and then each part into 10; so shall the quadrant be divided into 90 equal parts or degrees. Draw the right line *AFB* for the chord of this quadrant, and, setting one foot of the compasses in the point *A*, extend the other to the several divisions of the quadrant, and transfer these

divisions to the line *AFB* by the arcs 10 10, 20 20, &c. and this will be a line of chords, divided into 90 unequal parts; which, if transferred from the line back again to the quadrant, will divide it equally. It is plain by the figure, that the distance from *A* to 60 in the line of chords is just equal to *AE*, the radius of the circle from which that line is made: for, if the arc 60 60 be continued, of which *A* is the centre, it goes exactly through the centre *E* of the arc *AB*.

And, therefore, in laying down any number of degrees on a circle, by the line of chords, you must first open the compasses, so as to take in just 60 degrees upon that line, as from *A* to 60: and then, with that extent, as a radius, describe a circle which will be exactly of the same size with that from which the line was divided: which done, set one foot of the compasses in the beginning of the chord-line, as at *A*, and extend the other to the number of degrees you want upon the line, which extent, applied to the circle, will include the like number of degrees upon it.

Divide the quadrant *CD* into 90 equal parts, and from each point of division draw right lines as *i*, *k*, *l*, &c. to the line *CE*; all perpendicular to that line, and parallel to *DE*, which will divide *EC* into a line of sines; and although these are seldom put among the dialing-lines on a scale, yet they assist in drawing the line of latitudes. For, if a rule be laid upon the point *D*, and over each division in the line of sines, it will divide the quadrant *CB* into 90 unequal parts, as *Ba*, *ab*, &c. shewn by the right lines 10 *a*, 20 *b*, 30 *c*, &c. drawn along the edge of the rule. If the right line *BC* be

drawn, subtending this quadrant, and the nearest distances *Ba*, *Bb*, *Cc*, &c. be taken in the compasses from *B*, and set upon this line in the same manner as directed for the line of chords, it will make a line of latitudes *BC*, equal in length to the line of chords *AB*, and of an equal number of divisions, but very unequal as to their lengths. PLATE XXII.

Draw the right line *DGA*, subtending the quadrant *DA*; and parallel to it, draw the right line *rs*, touching the quadrant *DA* at the numeral figure 3. Divide this quadrant into six equal parts, as 1, 2, 3, &c. and through these points of division draw right lines from the centre *E* to the line *rs*, which will divide it at the points where the six hours are to be placed, as in the figure. If every sixth part of the quadrant be subdivided into four equal parts, right lines drawn from the centre through these points of division, and continued to the line *rs*, will divide each hour upon it into quarters.

A dial on card. Fig. 2.

In Fig. 2. we have the representation of a portable dial, which may be easily drawn on a card, and carried in a pocket-book. The lines *ad*, *ab*, and *bc*, of the gnomon, must be cut quite through the card; and as the end *ab* of the gnomon is raised occasionally above the plane of the dial, it turns upon the uncut line *cd* as on a hinge. The dotted line *AB* must be slit quite through the card, and the thread must be put through the slit, and have a knot tied behind, to keep it from being easily drawn out. On the other end of this thread is a small plummet *D*, and on the middle of it a small bead for shewing the time of the day.

To rectify this dial, set the thread in the slit right against the day of the month, and stretch

PLATE XXII. the thread from the day of the month over the angular point where the curve-lines meet at XII, then shift the bead to that point on the thread, and the dial will be rectified.

To find the hour of the day, raise the gnomon, (no matter how much or how little) and hold the edge of the dial next the gnomon toward the sun, so as the uppermost edge of the shadow of the gnomon may just cover the *shadow-line;* and the bead then playing freely on the face of the dial, by the weight of the plummet, will shew the time of the day among the hour-lines, as it is forenoon or afternoon.

To find the time of sun-rising and setting, move the thread among the hour-lines, until it either covers some one of them, or lies parallel betwixt any two; and then it will cut the time of sun-rising among the forenoon-hours, and of sun-setting among the afternoon-hours, on that day of the year for which the thread is set in the scale of months.

To find the sun's declination, stretch the thread from the day of the month over the angular point at XII, and it will cut the sun's declination, as it is north or south, for that day, in the arched scale of north and south declination.

To find on what day the sun enters the signs; when the bead, as above rectified, moves along any of the curve-lines which have the signs of the zodiac marked upon them, the sun enters those signs on the days pointed out by the thread in the scale of months respectively.

The construction of this dial is very easy, especially if the reader compares it all along with Fig. 3. as he reads the following explanation of that figure.

Draw the occult line *AB* parallel to the top of the card, and cross it at right angles with the six o'clock line *ECD*; then upon *C*, as a centre, with the radius *CA*, describe the semicircle *AEL*, and divide it into 12 equal parts (beginning at *A*) as *Ar*, *rs*, &c. and from these points of division, draw the hour-lines *r*, *s*, *t*, *u*, *v*, *E*, *w*, and *x*, all parallel to the six o'clock line *EC*. If each part of the semicircle be divided into four equal parts, they will give the half-hour-lines and quarters, as in Fig. 2. Draw the right line *ASDo*, making the angle *SAB* equal to the latitude of your place. Upon the centre *A* describe the arch *RST*, and set off upon it the arcs *SR* and *ST*, each equal to 23½ degrees, for the sun's greatest declination; and divide them into 23½ equal parts, as in Fig. 2. Through the intersection *D* of the lines *ECD* and *ADo*, draw the right line *FDG* at right angles to *ADo*. Lay a rule to the points *A* and *R*, and draw the line *ARF* through 23½ degrees of south declination in the arc *SR*; and then laying the rule to the points *A* and *T*, draw the line *ATG* through 23½ degrees of north declination in the arc *ST*: so shall the lines *ARF* and *ATG* cut the line *FDG* in the proper length for the scale of months. Upon the centre *D*, with the radius *DF*, describe the semicircle *FoG*; and divide it into six equal parts, *Fm*, *mn*, *no*, &c. and from these points of division draw the right lines *mh*, *ni*, *pk*, and *ql*, each parallel to *oD*. Then, setting one foot of the compasses in the point *F*, extend the other to *A*, and describe the arc *AzH* for the tropic of ♑: with the same extent, setting one foot in *G*, describe the arc *AEO* for the tropic of ♋. Next setting one foot in the point *h*, and extending

Plate XXII. Fig. 3.

PLATE XXII. the other to *A*, describe the arc *ACI* for the beginnings of the signs ♒ and ♐; and with the same extent, setting one foot in the point *l*, describe the arc *AN* for the beginnings of the signs ♊ and ♌. Set one foot in the point *i*, and having extended the other to *A*, describe the arc *AK* for the beginnings of the signs ♓ and ♏; and with the same extent, set one foot in *k*, and describe the arc *AM* for the beginnings of the signs ♉ and ♍. Then, setting one foot in the point *D*, and extending the other to *A*, describe the curve *AL* for the beginnings of ♈ and ♎; and thus the signs will be finished.—This done, lay a rule from the point *A* over the sun's declination in the arch *RST*, (found by the following table) for every fifth day of the year; and where the rule cuts the line *FDG*, make marks; and place the days of the months right against these marks, in the manner shewn by Fig. 2. Lastly, draw the shadow-line *PQ* parallel to the occult line *AB*; make the gnomon, and set the hours to their respective lines, as in Fig. 2. and the dial will be finished.

Fig. 4. There are several kinds of dials, which are called *universal*, because they serve for all latitudes. Of these, the best one that I know, is Mr. Pardie's, which consists of three principal parts: the first whereof is called the *horizontal*

A universal dial. *plane (A)*, because when the dial is used, it must be parallel to the horizon. In this place is fixed an upright pin, which enters into the edge of the second part *BD*, called the *meridional plane*; which is made of two pieces, the lowest whereof *(B)* is called the *quadrant*, because it contains a quarter of a circle, divided into 90 degreees; and it is only in this part, near *B*, that the pin enters. The other piece is a *semicircle*

(D), adjusted to the quadrant, and turning in it by a groove, for raising or depressing the diameter *(EF)* of the semicircle, which diameter is called the *axis* of the instrument. The third piece is a *circle (G)*, divided on both sides into 24 equal parts, which are the hours. This circle is put upon the meridional plane so, that the axis *(EF)* may be perpendicular to the circle; and the point *C*, the common centre of the circle, semicircle, and quadrant. The straight edge of the semicircle is chamfered on both sides to a sharp edge, which passes through the centre of the circle. On one side of the chamfered part, the first six months of the year are laid down, according to the sun's declination for their respective days, and on the other side the last six months. And against the days on which the sun enters the signs respectively, there are straight lines drawn upon the semicircle, with the characters of the signs marked upon them. There is a black line drawn along the middle of the upright edge of the quadrant, over which hangs a thread *(H)* with its plummet *(I)* for levelling the instrument. *N. B.* From the 22d of September to the 20th of March, the upper surface of the circle must touch both the centre *C* of the semicircle, and the line of ♈ and ♎; and from the 20th of March to the 22d of September, the lower surface of the circle must touch that centre and line.

To find the time of the day by this dial.—Having set it on a level place in sun-shine, and adjusted it by the levelling screws *k* and *l*, until the plumb-line hangs over the back-line upon the edge of the quadrant, and parallel to the said edge; move the semicircle in the quadrant,

PLATE XXII. until the line of ♈ and ♎, (where the circle touches) comes to the latitude of your place in the quadrant: then, turn the whole meridional plane *BD*, with its circle *G*, upon the horizontal plane *A*, until the edge of the shadow of the circle fall precisely on the day of the month in the semi-circle; and then, the meridional plane will be due north and south, the the axis *EF* will be parallel to the axis of the world, and will cast a shadow upon the true time of the day, among the hours on the circle.

N. B. As, when the instrument is thus rectified, the quadrant and semicircle are in the plane of the meridian, so the circle is then in the plane of the equinoctial. Therefore, as the sun is above the equinoctial in summer, (in northern latitudes) and below it in winter; the axis of the semicircle will cast a shadow on the hour of the day, on the upper surface of the circle, from the 20th of March to the 22d of September: and from the 22d of September to the 20th of March, the hour of the day will be determined by the shadow of the semicircle, upon the lower surface of the circle. In the former case, the shadow of the circle falls upon the day of the month, on the lower part of the diameter of the semicircle; and in the latter case on the upper part.

Fig. 5. The method of laying down the months and signs upon the semicircle, is as follows. Draw the right line *ACB*, equal to the diameter of the semicircle *ADB*, and cross it in the middle at right angles with the line *ECD*, equal in length to *ACB*; then *EC* will be the radius of the circle *FCG*, which is the same as that of the semicircle. Upon *E* as a centre, de-

scribe the circle *FCG*, on which set off the arcs *Ch* and *Ci*, each equal to $23\frac{1}{2}$ degrees, and divide them accordingly into that number for the sun's declination. Then, laying the edge of a rule over the centre *E*, and also over the sun's declination for every fifth day* of each month, (as in the card-dial) mark the points on the diameter *AB* of the semicircle from *a* to *g*, which are cut by the rule; and there place the days of the months accordingly, answering to the sun's declination. This done, setting one foot of the compasses in *C*, and extending the other to *a* or *g*, describe the semicircle *a b c d e f g*; which divide into six equal parts, and through the points of division draw right lines, parallel to *CD*, for the beginning of the signs, (of which one half are on one side of the semicircle, and the other half on the other side) and set the characters of the signs to their proper lines, as in the figure.

The following table shews the sun's place and declination, in degrees and minutes, at the noon of every day of the second year after leap-year; which is a mean between those of leap-year itself, and the first and third years after. It is useful for inscribing the months and their days on sun-dials; and also for finding the latitudes of places, ac-

* The intermediate days may be drawn in by hand, if the spaces be large enough to contain them.

cording to the methods prescribed after the table.*

* In this edition, the table of the sun's longitude and declination has been calculated anew, and adapted to the present improved state of the solar tables. The editor has also added an accurate table of the equation of time, which, he trusts, will be of great use to the practical dialist. The words *add* and *subtract*, at the head of the column, denote that the equation of time must be added to, or subtracted from, the apparent time, or that which is deduced from the motion of the sun, in order to obtain the equated or true time, as shewn by a well-regulated clock or watch. The table is calculated for the second after leap-year, and is as accurate as the difference between the civil and solar year will permit.—E. Ed.

A TABLE,

Shewing the Sun's place and declination.

January.

Days	Sun's Pl. D.	Sun's Pl. M.	Sun's De. D.	Sun's De. M.
1	10 ♑	27	23 S	3
2	11	28	22	58
3	12	29	22	53
4	13	31	22	47
5	14	32	22	40
6	15	33	22	34
7	16	34	22	26
8	17	35	22	19
9	18	37	22	10
10	19	38	22	2
11	20	39	21	53
12	21	40	21	43
13	22	41	21	33
14	23	42	21	23
15	24	43	21	12
16	25	44	21	1
17	26	46	20	50
18	27	47	20	38
19	28	48	20	25
20	29	49	20	13
21	0 ♒	50	20	0
22	1	51	19	46
23	2	52	19	32
24	3	53	19	18
25	4	54	19	4
26	5	55	18	49
27	6	56	18	34
28	7	57	18	18
29	8	58	18	2
30	9	58	17	46
31	10	59	17	30

February.

Days	Sun's Pl. D.	Sun's Pl. M.	Sun's De. D.	Sun's De. M.
1	12 ♒	0	17 S	13
2	13	1	16	56
3	14	2	16	38
4	15	3	16	20
5	16	4	16	2
6	17	4	15	44
7	18	5	15	26
8	19	6	15	7
9	20	7	14	48
10	21	7	14	28
11	22	8	14	9
12	23	9	13	49
13	24	9	13	29
14	25	10	13	9
15	26	10	12	49
16	27	11	12	28
17	28	11	12	7
18	29	12	11	46
19	0 ♓	12	11	25
20	1	12	11	3
21	2	13	10	42
22	3	13	10	20
23	4	13	9	58
24	5	14	9	36
25	6	14	9	14
26	7	14	8	52
27	8	14	8	29
28	9	15	8	7

In these tables N signifies north, and S south, declination.

A TABLE,

Shewing the Sun's place and declination.

	March.					*April.*			
Days	Sun's Pl.		Sun's De.		Days	Sun's Pl.		Sun's De.	
	D.	M.	D.	M.		D.	M.	D.	M.
1	10 ♓	15	7 S	44	1	11 ♈	3	4 N	23
2	11	15	7	21	2	12	2	4	46
3	12	15	6	58	3	13	1	5	9
4	13	15	6	35	4	14	0	5	32
5	14	15	6	12	5	14	59	5	55
6	15	15	5	49	6	15	58	6	17
7	16	15	5	26	7	16	57	6	40
8	17	15	5	2	8	17	56	7	3
9	18	15	4	39	9	18	55	7	25
10	19	15	4	16	10	19	54	7	47
11	20	15	3	52	11	20	53	8	9
12	21	15	3	29	12	21	51	8	31
13	22	14	3	5	13	22	50	8	53
14	23	14	2	41	14	23	49	9	15
15	24	14	2	18	15	24	47	9	37
16	25	13	1	54	16	25	46	9	58
17	26	13	1	30	17	26	44	10	19
18	27	12	1	7	18	27	43	10	40
19	28	12	0	43	19	28	41	11	1
20	29	11	0	19	20	29	40	11	22
21	0 ♈	11	0 N	4	21	0 ♉	38	11	43
22	1	10	0	28	22	1	37	12	3
23	2	10	0	52	23	2	35	12	23
24	3	9	1	15	24	3	33	12	43
25	4	9	1	39	25	4	32	13	3
26	5	8	2	2	26	5	30	13	22
27	6	7	2	26	27	6	28	13	42
28	7	6	2	50	28	7	27	14	1
29	8	6	3	13	29	8	25	14	20
30	9	5	3	36	30	9	23	14	38
31	10	4	3	59					

A TABLE,

Shewing the Sun's place and declination.

	May.					*June.*			
Days	Sun's Pl.		Sun's De.		Days	Sun's Pl.		Sun's De.	
	D.	M.	D.	M.		D.	M.	D.	M.
1	10 ♉	21	14 N	57	1	10 ♊	12	22 N	0
2	11	19	15	15	2	11	10	22	8
3	12	18	15	33	3	12	7	22	16
4	13	16	15	50	4	13	4	22	24
5	14	14	16	8	5	14	2	22	31
6	15	12	16	25	6	14	59	22	37
7	16	10	16	42	7	15	57	22	43
8	17	8	16	58	8	16	54	22	49
9	18	6	17	14	9	17	51	22	55
10	19	4	17	30	10	18	49	23	0
11	20	2	17	46	11	19	46	23	4
12	20	59	18	1	12	20	43	23	9
13	21	57	18	17	13	21	40	23	12
14	22	55	18	31	14	22	38	23	16
15	23	53	18	46	15	23	35	23	19
16	24	51	19	0	16	24	32	23	21
17	25	48	19	14	17	25	29	23	23
18	26	46	19	27	18	26	27	23	25
19	27	44	19	41	19	27	24	23	26
20	28	41	19	53	20	28	21	23	27
21	29	39	20	6	21	29	18	23	28
22	0 ♊	37	20	18	22	0 ♋	16	23	28
23	1	34	20	30	23	1	13	23	28
24	2	32	20	41	24	2	10	23	27
25	3	29	20	53	25	3	7	23	26
26	4	27	21	3	26	4	5	23	24
27	5	24	21	14	27	5	2	23	22
28	6	22	21	24	28	5	59	23	20
29	7	20	21	33	29	6	56	23	17
30	8	17	21	43	30	7	53	23	14
31	9	15	21	52					

A TABLE,

Shewing the Sun's place and declination.

	July.					*August.*			
Days	Sun's Pl.		Sun's De.		Days	Sun's Pl.		Sun's De.	
	D.	M.	D.	M.		D.	M.	D.	M.
1	8 ♋	51	23 N	10	1	8 ♌	26	18 N	10
2	9	48	23	6	2	9	24	17	55
3	10	45	23	2	3	10	21	17	40
4	11	42	22	57	4	11	19	17	24
5	12	40	22	52	5	12	16	17	8
6	13	37	22	46	6	13	14	16	52
7	14	34	22	40	7	14	11	16	35
8	15	31	22	34	8	15	9	16	19
9	16	28	22	27	9	16	6	16	2
10	17	26	22	20	10	17	4	15	44
11	18	23	22	12	11	18	2	15	27
12	19	20	22	4	12	18	59	15	9
13	20	17	21	56	13	19	57	14	51
14	21	14	21	47	14	20	54	14	33
15	22	12	21	38	15	21	52	14	14
16	23	9	21	29	16	22	50	13	55
17	24	6	21	19	17	23	47	13	36
18	25	3	21	9	18	24	45	13	17
19	26	1	20	58	19	25	43	12	58
20	26	58	20	47	20	26	41	12	58
21	27	55	20	36	21	27	39	12	18
22	28	52	20	24	22	28	36	11	58
23	29	50	20	13	23	29	34	11	38
24	0 ♌	47	20	0	24	0 ♍	32	11	18
25	1	44	19	48	25	1	30	10	57
26	2	42	19	35	26	2	28	10	36
27	3	39	19	22	27	3	26	10	15
28	4	37	19	8	28	4	24	9	54
29	5	34	18	54	29	5	22	9	33
30	6	31	18	40	30	6	20	9	12
31	7	29	18	25	31	7	18	8	50

A TABLE,

Shewing the Sun's place and declination.

September.					*October.*				
Days	Sun's Pl.		Sun's De.		Days	Sun's Pl.		Sun's De.	
	D.	M.	D.	M.		D.	M.	D.	M.
1	8 ♍	17	8 N	29	1	7 ♎	34	3 S	0
2	9	15	8	7	2	8	33	3	24
3	10	13	7	45	3	9	33	3	47
4	11	11	7	23	4	10	32	4	10
5	12	9	7	1	5	11	31	4	34
6	13	8	6	38	6	12	30	4	57
7	14	6	6	16	7	13	29	5	20
8	15	4	5	53	8	14	29	5	43
9	16	2	5	31	9	15	28	6	6
10	17	1	5	8	10	16	27	6	29
11	17	59	4	45	11	17	27	6	51
12	18	58	4	22	12	18	26	7	14
13	19	56	3	59	13	19	26	7	37
14	20	55	3	36	14	20	25	7	59
15	21	53	3	13	15	21	25	8	22
16	22	52	2	50	16	22	24	8	44
17	23	50	2	27	17	23	24	9	6
18	24	49	2	4	18	24	23	9	28
19	25	47	1	40	19	25	23	9	50
20	26	46	1	17	20	26	23	10	11
21	27	45	0	54	21	27	23	10	33
22	28	44	0	30	22	28	22	10	54
23	29	42	0	7	23	29	22	11	16
24	0 ♎	41	0 S	16	24	0 ♏	22	11	37
25	1	40	0	40	25	1	22	11	58
26	2	39	1	3	26	2	22	12	18
27	3	38	1	27	27	3	22	12	39
28	4	37	1	50	28	4	22	12	59
29	5	36	2	14	29	5	22	13	14
30	6	35	2	37	30	6	22	13	39
					31	7	22	13	59

A TABLE,

Shewing the Sun's place and declination.

	November.				*December.*				
Days	Sun's Pl.		Sun's De.		Days	Sun's Pl.		Sun's De.	
	D.	M.	D.	M.		D.	M.	D.	M.
1	8 ♏	22	14 S	19	1	8 ♐	39	21 S	46
2	9	22	14	38	2	9	39	21	55
3	10	23	14	57	3	10	40	22	4
4	11	23	15	16	4	11	41	22	13
5	12	23	15	34	5	12	42	22	21
6	13	23	15	53	6	13	43	22	28
7	14	24	16	11	7	14	44	22	35
8	15	24	16	28	8	15	45	22	42
9	16	24	16	46	9	16	46	22	48
10	17	24	17	3	10	17	47	22	54
11	18	25	17	20	11	18	48	23	0
12	19	25	17	36	12	19	49	23	5
13	20	26	17	53	13	20	50	23	9
14	21	26	18	9	14	21	51	23	13
15	22	27	18	24	15	22	52	23	16
16	23	27	18	39	16	23	53	23	19
17	24	28	18	54	17	24	55	23	22
18	25	28	19	9	18	25	56	23	24
19	26	29	19	23	19	26	57	23	26
20	27	30	19	37	20	27	58	23	27
21	28	30	19	51	21	28	59	23	28
22	29	31	20	4	22	0 ♑	0	23	28
23	0 ♐	32	20	17	23	1	2	23	28
24	1	33	20	30	24	2	3	23	27
25	2	33	20	42	25	3	4	23	26
26	3	34	20	53	26	4	5	23	24
27	4	35	21	5	27	5	6	23	22
28	5	36	21	16	28	6	8	23	19
29	6	37	21	26	29	7	9	23	16
30	7	38	21	36	30	8	10	23	13
					31	9	11	23	9

TABLE,
Of the Equation of Time.

Days	*Jan'ry.* M.	S.	*Feb'ry.* M.	S.	*March.* M.	S.	*April.* M.	S.
1	3 +	18	13 +	58	12 +	45	4 +	7
2	4	16	14	5	12	33	3	49
3	4	44	14	12	12	21	3	31
4	5	12	14	19	12	8	3	13
5	5	39	14	24	11	54	2	55
6	6	6	14	28	11	40	2	37
7	6	33	14	32	11	26	2	20
8	6	59	14	34	11	14	2	2
9	7	24	14	37	10	56	1	45
10	7	49	14	38	10	41	1	28
11	8	13	14	39	10	25	1	12
12	8	37	14	38	10	9	0	55
13	9	0	14	37	9	52	0	39
14	9	22	14	35	9	35	0	23
15	9	44	14	33	9	18	0	8
16	10	4	14	29	9	1	0 —	7
17	10	25	14	25	8	43	0	22
18	10	44	14	20	8	25	0	36
19	11	3	14	15	8	7	0	50
20	11	21	14	8	7	49	1	4
21	11	38	14	2	7	31	1	17
22	11	55	13	54	7	12	1	30
23	12	11	13	46	6	54	1	42
24	12	26	13	37	6	35	1	54
25	12	40	13	28	6	17	2	5
26	12	53	13	18	5	53	2	16
27	13	6	13	8	5	39	2	26
28	13	18	12	57	5	21	2	36
29	13	29			5	2	2	45
30	13	39			4	44	2	53
31	13	49			4	24		

TABLE,
Of the Equation of Time.

Days	May.		June.		July.		August.	
	M.	S.	M.	S.	M.	S.	M.	S.
1	3—	2	2—	12	3 +	13	5 +	57
2	3	10	2	34	3	25	5	54
3	3	17	2	24	3	36	5	50
4	3	23	2	14	3	48	5	46
5	3	29	2	4	3	58	5	40
6	3	35	1	54	4	9	5	35
7	3	40	1	43	4	19	5	28
8	3	44	1	32	4	29	5	21
9	3	48	1	21	4	38	5	14
10	3	51	1	10	4	47	5	5
11	3	54	0	58	4	56	4	57
12	3	56	0	46	5	4	4	47
13	3	57	0	34	5	11	4	37
14	3	58	0	22	5	18	4	27
15	3	59	0	9	5	25	4	16
16	3	59	0+	3	5	31	4	4
17	3	58	0	16	5	37	3	52
18	3	57	0	28	5	42	3	39
19	3	55	0	41	5	47	3	26
20	3	53	0	54	5	51	3	13
21	3	50	1	7	5	54	2	59
22	3	46	1	20	5	57	2	45
23	3	42	1	33	6	0	2	30
24	3	38	1	46	6	2	2	14
25	3	33	1	59	6	3	1	59
26	3	27	2	11	6	4	1	43
27	3	21	2	24	6	5	1	26
28	3	14	2	37	6	4	1	9
29	3	7	2	49	6	3	0	52
30	3	59	3	1	6	2	0	34
31	2	51			6	0	0	17

TABLE,
Of the Equation of Time.

Days	*Sept'r.* M.	S.	*Oct'r.* M.	S.	*Nov'r.* M.	S.	*Dec'r.* M.	S.
1	0—	2	10—	10	16—	13	10—	49
2	0	20	10	29	16	14	10	27
3	0	39	10	47	16	14	10	3
4	0	58	11	6	16	14	9	39
5	1	18	11	24	16	13	9	15
6	1	37	11	42	16	11	8	50
7	1	57	11	59	16	8	8	24
8	2	18	12	16	16	4	7	58
9	2	38	12	32	16	0	7	31
10	2	58	12	48	15	54	7	4
11	3	19	13	4	15	48	6	37
12	3	40	13	19	15	41	6	9
13	4	1	13	34	15	33	5	41
14	4	22	13	48	15	25	5	13
15	4	43	14	2	15	15	4	44
16	4	4	14	15	15	5	4	15
17	5	25	14	27	14	53	3	45
18	5	46	14	39	14	41	3	16
19	6	7	14	50	14	28	2	46
20	6	28	15	1	14	14	2	16
21	6	49	15	11	13	59	1	46
22	6	10	15	20	13	44	1	16
23	7	30	15	28	13	27	0	45
24	7	51	15	36	13	10	0	15
25	8	11	15	43	12	52	0+	15
26	8	32	15	50	12	33	0	45
27	8	52	15	55	12	14	1	15
28	9	12	16	0	11	54	1	45
29	9	31	16	5	11	33	2	14
30	9	51	16	8	11	12	2	43
31			16	11			3	13

Explanation of the Table of the Equation of Time.

AS our author has already given a familiar explanation of the equation of time, it may be sufficient to observe, that the preceding table contains the difference between *true* and apparent time, for every day of the year at 12 o'clock noon, when the sun is in the meridian; and is adapted to the second year after leap-year. When apparent, or solar, time is to be converted into true time, as shewn by a well-regulated clock or watch, the equation of time must be added to the apparent time, when it has the sign +, and subtracted from it when it has the sign —: but when true is to be converted into apparent time, the equation must be applied with contrary signs. If the equation be required for any intermediate hour, take the difference during a day, and say, as 24 hours is to this difference, so is the number of hours which the intermediate hour is from the preceding noon, to a third proportional, which, added to, or subtracted from, the equation of time at noon, according as it is increasing or decreasing, will give the equation of time for the given hour. If the equation of time be wanted, at a time when the signs change from + to —, or from — to +, the difference for 24 hours may be found by adding the equations of time for the noon preceding and following the given hour. Thus, if the equation of time be required for the 24th December at 12 o'clock midnight, the equation for the 24th at noon is — 15″, and for the 25th at noon + 15″, the difference of which is + 30″. Then, as 24h : + 30″ = 12h : + 15, which, subtracted from — 15 seconds, the equation for 24th noon, leaves 0; so that the hour, as shewn by the sun and clock, is the same on the 24th December at midnight. The equation thus found will be accurate for every second year after leap-year, and in other years will vary only a few seconds from the truth. In order, however, to determine the equation of time, with accuracy for any other year, find the difference between the equation of time for the given day, and that which precedes it: then,

1. *For leap-year*, take one half of this difference, and add it to the equation for the given time when increasing, but subtract when decreasing.

2. *For the first after leap-year*, take one fourth of the difference, and add it to the equation for the given time when increasing, but subtract it when decreasing.

3. *For the third after leap-year*, take one fourth of the difference, and subtract it from the equation for the given time, when in-

creasing, but add it when decreasing. Thus, to find the equation of time for the 2d May 1805, being the first after leap-year, the equation in the table is 3′ 10″, the daily difference is 8″, and the equation increases. Add, therefore, 2″ which is one fourth of the daily difference to 3′ 10″, and the sum 3′ 12″, will be the true equation of time for the 2d May 1805.—E. Ed.

To find the latitude of any place by observation.

The latitude of any place is equal to the elevation of the pole above the horizon of that place. Therefore, it is plain, that if a star were fixed in the pole, there would be nothing required to find the latitude, but to take the altitude of that star with a good instrument.—But although there is no star in the pole, yet the latitude may be found by taking the greatest and least altitude of any star that never sets; for if half the difference between these altitudes be added to the least altitude, or subtracted from the greatest, the sum or remainder will be equal to the altitude of the pole at the place of observation.

But because the length of the night must be more than 12 hours, in order to have two such observations; the sun's meridian-altitude and declination are generally made use of for finding the latitude. This may be done by the following general rule.—Subtract the meridian or greatest altitude from 90°, and the remainder will be the zenith-distance; which denominate north or south according as the zenith is towards the north or south of the body observed: then, if the zenith-distance and declination be both north or both south, take their sum; but, if one be north and the other south, take their difference; and this sum or difference will be the latitude of the place, and of the same name (north or south) with the greater of the two numbers from which it is found.

EXAMPLES.

1. Suppose ⊙'s true mer.

zen. dist.	47°	40′	N.	add.
Declination	10	15	N.	
Lat. of place	57	55	N.	

2. Suppose ⊙'s true mer.

zen. dist.	24°	50′	N.	sub.
Declination	15	30	S.	
Lat. of place	9	20	N.	

3. Suppose ⊙'s true mer.

zen. dist.	37°	30′	S.	sub.
Declination	20	10′	N.	
Lat. of place	17	20	S.	

4. Suppose ⊙'s true mer.

zen. dist.	0	0	
Declination	17	13	N.
Lat. of place	17	13	N.

5. Suppose ⊙'s true mer.

zen. dist.	18	16	S.
Declination	0	0	
Lat. of place	18	16	S.

If the meridian altitude of the sun or of a circum-polar star, be observed when below the pole, which may always be done when the polar-distance of the body is less than the latitude of the place, then, to this altitude, when

corrected, add 90°, and from the sum subtract the declination, and the remainder will be the latitude of the place, and of the same name with the declination.

Ex....Suppose ✳'s least mer. alt.	38°	50′	} add
	90		
	128	50	
✳'s Declination sub.	88	56	N.
Lat. of place	39	54	N.

Note 1. The apparent altitude of the sun, as observed with a quadrant, &c. will require to be corrected: 1. by subtracting the difference between the refraction and parallax: 2. (when the altitude of the lower limb is observed) by adding the apparent semidiameter, which on an average is about 16′; and 3. (when the altitude is observed at sea above the visible horizon) by subtracting the dip, corresponding to the height of the eye.

Note 2. The above general rule, with the explanatory examples, is substituted in place of seven or eight particular cases, with their appropriate rules, given by the author.—A. Ed.

LECTURE XI.

Of Dialing.

HAVING shewn in the preceding lecture how to make sun-dials by the assistance of a good globe, or of a dialing-scale, we shall now proceed to the method of constructing dials arithmetically; which will be more agreeable to those who have learned the elements of trigonometry, because globes and scales can never be so accurate as the logarithms, in finding the angular distances of the hours. Yet, as a globe may be found exact enough for some other requisites in dialing, we shall take it in occasionally.

The construction of sun-dials on all planes whatever, may be included in one general rule; which will be perfectly intelligible, if that of a horizontal dial for any given latitude be well understood. For there is no plane, however obliquely situated with respect to any given place, but what is parallel to the horizon of some other place; and, therefore, if we can find that other place by a problem on the terrestrial globe, or by a trigonometrical calculation, and construct a horizontal dial for it; that dial, applied to the plane where it is to serve, will be a true dial for that place.——Thus, an erect-direct south dial in $51\frac{1}{2}$ degrees north latitude, would be a horizontal dial on the same

meridian, 90 degrees southward of 51 degrees north latitude; which falls in with $38\frac{1}{2}$ degrees of south latitude; but if the upright plane decline from facing the south at the given place, it would still be a horizontal plane 90 degrees from that place; but for a different longitude: which would alter the reckoning of the hours accordingly.

CASE I.

1. Let us suppose that an upright plane at London declines 36 degrees westward from facing the south; and that it is required to find a place on the globe, to whose horizon the said plane is parallel; and also the difference of longitude between London and that place.

Rectify the globe to the latitude of London, and bring London to the zenith under the brazen meridian, then that point of the globe which lies in the horizon at the given degree of declination, (counted westward from the south point of the horizon) is the place at which the above-mentioned plane would be horizontal. Now, to find the latitude and longitude of that place, keep your eye upon the place, and turn the globe eastward until it comes under the graduated edge of the brazen meridian; then the degree of the brazen meridian that stands directly over the place, is its latitude; and the number of degrees on the equator, which are intercepted between the meridian of London and the brazen meridian, is the place's difference of longitude.

Thus, as the latitude of London is $51\frac{1}{2}$ degrees north, and the declination of the place is 36 degrees west; I elevate the north pole $51\frac{1}{2}$ degrees above the horizon, and turn the

globe until London comes to the zenith, or under the graduated edge of the meridian; then, I count 36 degrees on the horizon westward from the south point, and make a mark on that place of the globe over which the reckoning ends, and bringing the mark under the graduated edge of the brazen meridian, I find it to be under $30\frac{1}{4}$ degrees in south latitude; keeping it there, I count in the equator the number of degrees between the meridian of London and the brazen meridian, (which now becomes the meridian of the required place) and find it to be $42\frac{3}{4}$. Therefore, an upright plane at London, declining 36 degrees westward from the south, would be a horizontal plane at that place whose latitude is $30\frac{1}{4}$ degrees south of the equator, and longitude $42\frac{3}{4}$ degrees west of the meridian of London; which difference of longitude being converted into time, is 2 hours 51 minutes. PLATE XXIII.

The vertical dial declining westward 36 degrees at London, is therefore to be drawn in all respects as a horizontal dial for south latitude $30\frac{1}{4}$ degrees; save only, that the reckoning of the hours is to anticipate the reckoning on the horizontal dial, by 2 hours 51 minutes: for so much sooner will the sun come to the meridian of London, than to the meridian of any place whose longitude is $42\frac{3}{4}$ degrees west from London.

2. But to be more exact than by the globe, Fig. 1
we shall use a little trigonometry.

Let *NE SW* be the horizon of London, whose zenith is *Z*, and *P* the north pole of the sphere; and *Zh* be the position of a vertical plane at *Z*, declining westward from *S* (the south) by an angle of 36 degrees; on which

plane an erect dial for London at *Z* is to be described. Make the semidiameter *ZD* perpendicular to *Zh*, and it will cut the horizon in *D*, 36 degrees west of the south *S*. Then, a plane in the tangent *HD*, touching the sphere in *D*, will be parallel to the plane *Zh*; and the axis of the sphere will be equally inclined to both these planes.

Let *WQE* be the equinoctial, whose elevation above the horizon of *Z* (London) is $38\frac{1}{2}$ degrees; and *PRD* the meridian of the place *D*, cutting the equinoctial in *R*. Then, it is evident, that the arc *RD* is the latitude of the place *D*, (where the plane *Zh* would be horizontal) and the arc *RQ* is the difference of longitude of the planes *Zh* and *DH*.

In the spherical triangle *WDR*, the arc *WD* is given, for it is the complement of the plane's declination from *S*, the south; which complement is 54° (viz. 90°—36°): the angle at *R*, in which the meridian of the place *D* cuts the equator, is a right angle; and the angle *RWD* measures the elevation of the equinoctial above the horizon of *Z*, namely, $38\frac{1}{2}$ degrees. Say, therefore, as radius is to the co-sine of the plane's declination from the south, so is the co-sine of the latitude of *Z* to the sine of *RD* the latitude of *D*: which is of a different denomination from the latitude of *Z*, because *Z* and *D* are on different sides of the equator.

As radius	— — —	10.00000
To co-sine	36° 0′ = *RQ*	9.90796
So co-sine	51° 30′ = *QZ*	9.79415
To sine	34° 14′ = *DR*	(9.70211) =

the latitude of *D*, whose horizon is parallel to the vertical plane *Zh* at *Z*.

N. B. When radius is made the first term, it may be omitted, and then, by subtracting it, mentally, from the sum of the other two, the operation will be shortened. Thus, in the present case,

To the logarithmic sine of *WR* = *	54° 0′	—	9.90796
Add the logarithmic sine of *RD* = †	38° 30′	—	9.79415

Their sum—radius 9.70211
gives the same solution as above. And we shall observe this method in the following part of the work.

To find the difference of longitude of the places *D* and *Z*, say, as radius is to the co-sine of $38\frac{1}{2}$ degrees, the height of the equinoctial at *Z*, so is the co-tangent of 36 degrees, the plane's declination, to the co-tangent of the difference of longitude. Thus,

To the logarithmic sine of‡	51° 30′	9.89364
Add the logarithmic tang. of§	54° 0′	10.13874

Their sum—radius 10.03238
is the nearest tangent of 47° 8′=*WR*; which is the co-tangent of 42° 52′=*RQ*, the difference of longitude sought. Which difference being reduced to time, is 2 hours $51\frac{1}{2}$ minutes.

* The co-sine of 36° 0′, or of *RQ*.
† The co-sine of 51° 30′, or of *QZ*.
‡ The co-sine of 38° 30′, or of *WDR*.
§ The co-tangent of 36°, or of *DW*.

PLATE XXIII. 3. And thus having found the exact latitude and longitude of the place *D*, to whose horizon the vertical plane at *Z* is parallel, we shall proceed to the construction of a horizontal dial for the place *D*, whose latitude is 30° 14′ south; but anticipating the time at *D* by 2 hours 51 minutes, (neglecting the ½ minute in practice) because *D* is so far westward in longitude from the meridian of London; and this will be a true vertical dial at London, declining westward 36 degrees.

Fig. 2. Assume any right line *CSL* for the substile of the dial, and make the angle *KCP* equal to the latitude of the place (viz. 30° 14′) to whose horizon the plane of the dial is parallel; then *CRP* will be the axis of the stile, or edge that casts the shadow on the hours of the day, in the dial. This done, draw the contingent line *EQ*, cutting the substilar line at right angles in *K*; and from *K* make *KR* perpendicular to the axis *CRP*. Then *KG* (=*KR*) being made radius, that is equal to the chord of 60°, or tangent of 45°, on a good sector take 42° 52′, (the difference of longitude of the places *Z* and *D*) from the tangents, and having set it from *K* to *M*, draw *CM* for the hour-line of XII. Take *KN* equal to the tangent of an angle less by 15 degrees than *KM*; that is, the tangent 27° 52′; and through the point *N* draw *CN* for the hour-line of I. The tangent of 12° 52′, (which is 15° less than 27° 52′) set off the same way, will give a point between *K* and *N*, through which the hour-line of II is to be drawn. The tangent of 2° 8′, (the difference between 45° and 42° 52′) placed on the other side of *CL*, will determine the point through which the hour-line of III is to be drawn: to which 2° 8′, if the tangent of

15° be added, it will make 17° 8′; and this set off from *K* toward *Q*, on the line *EQ*, will give the point for the hour-line of IV; and so of the rest. The forenoon-hour-lines are drawn the same way, by the continual addition of the tangents 15°, 30°, 45°, &c. to 42° 52′, (= the tangent of *KM*) for the hours of XI, X, IX, &c. as far as necessary; that is, until there be five hours on each side of the substile. The sixth hour, accounted from that hour or part of the hour on which the substile falls, will be always in a line perpendicular to the substile, and drawn through the centre *C*.

4. In all erect dials, *CM*, the hour-line of XII, is perpendicular to the horizon of the place for which the dial is to serve: for that line is the intersection of a vertical plane with the plane of the meridian of the place, both which are perpendicular to the plane of the horizon: and any line *HO*, or *ho*, perpendicular to *CM*, will be a horizontal line on the plane of the dial, along which line the hours may be numbered: and *CM* being set perpendicular to the horizon, the dial will have its true position.

5. If the plane of the dial had declined by an equal angle toward the east, its description would have differed only in this, that the hour-line of XII would have fallen on the other side of the substile *CL*, and the line *HO* would have had a sub-contrary position to what it has in this figure.

6. And these two dials, with the upper points of their stiles turned toward the north pole, will serve for the other two planes parallel to them; the one declining from the north toward the east, and the other from the north toward the

PLATE XXIII. west, be the same quantity of angle. The like holds true of all dials in general, whatever be their declination and the obliquity of their planes to the horizon.

CASE II.

Fig. 3. 7. When the plane of the dial not only *declines*, but also *reclines*, or *inclines*. Suppose its declination from fronting the south *S* be equal to the arc *SD* on the horizon; and its reclination be equal to the arc *Dd* of the vertical circle *DZ*; then it is plain, that if the quadrant of altitude *ZdD*, on the globe, cut the point *D* in the horizon, and the reclination be counted upon the quadrant from *D* to *d*; the intersection of the hour-circle *PRd*, with the equinoctial *WQE*, will determine *Rd*, the latitude of the place *d*, whose horizon is parallel to the given plane *Zh* at *Z*; and *RQ* will be the difference in longitude of the planes at *d* and *Z*.

Trigonometrically thus: let a great circle pass through the three points *W*, *d*, *E*; and in the triangle *WDd*, right-angled at *D*, the sides *WD* and *Dd* are given; and thence the angle *DWd* is found, and so is the hypothenuse *Wd*. Again, the difference, or the sum of *DWd* and *DWR*, the elevation of the equinoctial above the horizon of *Z*, gives the angle *dWR*; and the hypothenuse of the triangle *WRd* was just now found; whence the sides *Rd* and *WR* are found, the former being the latitude of the place *d*, and the latter the complement of *RQ*, the difference of longitude sought.

Thus, if the latitude of the place *Z* be 52° 10′ north; the declination *SD* of the plane *Zh* (which would be horizontal at *d*) 36°, and the reclination 15°, or equal to the arc *Dd*; the south latitude of the place *d*, that is, the arc *Rd*, will be 15° 9′; and *RQ*, the difference of the longitude, 36° 2′. From these data, therefore, let the dial (Fig. 4.) be described, as in the former example. PLATE XXIII.

8. Only it is to be observed, that in the reclining or inclining dials, the horizontal line will not stand at right angles to the hour-line of XII, as in erect dials; but its position may be found as follows.

To the common substilar line *CKL*, on which the dial for the place *d* was described, draw the dial *Crpm* 12 for the place *D*, whose declination is the same as that of *d*, viz. the arc *SD*; and *HO*, perpendicular to *Cm*, the hour-line of XII on this dial, will be a horizontal line on the dial *CPRM* XII. For the declination of both dials being the same, the horizontal line remains parallel to itself, while the erect position of one dial is reclined or inclined with respect to the position of the other. Fig. 4

Or, the position of the dial may be found by applying it to its plane, so as to mark the true hour of the day by the sun, as shown by another dial; or, by a clock regulated by a true meridian-line and equation-table.

9. There are several other things requisite in the practice of dialing; the chief of which I shall give in the form of arithmetical rules, simple and easy to those who have learned the elements of trigonometry. For, in practical arts of this kind, arithmetic should be used as

PLATE XXIII. far as it can go; and scales never trusted to, except in the final construction, where they are absolutely necessary in laying down the calculated hour-distances on the plane of the dial. And although the inimitable artists of this metropolis have no occasion for such instructions, yet they may be of some use to students, and to private gentlemen, who amuse themselves this way.

RULE I.

To find the angles which the hour-lines on any dial make with the substile.

To the logarithmic sine of the given latitude, or of the stile's elevation above the plane of the dial, add the logarithmic tangent of the hour-distance* from the meridian, or from the substile;† and the sum *minus* radius will be the logarithmic tangent of the angle sought.

For, in Fig. 2. *KC* is to *KM* in the ratio compounded of the ratio of *KC* to *KG* (=*KR*) and of *KG* to *KM*; which, making *CK* the radius, 10,000000, or 10,0000, or 10, or 1, are the ratio of 10,000000, or of 10,0000, or of 10, or of 1, to $KG \times KM$.

Thus, in a horizontal dial, for latitude 51° 30′, to find the angular distance of XI in the forenoon, or I in the afternoon, from XII.

* That is, of 15, 30, 45 60, 75°, for the hours of I. II. III, IV, V, in the afternoon; and XI, X, IX, VIII, VII, in the forenoon.

† In all horizontal dials, and erect north or south dials, the substile and meridian are the same: but in all declining dials, the substile-line makes an angle with the meridian.

To the logarithmic sine of 51° 30′ 9.89354*
Add the logarithmic tang. of 51° 0′ 9.42805

The sum—radius is . . . . . 9.32159= the logarithmic tangent of 11° 50′, or of the angle which the hour-line of XI or I makes with the hour of XII.

And, by computing in this manner, with the sine of the latitude, and the tangents of 30, 45, 60, and 75°, for the hours of II, III, IV, and V, in the afternoon; or of X, IX, VIII, and VII, in the forenoon, you will find their angular distances from XII to be 24° 18′, 38° 3′, 53° 35′, and 71° 6′: which are all that there is occasion to compute for. And these distances may be set off from XII by a line of chords; or rather, by taking 1000 from a scale of equal parts, and setting that extent as a radius from *C* to XII: and then, taking 209 of the same parts, (which, in the tables, is the natural tangent of 11° 50′) and setting this from XII to XI and to I, on the line *ho*, which is perpendicular to *C* XII: and so for the rest of the Fig. 2. hour-lines, which, in the table of natural tangents, against the above distances, are 451, 782, 1355, and 2920, of such equal parts from XII, as the radius *C* XII contains 1000. And lastly, set off 1257, (the natural tangent of 51° 30′) for the angle of the stile's height, which is equal to the latitude of the place.†

The reason why I prefer the use of the tabular numbers, and of a scale decimally divid-

* In which case, the radins *CK* is supposed to be divided into 1000000 equal parts.

† See note 6, lect. X.—A. Ed.

ed, to that of the line of chords, is because there is the least chance of mistake and error in this way; and likewise, because, in some cases, it gives us the advantage of a *nonius* division.*

In the universal ring-dial, for instance, the divisions on the axis are the tangents of the angles of the sun's declination, placed on either side of the centre. But, instead of laying them down from a line of tangents, I would make a scale of equal parts, whereof 1000 should answer exactly to the length of the semi-axis, from the centre to the inside of the equinoctial ring; and then lay down 434 of these parts toward each end from the centre, which would limit all the divisions on the axis, because 434 is the natural tangent of 23° 29′. And thus, by a *nonius* affixed to the sliding-piece, and taking the sun's declination from an ephemeris, and the tangent of that declination from the table of natural tangents, the slider might be always set true to within two minutes of a degree.

And this scale of 434 equal parts might be placed right against the $23\frac{1}{2}$ degrees of the sun's declination, on the axis, instead of the sun's place, which is there of very little use. For then, the slider might be set in the usual way, to the day of the month, for common use; but to the natural tangent of the declination, when great accuracy is required.

* This scale for sub-dividing the limbs of quadrants, and the divisions of other mathematical instruments, is improperly called a *Nonius*, from one Nonius, who is supposed to be its inventor. The honour of the invention is due to Peter Vernier, a French gentleman, from whom it frequently receives its name The simplest and most perspicuous explanation of the nature and construction of the Vernier scale, that I have met with, is in Smith's Optics, vol. ii. p. 338, 339.—E Ed.

The like may be done wherever a scale of sines or tangents is required on any instrument. PLATE XXIII.

RULE II.

The latitude of the place, the sun's declination, and his hour-distance from the meridian, being given, to find (1.) *his altitude;* (2.) *his azimuth.*

1. Let d be the sun's place, dR his declination: and in the triangle PZd, Pd the sum, or the difference, of dR, and the quadrant PR being given by the supposition, as also the complement of the latitude PZ, and the angle dPZ, which measures the horary distance of d from the meridian; we shall (by case 4, of Keill's oblique spheric trigonometry) find the base Zd, which is the sun's distance from the zenith, or the complement of his altitude. Fig. 3.

And 2. As sine Zd : sine Pd : : sine dPZ : sine dZP, or of its supplement DZS, the azimuthal distance from the south.

Or, the practical rule may be as follows:

Write A for the sine of the sun's altitude, L and l for the sine and co-sine of the latitude, D and d for the sine and co-sine of the sun's declination, and H for the sine of the horary distance from VI.

Then the relation of H to A will have three varieties.

1. When the declination is toward the elevated pole, and the hour of the day is between XII and VI; it is $A = LD + HlD$, and

$$H = \frac{A - LD.}{ld}$$

2. When the hour is after VI, it is $A = LD - Hld$, and $H = \dfrac{LD \times A}{ld}$.

3. When the declination is toward the depressed pole, we have $A = Hld - LD$, and $H = \dfrac{A \times LD}{ld}$.

These theorems will be found useful, and sufficiently expeditious for solving those problems in geography and dialing, which depend on the relation of the sun's altitude to the hour of the day.

EXAMPLE I.

Suppose the latitude of the place to be $51\frac{1}{2}$ degrees north; the time five hours distant from XII, that is, an hour after VI in the morning, or before VI in the evening; and the sun's declination 20° north.—*Required the sun's altitude?*

Then, to long. L = long. sine 51° 38′ 1.89354*
add log. D = log. sine 20° 0′ 1.53405

Their sum 1.42759
gives LD = logarithm of 0.267664, in the natural sines.

* Here we consider the radius as unity, and not 10,00000, by which, instead of the index 9, we have—1, as above: which is of no further use, than making the work a little easier.

And, to	log. H = log. sine* 15° 0′	1.41300
add	log. l = log. sine† 38° 0′	1.79414
	log. d = log. sine‡ 70° 0′	1.97300
Their sum	. . . .	1.18014

gives Hld = logarithm of 0.151408, in the natural sines.

And these two numbers (0.267664 and 0.151408) make 0.419072 = $\mathcal{A}$; which, in the table, is the nearest natural sine of 24° 47′, the sun's altitude sought.

The same hour-distance being assumed on the other side of VI, then $LD—Hld$ is 0.116256, the sine of 6° 40½′; which is the sun's altitude at V in the morning, or VII in the evening, when his north declination is 20°.

But when the declination is 20° south, (or toward the depressed pole) the difference $Hld—LD$ becomes negative, and thereby shows that, an hour before VI in the morning, or past VI in the evening, the sun's centre is 6° 40½′ below the horizon.

EXAMPLE II.

In the same latitude and north declination, from the altitude given, to find the hour.

Let the altitude be 48°; and because, in this case $H = \frac{\mathcal{A}—LD,}{ld}$ and $\mathcal{A}$ (the natural sine of 48°) = .743145, and LD = .267664, $\mathcal{A}—LD$

* The distance of one hour from VI.
† The co-latitude of the place.
‡ The co-declination of the sun.

will be 0.475481, whose logarithmic sine is 1.6771331
from which taking the logarithmic sine of $l \times d =$ 1.7671345

Remains 1.9099977
the logarithmic sine of the hour-distance sought, viz. of 54° 22′; which, reduced to time, is 3 hours $37\frac{1}{2}$ min. that is, IX h. $37\frac{1}{2}$ min. in the forenoon, or II h. $22\frac{1}{2}$ min. in the afternoon.

Put the altitude = 18°, whose natural sine is .3090170; and thence $A—LD$ will be = .0491953; which, divided by lDd, gives .0717179, the sine of 4° $6\frac{1}{2}$′, in time, $16\frac{1}{2}$ minutes nearly, before VI in the morning, or after VI in the evening, when the sun's altitude is 18°.

And, if the declination 20° had been toward the south pole, the sun would have been depressed 18° below the horizon at $16\frac{1}{2}$ minutes after VI in the evening; at which time, the twilight would end; which happens about the 22d of November, and 19th of January, in the latitude of $51\frac{1}{2}$° north. The same way may the end of twilight, or beginning of dawn, be found for any time of the year.

NOTE 1.—If in theorem 2 and 3 (page 54) A be put = 4, and the value of H computed, we have the hour of sun's rising and setting for any latitude, and time of the year.—And, if we put $H=0$, and compute A, we have the sun's altitude or depression at the hour of VI. And, lastly, if H, A, and D, be given, the latitude may be found by the resolution of a quadratic equation; for $l = \sqrt{1—L2}$.

NOTE 2.—When A is equal to 0, H is equal $\frac{LD}{ld} = TL \times TD$, the tangent of the latitude multiplied by the tangent of the declination.

As, if it was required, *what is the greatest length of day in latitude* 51° 30′?

To the log. tangent of 51° 30′	0.0993948
Add the log. tangent of 23° 29′	1.6379564
Their sum	1.7373511 is

the log. sine of the hour-distance 33° 7′; in time, 2 h. $12\frac{1}{2}$ m. The longest day, therefore, is 12 h. + 4 h. 25 m. = 16 h. 25 m. And the shortest day is 12 h. — 4 h. 25 m. = 7 h. 35 m.

And if the longest day be given, the latitude of the place is found; $\frac{H}{TD}$ being equal to TL.

Thus, if the longest day be $13\frac{1}{2}$ hours = 2 × 6 h. 45 m. and 45 minutes in time being equal to $11\frac{1}{4}$ degrees:

From the log. sine of 11° 15′	1.2902357
Take the log. tang. of 23° 29′	1.6379562
Remains	1.6522795 =

the logarithmic tangent of lat. 24° 11′.

In the same way, the latitudes where the several geographical *climates* and parallels begin, may be found; and the latitudes of places, that are assigned in authors from the length of their days, may be examined and corrected.

NOTE 3.—The same rule for finding the longest day, in a given latitude, distinguishes the hour-lines that are necessary to be drawn on any dial from those which would be superfluous.

PLATE XXIII. In lat. 52° 10′ the longest day is 16 h. 32 m. and the hour-lines are to be marked from 44 m. after III in the morning, to 16 m. after VIII in the evening.

In the same latitude, let the dial of Art. 7. Fig. 4. be proposed; and the elevation of its stile, (or the latitude of the place *d*, whose horizon is parallel to the plane of the dial) being 15° 9′; the longest day at *d*, that is, the longest time that the sun can illuminate the plane of the dial, will (by the rule $H = TL \times TD$) be twice 6 hours 27 minutes = 12 h. 54 m. The difference of longitude of the planes *d* and *Z* was found in the same example to be 36° 2′; in time, 2 hours 24 minutes; and the declination of the plane was from the south toward the west. Adding, therefore, 2 h. 24 min. to 5 h. 33 m. the earliest sun-rising on a horizontal dial at *d*, the sum 7 h. 57 m. shows that the Fig. 3. morning-hours, or the parallel dial at Z, ought to begin at 3 min. before VIII. And to the latest sun-setting at *d*, which is 6 h. 27 m. adding the same 2 h. 24 m. the sum 8 h. 51 m. exceeding 6 h. 16 m. the latest sun-setting at Z, by 35 m. shows that none of the afternoon-hour-lines are superfluous. And the 4 h. 13 m. from III h. 44 m. the sun-rising at Z to VII h. 57 m. the sun-rising at *d*, belong to the other face of the dial; that is, to a dial declining 36° from north to east, and inclining 15°.

EXAMPLE III.

From the same data *to find the sun's* azimuth.

If *H*, *L*, and *D*, be given, then, (by Art. 2. of Rule II) from *H*, having found the altitude and its complement *Zd*; and the arc *PD* (the

distance from the pole) being given, say, as the co-sine of the altitude is to the sine of the distance from the pole, so is the sine of the hour-distance from the meridian to the sine of the azimuth-distance from the meridian.

Let the latitude be 51° 30′ north, the declination 15° 9′ south, and the time II h. 24 m. in the afternoon, when the sun begins to illuminate a vertical wall, and it is required to find the position the wall.

Then, by the foregoing theorems, the complement of the altitude will be 81° 32½′, and *Pd* the distance from the pole being 109° 5′, and the horary distance from the meridian, or the angle of *dPZ*, 36°.

To log. sine	74° 51′	1.98464
Add log. sine	36° 0′	1.76922
And from the sum		1.75386
Take the log. sine 81° 32½′		1.99525

Remains . . . . 1.75861 = log. sine 35°, the azimuth-distance from the south.

When the altitude is given, find from thence the hour, and proceed as above.

This praxis, in finding the declination of vertical planes more exactly than in the common way, is of singular use on many occasions; especially if the transit of the sun's centre is observed by applying a rule with sights, either plane or telescopical, to the wall or plane whose declination is required. As in drawing a meridian-line, and finding the magnetic variation—in finding the bearings of places in terrestrial surveys; the transits of the sun over any place, or his horizontal distance from it being observed, together with the altitude and hour——and

thence determining small differences of longitude—in observing the variation at sea, &c.

The learned Mr. Andrew Reid invented an instrument several years ago, for finding the latitude at sea, from two altitudes of the sun, observed on the same day, and the interval of the observations, measured by a common watch. This instrument, whose only fault was that of its being somewhat expensive, was made by Mr. Jackson. Tables have been lately computed for the same purpose.

But we may often, from the foregoing rules, resolve the same problem without much trouble; especially if we suppose the master of the ship to know within 2 or 3 degrees what his latitude is—thus,

Assume the two nearest probable limits of the latitude, and by the theorem $H = \frac{A + LD}{l\,d}$, compute the hours of observation for both suppositions. If one interval of those computed hours coincide with the interval observed, the question is solved. If not, the two distances of the intervals computed, from the true interval, will give a proportional part to be added to, or subtracted from, one of the latitudes assumed. And if more exactness be required, the operation may be repeated with the latitude already found.

But whichever way the question is solved, a proper allowance is to be made for the difference of latitude arising from the ship's course in the time between the two observations.

Plate XXIII.

Of the double horizontal dial, and the Babylonian and Italian dials.

To the *gnomonic* projection, there is sometimes added a *stereographic* projection of the hour-circles, and the parallels of the sun's declination, on the same horizontal plane; the upright side of the gnomon being sloped into an edge, standing perpendicularly over the centre of the projection: so that the dial being in its due position, the shadow of *that* perpendicular edge is a vertical circle passing through the sun, in the stereographic projection.

The months being duly marked on the dial, the sun's declination, and the length of the day, at any time, are had by inspection; as also his altitude, by means of a scale of tangents. But its chief property is, that it may be placed true, whenever the sun shines, without the help of any other instrument.

Let *d* be the sun's place in the stereographic projection, *xdyz* the parallel of the sun's declination, *Zd* a vertical circle through the sun's centre, *Pd* the hour-circle; and, it is evident, that the diameter *NS* of this projection being placed duly north and south, these three circles will pass through the point *d*. And, therefore, to give the dial its due position, we have only to turn its gnomon toward the sun, on a horizontal plane, until the hour on the common gnomonic projection coincides with that marked by the hour-circle *Pd*, which passes through the intersection of the shadow *Zd* with the circle of the sun's present declination. Fig. 3.

The Babylonian and Italian dials reckon the hours, not from the meridian, as with us,

PLATE XXIII. but from the sun's rising and setting. Thus, in Italy, one hour before sun-set is reckoned the 23d hour; two hours before sun-set the 22d hour; and so of the rest. And the shadow that marks them on the hour-lines, is *that* of the point of a stile. This occasions a perpetual variation between their dials and clocks, which they must correct from time to time, before it rises to any sensible quantity, by setting their clocks so much faster or slower. And in Italy they begin their day, and regulate their clocks, not from sun-set, but from about mid-twilight, when the *Ave Maria* is said; which corrects the difference that would otherwise be between the clock and the dial.

The improvements which have been made in all sorts of instruments and machines for measuring time, have rendered such dials of little account. Yet, as the theory of them is ingenious, and they are really, in some respects, the best contrived of any for vulgar use, a general idea of their description may not be unacceptable.

Let Fig. 5. represent an erect-direct south wall, on which a *Babylonian dial* is to be drawn, showing the hours from sun-rising; the latitude of the place, whose horizon is parallel to the wall, being equal to the angle *KCR*. Make, as for a common dial, *KG*=*KR*, (which is perpendicular to *CR*) the radius of the equinoctial *ÆQ*, and draw *RS* perpendicular to *CK* for the stile of the dial; the shadow of whose point *R* is to mark the hours, when *SR* is set upright on the plane of the dial,

Then it is evident, that in the contingent line *ÆQ*, the spaces *K*1, *K*2, *K*3, &c. being taken equal to the tangents of the hour-distances from

the meridian, to the radius *KG*, one, two, three, &c. hours after sun-rising, on the equinoctial day, the shadow of the point *R* will be found, at these times, respectively in the points 1, 2, 3, &c.

Draw, for the like hours after sun-rising, when the sun is in the tropic of Capricorn ♑ *v*, the like common lines *CD*, *CE*, *CF*, &c. and at these hours the shadow of the point *R* will be found in those lines respectively. Find the sun's altitudes above the plane of the dial at these hours, and with their co-tangents *Sd*, *Se*, *Sf*, &c. to radius *SR*, describe arcs intersecting the hour-lines in the points *d*, *e*, *f*, &c. so shall the right lines 1 *d*, 2 *e*, 3 *f*, &c. be the lines of I, II, III, &c. hours after sun-rising.

The construction is the same in every other case, due regard being had to the difference of longitude of the place at which the dial would be horizontal, and the place for which it is to serve. And, likewise, taking care to draw no lines but what are necessary; which may be done, partly by the rules already given for determining the time that the sun shines on any plane, and partly from this, that on the tropical days the hyperbola described by the shadow of the point *R* limits the extent of all the hour-lines.

The most useful, however, as well as the simplest, of such dials, is that which is described on the two sides of the meridian-plane.

That the Babylonian and Italic hours are truly enough marked by right lines, is easily shown. Mark the three points on a globe, where the horizon cuts the equinoctial, and the two tropics, toward the east or west: and turn the globe on its axis 15°, or 1 hour; and it is

plain that the three points which were in a great circle, (viz. the horizon) will be in a great circle still; which will be projected geometrically into a straight line. But these three points are universally the sun's places, one hour after sun-set, (or one hour before sun-rise) on the equinoctial and solstitial days. The like is true of all other circles of declination, as well as the tropics; and therefore the hours on such dials are truly marked by straight lines limitted by the projections of the tropics; and which are rightly drawn, as in the foregoing example.

Note 1.—The same dials may be delineated without the hour-lines, *CD*, *CE*, *CF*, &c. by setting off the sun's azimuths on the plane of the dial, from the centre *S*, on either side of the substile *CSK*, and the corresponding co-tangents of altitude from the same centre *S*, for I, II, III, &c. hours before or after the sun is in the horizon of the place for which the dial is to serve, on the equinoctial and solstitial days.

2.—One of these dials has its name from the hours being reckoned from sun-rising, the beginning of the Babylonian day. But we are not thence to imagine that the *equal* hours, which it shows, were those in which the astronomers of that country marked their observations.—These, we know with certainty, were unequal like the Jewish, as being twelfth parts of the natural day: and an hour of the night was, in like manner, a twelfth part of the night; longer or shorter, according to the season of the year. So that an hour of the day, and an hour of the night, at the same place, would always make $\frac{1}{12}$ of 24, or 2 equinoctial hours. In Palestine, among the Romans, and in several other coun-

tries, three of these unequal nocturnal hours were a *vigilia*, or *watch*. And the reduction of equal and unequal hours into one another is extremely easy. If, for instance, it is found, by a foregoing rule, that, in a certain latitude, at a given time of the year, the length of a day is 14 equinoctial hours, the unequal hours is then $\frac{14}{12}$ or $\frac{7}{6}$ of an hour, that is, 70 minutes; and the nocturnal hour is 50 minutes. The first watch begins at VII (sun-set); the second at 3 times 50 minutes after; viz. IX h. 30 m. the third always at midnight; the morning watch at half an hour past II.

If it were required to draw a dial for shewing these unequal hours, or 12th parts of the day, we must take as many declinations of the sun as are thought necessary, from the equator toward each tropic: and having computed the sun's altitude and azimuth for $\frac{1}{12}$, $\frac{2}{12}$, $\frac{3}{12}$th parts, &c. of each of the diurnal arcs belonging to the declinations assumed: by these, the several points in the circles of declination, where the shadow of the stile's point falls, are determined; and curve-lines drawn through the points of a homologous division will be the hour-lines required.*

Of the right placing of dials, and having a true meridian-line for the regulating of clocks and watches.†

The plane on which the dial is to rest, being duly prepared, and every thing necessary

* For a description of a curious Analemmatic dial, which can be properly placed without a mariner's needle, or a meridian-line, and which can be drawn in a garden, the spectator being its stile, See Appendix, vol. ii.—E. Ed.

† In another work, when speaking upon the placing of sun-dials, our author observes, 'that if the dial be made according to

for fixing it, you may find the hour tolerably exact by a large equinoctial ring-dial, and set your watch to it. And then the dial may be fixed by the watch at your leisure.

If you would be more exact, take the sun's altitude by a good quadrant, noting the precise time of observation by a clock or watch. Then, compute the time for the altitude observed, (by the rule, page 73) and set the watch to agree with that time, according to the sun. A Hadley's quadrant is very convenient for this purpose; for, by it you may take the angle between the sun and his image, reflected from a bason of water: the half of which angle, subtracting the refraction, is the altitude required. This is best done in summer, and the nearer the sun is to the prime vertical (the east or west azimuth) when the observation is made, so much the better.

'the strictest rules of calculation, and be truly set the instant 'when the sun's centre is on the meridian, it will be a minute 'too fast in the forenoon, and a minute too slow in the afternoon, 'by the shadow of the stile; for the edge of the shadow that 'shews the time is even with the sun's foremost edge all the 'time before noon, and even with his hindermost all the after-'noon on the dial. But it is the sun's centre that determines the 'time in the supposed hour-circles of the heavens: and as the 'sun is half a degree in breadth, he takes two minutes to move 'through a space equal to his breadth, so that there will be two 'minutes at noon in which the shadow will have no motion at 'all on the dial. Consequently, if the dial be set true by the sun 'in the forenoon, it will be two minutes too slow in the afternoon, 'and if it be set true in the afternoon, it will be two minutes too 'fast in the forenoon. The only way that I know of to remedy 'this is, to set every hour and minute division on the dial one 'minute nearer 12 than the calculation makes it to be.' Tables and Tracts, 2d ed. p. 73.

These observations are new, and just enough in themselves; but the evil which the author points out may be remedied by observing the middle of the shadows penumbra, which corresponds with the sun's centre, instead of the border of the real shadow: and I believe it will be found, that every person naturally does this when he determines the hour of the day upon a sun-dial.—E. Ed.

Or, in summer, take two equal altitudes of the sun on the same day; one any time between VII and X in the morning, the other between II and V in the afternoon, noting the moments of these two observations by a clock or watch: and if the watch shows the observations to be at equal distances from noon, it agrees exactly with the sun; if not, the watch must be corrected by half the difference of the forenoon and afternoon-intervals; and then the dial may be set true by the watch.

Thus, for example, suppose you have taken the sun's altitude when it was 20 minutes past VIII in the morning by the watch, and found, by observing in the afternoon, that the sun had the same altitude 10 minutes before IV, then it is plain, that the watch was 5 minutes too fast for the sun; for 5 minutes after XII is the middle time between VIII h. 20 m. in the morning, and III h. 50 m. in the afternoon; and, therefore, to make the watch agree with the sun, it must be set back five minutes.*

A good *meridian-line*, for regulating clocks or watches, may be had by the following method: *A meridian-line.*

Make a round hole, almost a quarter of an inch diameter, in a thin plate of metal; and

* The above method of finding the hour of the day by corresponding altitudes of the sun or stars, is the easiest and most correct that can be employed. Owing, however, to the change that takes place in the sun's declination before the afternoon-altitude is taken, it is liable to an error, which, at a maximum, amounts to 30 seconds at the time of the equinoxes. A table containing this correction, which depends upon the interval between the altitudes, and upon the declination of the sun, may be seen in the Astronomie de La Lande, and in the Tables de Berlin, tom. i, p 291.

fix the plate in the top of a south window, in such a manner, that it may recline from the zenith at an angle equal to the co-latitude of your place, as nearly as you can guess; for then the plate will face the sun directly at noon on the equinoctial days. Let the sun shine freely through the hole into the room; and hang a plumb-line to the ceiling of the room; at least five or six feet from the window, in such a place as that the sun's rays, transmitted through the hole, may fall upon the line when it is noon by the clock: and having marked the said place on the ceiling, take away the line.

Having adjusted a sliding bar to a dove-tail groove, in a piece of wood about 18 inches long, and fixed a hook in the middle of the bar, nail the wood to the above-mentioned place on the ceiling, parallel to the side of the room in which the window is; the groove and bar being toward the window. Then hang the plumb-line upon the hook of the bar, the weight or plummet reaching almost to the floor; and the whole will be prepared for farther and proper adjustment.

This done, find the true solar time by either of the two last methods, and thereby regulate your clock. Then, at the moment of next noon by the clock, when the sun shines, move the sliding-bar in the groove until the shadow of the plumb-line bisects the image of the sun (made by his rays transmitted through the hole) on the floor, wall, or on a white screen placed on the north-side of the line; the plummet or weight at the end of the line hanging freely in a pail of water placed below it on the floor. But because this may not be quite

correct for the first time, on account that the plummet will not settle immediately, even in water; it may be farther corrected on the following days, by the above method, with the sun and clock, and so brought to a very great exactness.

N. B.—The rays transmitted through the hole will cast but a faint image of the sun, even on a white screen, unless the room be so darkened that no sun-shine may be allowed to enter but what comes through the small hole in the plate. And always, for some time before the observation is made, the plummet ought to be immersed in a jar of water, where it may hang freely; by which means the line will soon become steady, which otherwise would be apt to continue swinging.

As this meridian-line will not only be sufficient for regulating the clocks and watches to the true time by equation-tables, but also for most astronomical purposes, I shall say nothing of the magnificent and expensive meridian-lines at Bologne and Rome, nor of the better methods by which astronomers observe precisely the transits of the heavenly bodies on the meridian.*

* For farther information upon dialing, the reader may consult Orontii Finei opera, Fol. lib. iii. De horologiis Sciothericis à Joanne Voello Turoni 1608. Horologiographia per Sabastianum Monsterum 1533. Christ. Clavii Bambergensis horologiorum nova descriptio.—Demonstratio et constructio horologiorum novorum auctore Georgio Schombergero. Gnomonice Schoner qto. Wolfii oper. Mathemat. tom. ii. 7. 787, our author's Select Exercises, Leybourn's Dialing, Leadbetter's Dialing, and an excellent treatise by the celebrated Deparcieux, published at the end of his *Traite de Trigonometrie rectiligne et spherique.*—E. En

LECTURE XII.

Showing how to calculate the mean time of any new or full Moon, or Eclipse, from the Creation of the World to the year of Christ 5800.

IN the following tables, the mean lunation is about a 20th part of a second of time longer than its measure as now printed in the last edition of my astronomy; which makes the difference of an hour and thirty minutes in 3000 years. But this is not material, when only the mean times are required.

PRECEPTS.

To find the mean time of any new or full Moon in any given year and month after the Christian Æra.

1. If the given year be found in the third column of the *table of the moon's mean motion from the sun*, unter the title, *years before and after Christ;* write out that year, with the mean motions belonging to it, and thereto join the given month with its mean motions. But, if the given year be not in the table, take out the next lesser one to it that you find, in the same column; and thereto add as many *com-*

plete years, as will make up the given year: then, join the month and all the respective mean motions.

2. Collect these mean motions into one sum of signs, degrees, minutes, and seconds; remembering that 60 seconds (″) make a minute, 60 minutes (′) a degree; 30 degrees (°) a sign, and 12 signs (^{s}) a circle. When the signs exceed 12, or 24, or 36 (which are whole circles), reject them, and set down only the remainder; which, together with the odd degrees, minutes, and seconds, already set down, must be reckoned the whole sum of the collection.

3. Subtract the result, or sum of this collection, from 12 signs; and write down the remainder. Then look in the table, under *days*, for the next less mean motions to this remainder, and subtract them from it, writing down their remainder.

This done, look in the table under *hours*, (marked H.) for the next less mean motions to this last remainder, and subtract them from it, writing down their remainder.

Then look in the table under *minutes*, (marked M.) for the next less mean motions to this remainder, and subtract them from it, writing down their remainder.

Lastly, look in the table under *seconds*, (marked S.) for the next less mean motion to this remainder, either greater or less; and against it you have the seconds answering thereto.

4. And these times collected, will give the mean time of the *required new moon*; which will be right in common years; and also in January and February in leap-years; but always one day too late in leap-years after February.

EXAMPLE I.

Required the time of new moon in September, 1764.

[A Year not inserted in the Table.]

		Moon from Sun. s	°	′	″
To the year after Christ's birth	1753	10	9	24	56
Add complete years	11	0	10	14	20
(sum 1764)					
And join September		2	22	21	8
The sum of these mean motions is		1	12	0	24
Which, being subtracted from a circle, or		12	0	0	0
Leaves remaining		10	17	59	36
Next less mean motion for twenty-six days, subtract		10	16	57	34
And there remains			1	2	2
Next less mean motion for two hours, subtract			1	0	57
And the remainder will be				1	5
Next less mean motion for two minutes, subtract				1	1
Remains the mean motion of 12 seconds					4

These times being collected, would show the mean time of the required new moon in September 1764, to be on the 26th day, at 2 hours, 2 minutes, 12 seconds past noon. But, as it is in a leap-year, and after February, the time is one day too late. So the true mean time is September the 25th, at 2 minutes, 12 seconds past II in the afternoon.

N. B.—The tables always begin the day at noon, and reckon thence forward, to the noon of the day following.

To find the mean time of full moon in any given year and month after the Christian Æra.

Having collected the moon's mean motion from the sun for the beginning of the given year and month, and subtracted their sum from twelve signs, (as in the former example) add six signs to the remainder, and then proceed in all respects as above.

EXAMPLE II.

Required the mean time of full moon in September, 1764.

		Moon from Sun s	°	′	″
To the year after Christ's birth	1753	10	9	24	56
Add complete years	11	0	10	14	20
	(sum 1764)				
And join September		2	22	21	8
The sum of these mean motions is		1	12	0	24
Which, being subtracted from a circle, or		12	0	0	0
Leaves remaining		10	17	59	36
To which remainder add		6	0	0	0
And the sum will be		4	17	59	36
Next less mean motion for eleven days, subtract		4	14	5	54
And there remains			3	53	42
Next less mean motion for seven hours, subtract			3	33	20
And the remainder will be				20	22
Next less mean motion for forty minutes, subtract				20	19
Remains the mean motion for eight seconds					3

So, the mean time, according to the tables, is the 11th of September, at 7 hours 40 minutes 8 seconds past noon: One day too late, being after February in a leap-year.

And thus may the mean time of any new or full moon be found, in any year after the Christian æra.

To find the mean time of new or full moon in any given year and month before the Christian Æra.

If the given year before the year of Christ 1 be found in the third column of the table, under the title of *years before and after Christ*, write it out, together with the given month, and join the mean motions. But, if the given year be not in the table, take out the next greater one to it that you find; which being still farther back than the given year, add as many complete years to it as will bring the time forward to the given year; then join the month, and proceed in all respects as above.

EXAMPLE III.

Required the mean time of a new moon in May, the year before Christ 585.

The next greater year in the table is 600; which being 15 years before the given year, add the mean motions for 15 years to those of 600, together with those for the beginning of May.

	MOON FROM SUN. s	°	′	″
To the year before Christ 600	5	11	6	16
Add complete year's motion 15	6	0	55	24
And the mean motion for May	0	22	53	23
The whole sum is	0	4	55	3
Which being subtracted from a circle, or	12	0	0	0
Leaves remaining	11	25	4	57
Next less mean motion for twenty-nine days, subtract	11	23	31	54
And there remains		1	33	3
Next less mean motion for three hours, subtract		1	31	26
And the remainder will be			1	37
Next less mean motion for three minutes, subtract			1	31
Remains the mean motion of fourteen seconds				6

So the mean time, by the tables, was the 29th of May, at 3 hours 3 minutes 14 seconds past noon: a day later than the truth, on account of its being in a leap-year. For, as the year of Christ 1 was the first after a leap-year, the year 585 before the year 1 was a leap-year of course.

If the given year be after the Christian æra, divide its date by 4, and if nothing remain, it is a leap-year in the old style. But if the given year was before the Christian æra, (or year of Christ 1) subtract one from its date, and divide the remainder by 4; then, if nothing remain, it was a leap-year; otherwise not.

To find whether the sun is eclipsed at the time of any given change, or the moon at any given full.

From the *table of the sun's mean motion,* (or distance) *from the moon's ascending node,* collect the mean motions answering to the given time; and if the result show the sun to be within 18 degrees of either of the nodes at the time of new moon, the sun will be eclipsed at that time. Or, if the result show the sun to be within 12 degrees of either of the nodes at the time of full moon, the moon will be eclipsed at that time, in or near the contrary node, otherwise not. Of eclipses.

EXAMPLE IV.

The moon changed on the 26th of September, 1764, at 2 h. 2 m. (neglecting the seconds) afternoon. (See Example I.) Qu. *Whether the sun was eclipsed at that time?*

		SUN FROM NODE. s	°	′	″
To the year after Christ's birth	1753	1	28	0	19
Add complete years	11	7	2	3	56
(sum)	1764				
And September		8	12	22	49
26 days			27	0	13
2 hours				5	12
2 minutes					5
Sun's distance from the ascending node		6	9	32	34

Now, as the descending node is just opposite to the ascending, (viz. six signs distant from it) and the tables show only how far the sun has gone from the ascending node, which, by this example, appears to be 6 signs 9 degrees 32 minutes 34 seconds, it is plain that he must have then been eclipsed; as he was then only 9° 32′ 34″ short of the descending node.

EXAMPLE V.

The moon was full on the 11th of September, 1764, at 7 h. 40 m. past noon. (See Example II.) Qu. *Whether she was eclipsed at that time?*

		SUN FROM NODE. s	°	′	″
To the year after Christ's birth	1753	1	28	0	19
Add complete years	11	7	2	3	56
	(sum 1764)				
And September		8	12	22	49
11 days			11	25	29
7 hours				18	11
40 minutes				1	44
Sun's distance from the ascending node		5	24	12	28

Which being subtracted from 6 signs, leaves only 5° 47′ 32″ remaining; and this being all the space that the sun was short of the descending node, it is plain that the moon must then have been eclipsed, because she was just as near the contrary node.

EXAMPLE VI.

Q. *Whether the sun was eclipsed in May, the year before Christ 585?* (See Example III.)

	SUN FROM NODE.			
	s	°	′	″
To the year before Christ 600	9	9	23	51
Add the mean motion of fifteen complete years	9	19	27	49
And May	4	4	37	57
26 days	1	0	7	10
3 hours			7	48
8 minutes (neglecting the seconds)				8
Sun's distance from the ascending node	0	3	44	43

Which, being less than 18 degrees, shows that the sun was eclipsed at that time.

This eclipse was foretold by Thales, and is thought to be the eclipse which put an end to the war between the Medes and Lydians. Thales's eclipse.

The times of the *sun's conjunction with the nodes,* and consequently the *eclipse-months* of any given year, are easily found by the *tables of the sun's mean motion from the moon's ascending node;* and much in the same way as the mean conjunctions of the sun and moon are found by the table of the moon's mean motion from the sun. For, collect the sun's mean motion from the node, (which is the same as his distance gone from it) for the beginning of any given *year,* and subtract it from 12 signs; then, from the remainder, subtract the less When eclipses must happen.

mean motions belonging to whatever *month* you find them in the table; and from their remainder subtract the next less mean motion for *days*, and so on for *hours* and *minutes;* the result of all which, will show the time of the sun's mean conjunction with the *ascending node* of the moon's orbit.

EXAMPLE VII.

Required the time of the sun's conjunction with the ascending node in the year 1764.

		SUN FROM NODE. s	°	′	″
To the year after Christ's birth	1753	1	28	0	19
Add complete years	11	7	2	3	56
Mean distance at beginning of A. D.	1764	9	0	4	15
Subtract this distance from a circle, or		12	0	0	0
And there remains		2	29	55	45
Next less mean motion for March, subtract		2	1	16	39
And the remainder will be			28	39	6
Next less mean motion for 27 days, subtract			28	2	32
And there remains				36	34
Next less mean motion for 14 hours, subtracted				36	21
Remains, nearly, the mean motion of 5 minutes					13

Hence, it appears, that the sun will pass by the moon's *ascending node* on the 27th of March at 14 hours 5 minutes past noon, viz. on the 28th day, at 5 minutes past II in the morning, according to the tables; but this being in a leap-year, and after February, the time is one day too late. Consequently, the true time is at 5 minutes past II in the morning on the 27th day; at which time the descending node will be directly opposite to the sun.

If six signs be added to the remainder arising from the first subtraction, (viz. from twelve signs) and then the work carried on as in the last example, the result will give the mean time of the sun's conjunction with the descending node. Thus, in

EXAMPLE VIII.

To find when the sun will be in conjunction with the descending node in the year 1764.

		Sun from node. s	°	′	″
To the year after Christ's birth	1753	1	28	0	19
Add complete years	11	7	2	3	56
Mean distance from ascending node at beginning of	1764	9	0	4	15
Subtract this distance from a circle, or		12	0	0	0
And the remainder will be		2	29	55	45
To which add half a circle, or		6	0	0	0
And the sum will be		8	29	55	45
Next less mean motion for September subtracted		8	12	22	49
And there remains			17	32	56
Next less mean motion for 16 days subtracted			16	37	4
And the remainder will be				55	52
Next less mean motion for 21 hours, subtracted				54	32
Remains, nearly, the mean motion of 31 minutes				1	20

So that, according to the tables, the sun will be in conjunction with the *descending node* on the 16th of September, at 21 hours 31 minutes past noon: one day later than the truth, on account of the leap-year.

The limits of eclipses.

When the moon changes within 18 days before or after the sun's conjunction with either of the nodes, the sun will be eclipsed at that change: and when the moon is full within 12 days before or after the time of the sun's conjunction with either of the nodes, she will be eclipsed at the full: otherwise not.

Their period and restitution.

If to the mean time of any eclipse, either of the sun or moon, we add 557 Julian years 21 days 18 hours 11 minutes and 51 seconds, (in which there are exactly 6890 mean lunations) we shall have the mean time of another eclipse.* For, at the end of that time, the moon will be either new or full, according as we add it to the time of new or full moon; and the sun will be only 45″ farther from the same node, at the

* Dr. Halley's period of eclipses contains only 18 years 11 days 7 hours 43 minutes 20 seconds; in which time, according to his tables, there are just 223 mean lunations: but, as in that time, the sun's mean motion from the node is no more than 11s 29° 31′ 49″, which wants 28′ 11″ of being as nearly in conjunction with the same node at the end of the period as it was at the beginning, this period cannot be of constant duration for finding eclipses, because it will in time fall quite without their limits. The following tables make this period 31 seconds shorter, as appears by the following calculation.

The period.		Moon from Sun. s	°	′	″	Sun from node. s	°	′	″
Complete years	18	7	11	59	4	11	17	46	18
——— days	11	4	14	5	54		11	25	29
——— hours	7		3	33	20			18	11
——— minutes	42			21	20			1	49
——— seconds	44				22				2
Mean motions		0	0	0	0	11	29	31	49

end of the said time, than he was at the beginning of it; as appears by the following example.

THE PERIOD.		MOON FROM SUN.				SUN FROM NODE.			
		s	°	'	"	s	°	'	"
Complete years.	500—	3	3	32	47—	10	14	45	8
	40—	8	26	50	37—	1	23	58	49
	17—	3	2	21	39—	10	28	40	15
days	21—	8	16	0	21—		21	48	38
hours	18—		9	8	35—			46	44
minutes	11—			5	35—				29
seconds	51—				26—				2
Mean motions	—	0	0	0	0—	0	0	0	45

And this period is so very near, that in 6000 years it will vary no more from the truth as to the restitution of eclipses, than $8\frac{1}{4}$ minutes of a degree; which may be reckoned next to nothing. It is the shortest in which, after many trials, I can find so near a conjunction of the sun, moon, and the same node.

A Table of Mean Lunations.

This Table is made by the continued addition of a mean lunation, viz. $29^{d}\ 12^{h}\ 44^{m}\ 3^{s}\ 6^{th}\ 21^{iv}\ 14^{v}\ 24^{vi}\ 0^{vii}$.

Lun.		Days.	H.	M.	S.	Th.
1	Contain	29	12	44	3	6
2		59	1	28	6	13
3		88	14	12	9	19
4		118	2	56	12	25
5		147	15	40	15	32
6		177	4	24	18	38
7		206	17	8	21	44
8		236	5	52	24	51
9		265	18	36	27	57
10		295	7	20	31	3
20		590	14	41	2	7
30		885	22	1	33	11
40		1181	5	22	4	14
50		1476	12	42	35	18
100		2953	1	25	10	35
200		5906	2	50	21	11
300		8859	4	15	31	46
400		11812	5	40	42	22
500		14756	7	5	52	57
1000		29530	14	11	45	54
2000		59661	4	23	31	48
3000		88591	18	35	17	42
4000		118122	8	47	3	36
5000		147652	22	58	49	30
10000		295305	21	57	39	0
20000		590611	19	55	18	0
30000		885917	17	52	57	0
40000		1181223	15	50	36	0
50000		1476529	13	48	15	0
100000		2953059	3	36	39	0

In 100000 mean lunations there are 8085 Julian years 12 days 21 hours 36 minutes 30 seconds = 2953059 days 3 hours 36 minutes 30 seconds.

PROOF OF THE TABLE.

	In	Moon from Sun. s	°	'	''
Jul. years	4000	1	14	22	12
	4000	1	14	22	12
	80	5	23	41	15
	5	10	0	18	28
Days	12	4	26	17	20
Hours	21		10	40	1
Min.	36			18	17
Sec.	20				15
M. from S.		0	0	0	0

Having by the former precepts computed the mean time of new moon in January, for any given year, it is easy, by this table, to find the mean time of new moon in January for any number of years afterwards: and by means of a small table of lunations for 12 or 13 months, to make a general table for finding the mean time of new or full moon in any given year and month whatever.

	D.	H.	M.	S.	Th.
In 11 lunations there are	324	20	4	34	10
In 12 lunations	354	8	48	37	16
In 13 lunations	383	21	32	40	23

But then it would be best to begin with March, to avoid the inconvenience of losing a day by mistake in leap-year.

A Table of the Moon's mean motion from the Sun.

Years of the Jul. period.	Years of the World.	Years before and after Christ.	Moon from Sun. s	°	′	″	Complete Years.	Moon from Sun. s	°	′	″
706	0	4008	5	28	1	17	11	0	10	14	20
714	8	4000	5	9	23	24	12	5	2	3	11
1714	1008	3000	11	20	28	57	13	9	11	40	35
2714	2008	2000	6	1	34	30	14	1	21	18	0
3714	3008	1000	0	12	40	3	15	6	0	55	24
3814	3108	900	10	19	46	36	16	10	22	44	15
3914	3208	800	8	26	53	9	17	3	2	21	39
4014	3308	700	7	3	59	43	18	7	11	59	4
4114	3408	600	5	11	6	16	19	11	21	36	27
4214	3508	500	8	18	12	49	20	4	13	25	19
4314	3608	400	1	25	19	23	40	8	26	50	37
4414	3708	300	0	2	25	56	60	1	10	15	56
4514	3808	200	10	9	32	29	80	5	23	41	15
4614	3908	100	8	16	39	3	100	10	7	6	33
4714	4008	1	6	23	45	36	200	8	14	13	7
4814	4108	101	5	0	52	9	300	6	21	19	40
4974	4208	201	3	7	58	45	400	4	28	26	13
5014	4308	301	1	15	5	16	500	3	5	32	47
5114	4408	401	11	22	11	49	1000	6	11	5	33
5214	4508	501	9	29	18	23	2000	0	22	11	6
5714	5008	1001	1	4	51	9	3000	7	3	16	39
6414	5708	1701	0	24	37	2	4000	1	14	22	12
6466	5760	1753	10	9	24	56					
6514	5808	1801	6	5	26	15					

Before the Year of Christ 1.—After Christ.

Complete years.	Moon from sun s	°	′	″	Months	Moon from sun s	°	′	″
					Jan.	0	0	0	0
					Feb.	0	17	54	48
1	4	9	37	24	Mar.	11	29	15	16
2	8	19	14	8	April	0	17	16	3
3	0	28	52	13	May	0	22	53	23
4	5	20	41	4	June	1	10	48	11
5	10	0	18	28	July	1	16	31	32
6	2	9	55	52	Aug.	2	4	26	20
7	6	19	33	17	Sept.	2	22	21	8
8	11	11	22	7	Oct.	2	28	4	29
9	3	20	59	32	Nov.	3	15	59	17
10	8	0	36	55	Dec.	3	21	42	27

Julian years, 3 of which have 365 days, and the 4th 366.

The 4008th year before the year of Christ 1, was the 4007th year before the year of his birth; and is supposed to have been the year of the creation.

This table agrees with the *old style* until the year 1753; and after that with the *new*.

A Table of the Moon's mean motion from the Sun.

Days.	Moon from Sun. s	°	′	″	Moon from Sun. H. M. S.	° ′ ″	′ ″ ‴	″ ‴ ⁗	Moon from Sun. M. S. TH.	′ ″ ‴	″ ‴ ⁗	‴ ⁗ v
1	0	12	11	27								
2	0	24	22	53								
3	1	6	34	20	1	0	30	29	31	15	44	47
4	1	18	45	47	2	1	0	57	32	16	15	16
5	2	0	57	13	3	1	31	26	33	16	45	44
6	2	13	8	40	4	2	1	54	34	17	16	13
7	2	25	20	7	5	2	32	23	35	17	46	42
8	3	7	31	34	6	3	2	52	36	18	17	10
9	3	19	43	0	7	3	33	20	37	18	47	39
10	4	1	54	27	8	4	3	49	38	19	18	7
11	4	14	5	54	9	4	34	18	39	19	48	36
12	4	26	17	20	10	5	4	46	40	20	19	5
13	5	8	28	47	11	5	35	15	41	20	49	33
14	5	20	40	14	12	6	5	43	42	21	20	2
15	6	2	51	40	13	6	36	12	43	21	50	31
16	6	15	3	7	14	7	6	41	44	22	20	59
17	6	27	14	34	15	7	37	9	45	22	51	28
18	7	9	26	0	16	8	7	38	46	23	21	56
19	7	21	37	27	17	8	38	6	47	23	52	25
20	8	3	48	54	18	9	8	35	48	24	22	54
21	8	16	0	21	19	9	39	4	49	24	53	22
22	8	28	11	47	20	10	9	32	50	25	23	51
23	9	10	23	14	21	10	40	1	51	25	54	19
24	9	22	34	41	22	11	10	30	52	26	24	48
25	10	4	46	7	23	11	40	58	53	26	55	17
26	10	16	57	34	24	12	11	27	54	27	25	45
27	10	29	9	1	25	12	41	55	55	27	56	14
28	11	11	20	27	26	13	12	24	56	28	26	43
29	11	23	31	54	27	13	42	53	57	28	57	11
30	0	5	43	21	28	14	13	21	58	29	27	40
31	0	17	54	47	29	14	43	50	59	29	58	8
32	1	0	6	15	30	15	14	18	60	30	28	37

1 Lunation=29d 12h 44m 3s 6th 21iv 14v 24vi 0vii.

In leap-years, after February, a day and its motion must be added to the time for which the moon's mean distance from the sun is given. But when the mean time of any new or full moon is required in leap-year after February, a day must be subtracted from the mean time thereof, as found by the tables. In common years they give the day right.

A Table of the Sun's mean motion from the Moon's ascending node.

Years of the Jul. period.	Years of the World	Years before and after Christ.	Sun from node. s	°	′	″
706	0	4008	7	6	17	9
714	8	4000	0	11	4	55
1714	1008	3000	9	10	35	11
2714	2008	2000	6	10	5	28
3714	3008	1000	3	9	35	44
3814	3108	900	7	24	32	46
3914	3208	800	0	9	29	48
4014	3308	700	4	24	26	49
4114	3408	600	9	9	23	51
4214	3508	500	1	24	20	53
4314	3608	400	6	9	17	54
4414	3708	300	10	24	14	56
4514	3808	200	3	9	11	58
4614	3908	100	7	24	8	59
4714	4008	1	0	9	6	1
4814	4108	101	4	24	3	3
4914	4208	201	9	9	0	4
5014	4308	301	1	23	57	6
5114	4408	401	6	8	54	8
5214	4508	501	10	23	51	9
5714	5708	1001	9	8	36	18
6414	5708	1701	4	23	15	30
6466	5760	1753	1	28	0	19
6514	5808	1801	8	25	44	44

Before the Year of Christ 1.—After Christ.

The 4008th year before the year of Christ 1, was the 4007th year before the year of his birth, and is supposed to have been the year of the creation.

Complete years.	Sun from node s	°	′	″
1	0	19	5	23
2	1	8	10	47
3	1	27	16	10
4	2	17	23	53
5	3	6	29	16
6	3	25	34	40
7	4	14	40	3
8	5	4	47	46
9	5	23	53	9
10	6	12	58	33

Julian years, 3 of which have 365 days, and the 4th 366.

Complete Years.	Sun from node. ″	′	s	°
11	7	2	3	56
12	7	22	11	39
13	8	11	17	2
14	9	0	22	25
15	9	19	27	49
16	10	9	35	31
17	10	28	40	55
18	11	17	46	18
19	0	6	51	43
20	0	26	59	24
40	1	23	58	49
60	2	20	58	13
80	3	17	57	37
100	4	14	37	2
200	8	29	54	3
300	1	14	51	6
400	5	29	48	7
500	10	14	45	8
1000	8	29	30	17
2000	5	29	0	33
3000	2	28	30	50
4000	11	28	1	6

Months	Sun from node s	°	′	″
Jan.	0	0	0	0
Feb.	1	2	11	48
Mar.	2	1	16	39
April	3	3	28	27
May	4	4	37	57
June	5	6	49	45
July	6	7	59	14
Aug.	7	9	11	1
Sept.	8	12	22	49
Oct.	9	13	33	18
Nov.	10	15	44	5
Dec.	11	16	53	34

This table agrees with the *old style* until the year 1753, and after that with the *new*.

A Table of the Sun's mean motion from the Moon's ascending node.

Days.	Sun from Node.				H. M. S.	Sun from Node.			M. S. TH.	Sun from Node.		
	s	°	′	″		° / ′ / ″	′ / ″ / ‴	″ / ‴ / ⁗		′ / ″ / ‴	″ / ‴ / ⁗	‴ / ⁗ / v
1	0	1	2	19	1	0	2	36	31	1	20	31
2	0	2	4	38	2	0	5	12	32	1	23	7
3	0	3	6	57	3	0	7	48	33	1	25	43
4	0	4	9	16	4	0	10	23	34	1	28	9
5	0	5	11	36	5	0	12	59	35	1	31	55
6	0	6	13	54	6	0	15	35	36	1	33	31
7	0	7	16	13	7	0	18	11	37	1	36	6
8	0	8	18	32	8	0	20	47	38	1	38	42
9	0	9	20	51	9	0	23	23	39	1	41	18
10	0	10	23	10	10	0	25	58	40	1	43	54
11	0	11	25	29	11	0	28	33	41	1	46	36
12	0	12	27	48	12	0	31	9	42	1	49	5
13	0	13	30	7	13	0	33	45	43	1	51	41
14	0	14	32	26	14	0	36	21	44	1	54	17
15	0	15	34	15	15	0	38	57	45	1	56	53
16	0	16	37	4	16	0	41	32	46	1	59	29
17	0	17	39	23	17	0	44	8	47	2	2	5
18	0	18	41	41	18	0	46	44	48	2	4	41
19	0	19	44	0	19	0	49	20	49	2	7	17
20	0	20	46	19	20	0	51	56	50	2	9	53
21	0	21	48	38	21	0	54	32	51	2	12	29
22	0	22	50	57	22	0	57	8	52	2	15	5
23	0	23	53	16	23	0	59	43	53	2	17	41
24	0	24	55	35	24	1	2	19	54	2	20	17
25	0	25	57	54	25	1	4	55	55	2	22	53
26	0	27	0	13	26	1	7	31	56	2	25	29
27	0	28	2	32	27	1	10	7	57	2	28	4
28	0	29	4	51	28	1	12	43	58	2	30	40
29	1	0	7	10	29	1	15	9	59	2	33	16
30	1	1	9	29	30	1	17	55	60	2	35	52
31	1	2	11	48								
32	1	3	14	47								

In leap-years, after February, add one day and one day's motion to the time at which the sun's mean distance from the ascending node is required.

A SUPPLEMENT

TO THE

PRECEDING LECTURES.

BY THE AUTHOR.

A SUPPLEMENT

TO THE PRECEDING LECTURES.

MECHANICS.

*The description of a new and safe Crane, which has four different powers, adapted to different weights.**

THE common crane consists only of a large wheel and axle; and the rope, by which goods are drawn up from ships, or let down to them from the quay, winds or coils round the axle, as the axle is turned by men walking in the wheel. But, as these engines have nothing to stop the weight from running down, if any of the men happen to trip or fall in the wheel, the weight descends, and turning the wheel

* Our author received a reward of fifty pounds for the invention of this crane, from the Society for the encouragement of Arts; and a description of it was honoured with a place among the Transactions of the Royal Society of London, see vol. xlv, p. 42.—E. Ed.

rapidly backward, tosses the men violently about within it; which has produced melancholy instances, not only of limbs broken, but even of lives lost, by this ill-judged construction of cranes. And besides, they have but one power for all sorts of weights; so that they generally spend as much time in raising a small weight as in raising a great one.

These imperfections and dangers induced me to think of a method for remedying them. And for that purpose, I contrived a crane with a proper stop to prevent the danger, and with different powers suited to different weights; so that there might be as little loss of time as possible: and also, that when heavy goods are let down into ships, the descent may be regular and deliberate.

This crane has four different powers: and, I believe, might be built in a room eight feet in width: the gib being on the outside of the room.

Three trundles, with different numbers of staves, are applied to the cogs of a horizontal wheel with an upright axle; and the rope that draws up the weight coils round the axle.—The wheel has 96 cogs, the largest trundle 24 staves, the next largest has 12, and the smallest has 6. So that the largest trundle makes 4 revolutions for 1 revolution of the wheel: the next makes 8, and the smallest makes 16. A winch is occasionally put upon the axis of either of these trundles, for turning it; that trundle being then used which gives a power best suited to the weight: and the handle of the winch describes a circle in every revolution equal to twice the circumference of the

axle of the wheel. So that the length of the winch doubles the power gained by each trundle.

As the power gained by any machine or engine whatever, is in direct proportion as the velocity of the power is to the velocity of the weight; the powers of this crane are easily estimated, and are as follows:

If the winch be put upon the axle of the largest trundle, and turned four times round, the wheel and axle will be turned once round: and the circle described by the power that turns the winch, being, in each revolution, double the circumference of the axle, when the thickness of the rope is added thereto; the power goes through eight times as much space as the weight rises through: and, therefore, (making some allowance for friction) a man will raise eight times as much weight by this crane as he would by his natural strength without it: the power, in this case, being as eight to one.

If the winch be put upon the axis of the next trundle, the power will be as 16 to 1, because it moves 16 times as fast as the weight moves.

If the winch be put upon the axis of the smallest trundle, and turned round, the power will be as 32 to 1.

But if the weight should be too great, even for this power to raise, the power may be doubled by drawing up the weight by one of the parts of a double rope, going under a pulley in the moveable block, which is hooked to the weight below the arm of the gib; and then the power will be as 64 to 1. That is, a man

could then raise 64 times as much weight by the crane as he could raise by his natural strength without it; because, for every inch that the weight rises, the working power will move through 64 inches.

By hanging a block with two pulleys to the arm of the gib, and having two pulleys in the moveable block that rises with the weight, the rope being doubled over and under the pullies, the power of the crane will be as 128 to one. And thus, by increasing the number of pullies, the power may be increased as much as you please: always remembering, that the larger the pullies are the less is their friction.

While the weight is drawing up, the ratch-teeth of a wheel slip round below a catch or click that falls successively into them, and so hinders the crane from turning backward, and detains the weight in any part of its ascent, if the man who works at the winch should accidentally happen to quit his hold, or choose to rest himself before the weight be quite drawn up.

In order to let down the weight, a man pulls down one end of a lever of the second kind, which lifts the catch of the ratchet-wheel, and gives the weight liberty to descend. But, if the descent be too quick, he pulls the lever a little farther down, so as to make it rub against the outer edge of a round wheel, by which means he lets down the weights as slowly as he pleases: and, by pulling a little harder, he may stop the weight, if needful, in any part of its descent. If he accidentally quits hold of the lever, the catch immediately falls, and stops both the weight and the whole machine.

This crane is represented in Plate XXIV, where *A* is the great wheel, and *B* its axle on which the rope *C* winds. This rope goes over a pulley *D* in the end of the arm of the gib *E*, and draws up the weight *F*, as the winch *G* is turned round. *H* is the largest trundle, *I* the next, and *K* is the axis of the smallest trundle, which is supposed to be hid from view by the upright supporter *L*. A trundle *M* is turned by the great wheel, and on the axis of this trundle is fixed the ratchet-wheel *N*, into the teeth of which the catch *O* falls. *P* is the lever, from which goes a rope *QQ*, over a pulley *R*, to the catch; one end of the rope being fixed to the lever, and the other end to the catch. *S* is an elastic bar of wood, one end of which is screwed to the floor; and, from the other end, goes a rope, (out of sight in the figure) to the farther end of the lever, beyond the pin or axis on which it turns in the upright supporter *T*. The use of this bar is to keep up the lever from rubbing against the edge of the wheel *U*, and to let the catch keep in the teeth of the ratchet-wheel: but a weight hung to the farther end of the lever would do full as well as the elastic bar and rope.

When the lever is pulled down, it lifts the catch out of the ratchet-wheel, by means of the rope *QQ*, and gives the weight *F* liberty to descend: but if the lever *P* be pulled a little farther down than what is sufficient to lift the catch *O* out of the ratchet-wheel *N*, it will rub against the edge of the wheel *U*, and thereby hinder the too quick descent of the weight; and will quite stop the weight if pulled hard. And if the man who pulls the lever, should happen inadvertently to let it go, the elastic bar will

suddenly pull it up, and the catch will fall down and stop the machine.

WW are two upright rollers above the axis or upper gudgeon of the gib *E*: their use is to let the rope *C* bend upon them, as the gib is turned to either side, in order to bring the weight over the place where it is intended to be let down.

N. B.—The rollers ought to be so placed, that, if the rope *C* be stretched close by their utmost sides, the half thickness of the rope may be perpendicularly over the centre of the upper gudgeon of the gib. For then, and in no other position of the rollers, the length of the rope between the pulley in the gib and the axle of the great wheel will be always the same, in all positions of the gib: and the gib will remain in any position to which it is turned.

When either of the trundles is not turned by the winch in working the crane, it may be drawn off from the wheel, after the pin near the axis of the trundle is drawn out, and the thick piece of wood is raised a little behind the outward supporter of the axis of the trundle. But this is not material; for, as the trundle has no friction on its axis but what is occasioned by its weight, it will be turned by the wheel without any sensible resistance in working the crane.

A pyrometer, that makes the expansion of metals by heat visible to the five-and-forty thousandth part of an inch.

The upper surface of this machine is represented by Fig. 1. of Plate XXV. Its frame *ABCD* is made of mahogany wood, on which

is a circle divided into 360 equal parts; and within that circle is another, divided into eight equal parts. If the short bar *E* be pushed one inch forward, (or toward the centre of the circle) the index *e* will be turned 125 times round the circle of 360 parts or degrees. As 125 times 360 is 45,000, it is evident, that if the bar *E* be moved only the 45,000th part of an inch, the index will move one degree of the circle. But, as in my pyrometer the circle is nine inches in diameter, the motion of the index is visible to half a degree, which answers to the ninety thousandth part of an inch in the motion or pushing of the short bar *E*.

One end of a long bar of metal *F* is laid into a hollow place in a piece of iron *G*, which is fixed to the frame of the machine; and the other end of this bar is laid against the end of the short bar *E*, over the supporting cross bar *HI*: and, as the end *f* of the long bar is placed close against the end of the short bar, it is plain, that when *F* expands, it will push *E* forward, and turn the index *e*.

The machine stands on four short pillars, high enough from a table, to let a spirit-lamp be put on the table under the bar *F*; and when that is done, the heat of the flame of the lamp expands the bar, and turns the index.

There are bars of different metals, as silver, brass, and iron, all of the same length as the bar *F*, for trying experiments on the different expansions of different metals, by equal degrees of heat applied to them for equal lengths of time; which may be measured by a pendulum, that swings seconds—thus,

Put on the brass bar *F*, and set the index to the 360th degree: then put the lighted lamp

under the bar, and count the number of seconds in which the index goes round the plate, from 360 to 360 again; and then blow out the lamp, and take away the bar.

This done, put on an iron bar *F* where the brass one was before, and then set the index to the 360th degree again. Light the lamp, and put it under the iron bar, and let it remain just as many seconds as it did under the brass one; and then blow it out, and you will see how many degrees the index has moved in the circle: and by that means you will know in what proportion the expansion of iron is to the expansion of brass; which, I find to be as 210 is to 360, or as 7 is to 12.—By this method, the relative expansions of different metals may be found.

The bars ought to be exactly of equal size; and to have them so, they should be drawn, like wire, through a hole.

When the lamp is blown out, you will see the index turn backward: which shows that the metal contracts as it cools.

The inside of this pyrometer is constructed as follows:

In Fig. 2. *Aa* is the short bar which moves between rollers; and, on the side *a* it has 15 teeth in an inch, which take into the leaves of a pinion *B*, (12 in number) on whose axis is the wheel *C* of 100 teeth, which take into the 10 leaves of the pinion *D*, on whose axis is the wheel *E* of 100 teeth, which take into the 10 leaves of the pinion *F*, on the top of whose axis is the index above-mentioned.

Now, as the wheels *C* and *E* have 100 teeth each; and the pinions *D* and *F* have 10 leaves each, it is plain, that if the wheel *C* turn once

round, the pinion *F*, and the index on its axis will turn 100 times round. But, as the first pinion *B* has only 12 leaves, and the bar *Aa* that turns it has 15 teeth in an inch, which is 12 and a fourth part more; one inch motion of the bar will cause the last pinion *F* to turn a hundred times round, and a fourth part of a hundred over and above, which is 25. So that if *Aa* be pushed one inch, *F* will be turned 125 times round.

A silk thread *b* is tied to the axis of the pinion *D*, and wound several times round it; and the other end of the thread is tied to a piece of slender watch-spring *G*, which is fixed into the stud *H*. So that as the bar *f* expands, and pushes the bar *Aa* forward, the thread winds round the axle, and draws out the spring: and as the bar contracts, the spring pulls back the thread, and turns the work the contrary way, which pushes back the short bar *Aa* against the long bar *f*. This spring always keeps the teeth of the wheels in contact with the leaves of the pinions, and so prevents any shake in the teeth.

In Fig. 1. the eight divisions of the inner circle are so many thousandth parts of an inch in the expansion or contraction of the bars; which is just one thousandth part of an inch for each division moved over by the index.

A water-mill, invented by Dr. Barker, that has neither wheel nor trundle.

This machine is represented by Fig. 1. of Plate XXVI, in which *A* is a pipe or channel that brings water to the upright tube *B*. The water runs down the tube, and thence into the

horizontal trunk *C*, and runs out through holes at *d* and *e*, near the ends of the trunks on the contrary sides thereof.

The upright spindle *D* is fixed in the bottom of the trunk, and screwed to it below by the nut *g*; and is fixed into the trunk by two cross bars at *f*: so that, if the tube *B* and trunk *C* be turned round, the spindle *D* will be turned also.

The top of the spindle goes square into the rynd of the upper mill-stone *H*, as in common mills; and, as the trunk, tube and spindle, turn round, the mill-stone is turned round thereby. The lower, or quiescent millstone, is represented by *I*; and *K* is the floor on which it rests, and wherein is the hole *L* for letting the meal run through, and fall down into a trough, which may be about *M*. The hoop or case that goes round the mill-stone rests on the floor *K*, and supports the hopper, in the common way. The lower end of the spindle turns in a hole in the bridge-tree *GF*, which supports the mill-stone, tube, spindle, and trunk. This tree is moveable on a pin at *h*, and its other end is supported by an iron rod *N* fixed into it, the top of the rod going through the fixed bracket *O*, and having a screw-nut *o* upon it, above the bracket. By turning this nut forward or backward, the mill-stone is raised or lowered at pleasure.

While the tube *B* is kept full of water from the pipe *A*, and the water continues to run out from the ends of the trunk; the upper mill-stone *H*, together with the trunk, tube, and spindle, turns round. But, if the holes in the trunk were stopped, no motion would ensue; even though the tube and trunk were full of water. For,

If there were no hole in the trunk, the pressure of the water would be equal against all parts of its sides within. But, when the water has free egress through the holes, its pressure there is entirely removed: and the pressure against the parts of the sides which are opposite to the holes, turns the machine.*

* This mill, which is sometimes called *Parent's Mill*, and which has lately been brought forward by a Germam Professor (Segner's *Exercitationes Hydraulicæ*) as an invention of his own, has exercised the ingenuity of Euler and Bernouilli, and its operation seems to be as complicated as its construction is simple. In the Journal de Physique for January and August 1775, an excellent improvement upon it is proposed by M. Mathon de la Cour. Instead of introducing the water at the top of the tube *B*, he bends the pipe *A*, that conducts the water from the reservoir down by the letters *O*, *N*, *G*, and introduces the water at the point *g* at the bottom of the trunk *C*, upon which is fixed the upright spindle *D*, which carries the mill-stone. When the water is thus conveyed into the horizontal arm *C*, it runs out at the holes *d* and *e*, and the motion is begun and continued by the unbalanced pressure on the opposite sides of the arms. In a machine of this kind, erected at Bourg Argental, the length of the horizontal trunk *C* is seven feet seven inches. The diameter of the orifices at *d* and *e* 1 1-6 inches. The height of the reservoir above the trunk *C* is twenty-one feet. The diameter of the pipe which conveyed the water into *C* was two inches at their junction, and was fitted into *C* by grinding. The reader will find some excellent remarks upon this mill in the article Water-works in the Enclycopædia Britannica, vol. xviii, p. 909, by the learned Dr. Robinson; also in the Transactions of the American Philosophical society, vol. iii. p. 185, where there is a dissertation by Mr. Waring, on the power and machinery of Barker's mill, with a description of it as improved by Mr. James Rumsey, and in Desagulier's Experimental philosophy, edit. 3, vol. ii, p. 459. Would not the force of this mill be considerably increased by adding another cylindrical trunk crossing *C* at right angles, without being liable to the objections which may be stated against the hollow conodial ring which Euler proposes to substitute instead of the horizontal arm?—E. Ed.

A machine for demonstrating, that, on equal bottoms, the pressure of fluids is in proportion to their perpendicular heights, without any regard to their quantities.

THIS is termed the *Hydrostatical Paradox:* and the machine for shewing it is represented in Fig. 2. of Plate XXVI. In which *A* is a box that holds about a pound of water, *abcde* a glass tube fixed in the top of the box, having a small wire within it; one end of the wire being hooked to the end *F* of the beam of a balance, and the other end of the wire fixed to a moveable bottom, on which the water lies, within the box; the bottom and wire being of equal weight with an empty scale (out of sight in the figure) hanging at the other end of the balance. If this scale be pulled down, the bottom will be drawn up within the box, and that motion will cause the water to rise in the glass tube.

Put one pound weight into the scale, which will move the bottom a little, and cause the water to appear just in the lower end of the tube at *a;* which shows that the water presses with the force of one pound on the bottom; put another pound into the scale, and the water will rise from *a* to *b* in the tube, just twice as high above the bottom as it was when at *a;* and then, as its pressure on the bottom, supports two pound weight in the scale, it is plain that the pressure on the bottom is then equal to two

pounds. Put a third pound weight in the scale; and the water will be raised from *b* to *c* in the tube, three times as high above the bottom as when it began to appear in the tube at *a*; which shows, that the same quantity of water that pressed but with the force of one pound on the bottom, when raised no higher than *a*, presses with the force of three pounds on the bottom when raised three times as high to *c* in the tube. Put a fourth pound weight into the scale, and it will cause the water to rise in the tube from *c* to *d*, four times as high as when it was all contained in the box, which shows that its pressure then upon the bottom is four times as great as when it lay all within the box. Put a fifth pound weight into the scale, and the water will rise in the tube from *d* to *e*, five times as high as it was above the bottom before it rose in the tube; which shows that its pressure on the bottom is then equal to five pounds, seeing that it supports so much weight in the scale;—and so on, if the tube were still longer: for it would still require an additional pound put into the scale, to raise the water in the tube to an additional height equal to the space *de*; even if the bore of the tube were so small as only to let the wire move freely within it, and leave room for any water to get round the wire.

Hence we infer, that if a long narrow pipe or tube were fixed in the top of a cask full of liquor, and if as much liquor were poured into the tube as would fill it, even though it were so small as not to hold an ounce weight of liquor, the pressure arising from the liquor in the tube would be as great upon the bottom, and the cask will be in as much danger of burst-

ing, as if it were continued up, in its full size, to the height of the tube, and filled with liquor.

In order to account for this surprising effect we must consider that fluids press equally in all directions; and, consequently, that they press just as strongly upward as they do downward. For, if another tube, as *f*, be put into a hole made in the top of the box, and the box be filled with water; and then, if water be poured in at the top of the tube *abcde*, it will rise in the tube *f* to the same height as it does in the other tube: and if you leave off pouring, when the water is at *c*, or any other place in the tube *abcde*, you will find it just as high in the tube *f*: and if you pour in water to fill the first tube, the second will be filled also.

Now, it is evident that the water rises in the tube *f*, from the downward pressure of the water in the tube *abcde*, on the surface of the water, contiguous to the inside of the top of the box; and as it will stand at equal heights in both tubes, the upward pressure in the tube *f* is equal to the downward pressure in the other tube. But if the tube *f* were put in any other part of the top of the box, the rising of the water in it would still be the same: or, if the top were full of holes, and a tube put into each of them, the water would rise as high in each tube as it was poured into the tube *abcde;* and then the moveable bottom would have the weight of the water in all the tubes to bear, beside the weight of all the water in the box.

And seeing that the water is pressed upward into each tube, it is evident that, if they be all taken away, except the tube *abcde*, and the holes in which they stood be stopped up; each

part thus stopped, will be pressed as much upward, as was equal to the weight of water in each tube. So that the inside of the top of the box, on every part equal in breadth to the width of the tube *abcde*, will be pressed upward with a force equal to the whole weight of water in the tube. And, consequently, the whole upward pressure against the top of the box, arising from the weight or downward pressure of the water in the tube, will be equal to the weight of a column of water of the same height with that in the tube, and of the same thickness as the width of the inside of the box: and this upward pressure against the top will re-act downward against the bottom, and be as great thereon, as would be equal to the weight of a column of water as thick as the moveable bottom is broad, and as high as the water stands in the tube. And thus, the paradox is solved.

The moveable bottom has no friction against the inside of the box, nor can any water get between it and the box. The method of making it so, is as follows:

In Fig. 3. *ABCD* represents a section of the box, and *abcd* is the lid or top thereof, which goes on tight, like the lid of a common paper snuff-box. *E* is the moveable bottom, with a groove around its edge, and it is put into a bladder *fg*, which is tied close around it in the groove by a stróng waxed thread; the bladder coming up like a purse within the box, and put over the top of it at *a* and *d* all round, and then the lid pressed on. So that, if water be poured in through the hole *ll* of the lid, it will lie upon the bottom *E*, and be contained in the space *fEgh* within the bladder; and the bottom may

be raised by pulling the wire *i*, which is fixed to it at *E*: and by thus pulling the wire, the water will be thus lifted up in the tube *k*, and as the bottom does not touch against the inside of the box, it moves without friction.

Now, suppose the diameter of this round bottom to be three inches, (in which case, the area thereof will be about seven inches) and the diameter of the bore of the tube to be a quarter of an inch; the whole area of the bottom will be 144 times as great as the area of the top of a pin that would fill the tube like a cork.

And hence it is plain, that if the moveable bottom be raised only the 144th part of an inch, the water will thereby be raised a whole inch in the tube; and, consequently, that if the bottom be raised one inch, it would raise the water to the top of a tube 144 inches, or twelve feet in height.

N. B.—The box must be open below the moveable bottom, to let in the air; otherwise, the pressure of the atmosphere would be so great upon the moveable bottom, if it be three inches in diameter, as to require 108 pounds in the scale, to balance that pressure, before the bottom could begin to move.

A machine, to be substituted in place of the common hydrostatical bellows.

IN Fig. 1. of Plate XXVII, *ABCD* is an oblong square box, in one end of which is a round groove, as at *a*, from top to bottom, for receiving the upright glass tube *I*, which is bent to a right angle at the lower end, (as at *i* in Fig. 2.) and to that part is tied the neck of a large bladder *K*, (Fig. 2.) which lies in the bottom of the box. Over this bladder is laid the moveable board *L*, (Fig. 1. and 3.) in which is fixed an upright wire *M*; and leaden weights *NN*, to the amount of 16 pounds, with holes in their middle, which are put upon the wire, over the board, and press upon it with all their force.

The cross-bar *p* is then put on, to secure the tube from falling, and keep it in an upright position: and then the piece *EFG* is to be put on, the part *G* sliding tight into the dove-tailed groove *H*, to keep the weights *NN* horizontal, and the wire *M* upright; there being a round hole *e* in the part *EF* for receiving the wire.

There are four upright pins in the four corners of the box within, each almost an inch long, for the board *L* to rest upon; to keep it from pressing the sides of the bladder below it close together.

The whole machine being thus put together, pour water into the tube at top; and the water

will run down the tube into the bladder below the board; and, after the bladder has been filled up to the board, continue pouring water into the tube, and the upward pressure which it will excite in the bladder, will raise the board with all the weight upon it, even though the bore of the tube should be so small, that less than an ounce of water would fill it.*

This machine acts upon the same principle, as the one last described, concerning the *hydrostatical paradox.* For, the upward pressure against every part of the board, (which the bladder touches) equal in area to the area of the bore of the tube, will be pressed upward with a force equal to the weight of the water in the tube; and the sum of all these pressures against so many areas of the board, will be sufficient to raise it with all the weights upon it.

In my opinion, nothing can exceed this simple machine, in making the upward pressure of fluids evident to sight.

* Upon this principle, it has been justly affirmed by some writers on natural philosophy, that a certain quantity of water, however small, may be rendered capable of exerting a force equal to any assignable one, by increasing the height of the column, and diminishing the base on which it presses. Dr. Goldsmith observes, that he has seen a strong hogshead split in this manner. A small, though strong tube of tin, twenty feet high, was inserted in the bung-hole of the hogshead. Water was then poured into the tube till the hogshead was filled, and the water had reached within a foot of the top of the tin tube. By the pressure of this column of water, the hogshead bursts with incredible force, and the water was scattered in every direction. By diminishing the area of the tube one half, and doubling its height, the same quantity of water would have a double force.—E. Ed.

*The cause of reciprocating springs, and of ebbing and flowing wells, explained.**

IN Fig. 1. of Plate XXVIII, let *abcd* be a hill, within which is a large cavern *AA* near the top, filled or fed by rains and melted snow on the top *a*, making their way through chinks and crannies into the said cavern, from which proceeds a small stream *CC* within the body of the hill, and issues out in a spring at *G* on the side of the hill, which will run constantly while the cavern is fed with water.

From the same cavern *AA*, let there be a small channel *D*, to carry water into the cavern *B*; and from that cavern let there be a bended channel *EeF*, larger than *D*, joining with the former channel *CC*, as at *f* before it comes to the side of the hill; and let the joining at *f* be below the level of the bottom of both these caverns.

As the water rises in the cavern *B*, it will rise as high in the channel *EeF*: and when it rises to the top of that channel at *e*, it will run down the part *eFG*, and make a swell in the spring *G*, which will continue till all the water be drawn off from the cavern *B*, by the natural syphon *EeF*, (which carries off the water faster from *B* than the channel *D* brings water to it) and then the swell will stop, and

* Dr. Atwell of Oxford seems to have been the first person that pointed out the cause of reciprocating springs. The theory of this gentleman, of which the article in the text is an abridgment, was published in number 424 of the Philosophical Transactions, and was suggested by the phenomena of *Laywell* spring at *Brixam* in Devonshire.—See Desagulier's Experimental philosophy, vol. ii, p. 173, and vol. i, of this work,, p. 134.—E. Ed.

only the small channel *CC* will carry water to the spring *G*, till the cavern *B* is filled again to *B* by the rill *D*; and then the water being at the top *e* of the channel *EeF*, that channel will again act as a syphon, and carry off all the water from *B* to the spring *G*, and so make a swelling flow of water at *G* as before.

To illustrate this by a machine, (Fig. 2.) let *A* be a large wooden box, filled with water; and let a small pipe *CC* (the upper end of which is fixed in the bottom of the box,) carry water from the box to *G*, where it will run off constantly, like a small spring. Let another small pipe *D* carry water from the same box to the box or well *B*, from which let a syphon *EeF* proceed, and join with the pipe *CC* at *f*: the bore of the syphon being larger than the bore of the feeding-pipe *D*. As the water from this pipe rises in the well *B*, it will also rise as high in the syphon *EeF*; and when the syphon is full to the top *e*, the water will run over the bend *e*, down the part *eF*, and go off at the mouth *G*; which will make a great stream at *G*: and that stream will continue, till the syphon has carried off all the water from the well *B*; the syphon carrying off the water faster from *B* than the pipe *D* brings water to it: and then the swell at *G* will cease, and only the water from the small pipe *CC* will run off at *G*, till the pipe *D* again fills the well *B*; and then the syphon will run, and make a swell at *G* as before.

And thus, we have an artificial representation of an ebbing and flowing well, and of a reciprocating spring, in a very natural and simple manner.

An account of the principles by which Mr. Blakey proposes to raise water from mines or from rivers, to supply towns and gentlemen's seats, by his new-invented fire-engine, for which he has received his majesty's patent.

Although I am not at liberty to describe the whole of this simple engine, yet I have the patentee's leave to describe such a one as will shew the principles by which it acts.

In Fig. 4. of Plate XXVII, let *A* be a large, strong, close vessel; immersed in water up to the cock *b*, and having a hole in the bottom, with a valve *a* upon it, opening upward within the vessel. A pipe *BC* rises from the bottom of this vessel, and has a cock *c* in it near the top, which is small there, for playing a very high jet *d*. *E* is the little boiler, (not so big as a common tea-kettle) which is connected with the vessel *A* by the steam-pipe *F*; and *G* is a funnel, through which a little water must be occasionally poured into the boiler, to yield a proper quantity of steam. And a small quantity of water will do for that purpose, because steam possesses upward of 14,000 times as much space or bulk as the water does from which it proceeds.

The vessel *A* being immersed in water up to the cock *b*, open that cock, and the water will rush in through the bottom of the vessel at *a*, and fill it as high up as the water stands on its outside; and the water, coming into the vessel,

will drive the air out of it, (as high as the water rises within it) through the cock *b*. When the water has done rushing into the vessel, shut the cock *b*, and the valve *a* will fall down, and hinder the water from being pushed out that way, by any force that presses on its surface. All the part of the vessel above *b* will be full of common air when the water rises to *b*.

Shut the cock *c*, and open the cocks *d* and *e*; then pour as much water into the boiler *E*, (through the funnel *G*) as will about half fill it; and then shut the cock *d*, and leave the cock *e* open.

This done, make a fire under the boiler *E*, and the heat thereof will raise a steam from the water in the boiler; and the steam will make its way thence, through the pipe *F* into the vessel *A*; and the steam will compress the air (above *b*) with a very great force upon the surface of the water in *A*.

When the top of the vessel *A* feels very hot by the steam under it, open the cock *c* in the pipe *C*; and the air being strongly compressed in *A*, between the steam and the water therein, will drive all the water out of the vessel *A*, up the pipe *BC*, from which it will fly up in a jet to a very great height. In my fountain, which is made in this manner, after Mr. Blakey's, three tea-cup fulls of water in the boiler will afford steam enough to play a jet 30 feet high.

When all the water is out of the vessel *A*, and the compressed air begins to follow the jet, open the cocks *b* and *d* to let the steam out of the boiler *E* and vessel *A*, and shut the cock *e* to prevent any more steam from getting into *A*; and the air will rush into the vessel *A*,

through the cock *b*, and the water through the valve *a*: and so the vessel will be filled up with water to the cock *b*, as before. Then shut the cock *b*, and the cocks *c* and *d*, and open the cock *e*; and then the next steam that rises in the boiler will make its way into the vessel *A* again; and the operation will go on, as above.

When all the water in the boiler is evaporated, and gone off into steam, pour a little more into the boiler, through the funnel *G*.

In order to make this engine raise water to any gentleman's house, if the house be on the bank of a river, the pipe *BC* may be continued up to the intended height, in the direction *HI*. Or, if the house be on the side or top of a hill, at a distance from the river, the pipe, through which the water is forced up, may be laid along on the hill, from the river, or spring, to the house.

The boiler may be fed by a small pipe *K*, from the water that rises in the main pipe *BCHI*; the pipe *K* being of a very small bore, so as to fill the funnel *G* with water in the time that the boiler *E* will require a fresh supply. And then, by turning the cock *d*, the water will fall from the funnel into the boiler. The funnel should hold as much water as will about half-fill the boiler.

When either of these methods of raising water, perpendicularly or obliquely, is used, there will be no occasion for having the cock *c* in the main pipe *BCHI*: for such a cock is requisite only when the engine is used as a fountain.

A contrivance may be very easily made, from a lever to the cocks *b*, *d*, and *e*; so that

by pulling the lever, the cocks *b* and *d* may be opened when the cock *e* must be shut; and the cock *e* be opened when *b* and *d* must be shut.

The boiler *E* should be inclosed in a brick wall, at a little distance from it, all around; to give liberty for the flames of the fire under the boiler to ascend round about it. By which means, (the wall not covering the funnel *G*) the force of the steam will be prodigiously increased by the heat round the boiler; and the funnel and water in it will be heated from the boiler; so that the boiler will not be chilled by letting cold water into it; and the rising of the steam will be so much the quicker.

Mr. Blakey is the only person who ever thought of making use of air as an intermediate body between steam and water: by which means, the steam is always kept from touching the water, and, consequently, from being condensed by it. And, on this new principle, he has obtained a patent: so that no one (vary the engine how he will) can make use of the air between steam and water, without infringing on the patent, and being subject to the penalties of the law.

The engine may be built for a trifling expense, in comparison of the common fire engine now in use. It will seldom need repairs, and will not consume half so much fuel. And, as it has no pumps with pistons, it is clear of all their friction: and the effect is equal to the whole strength or compressive force of the steam: which the effect of the common fire-engine never is, on account of the great frictions of the pistons in their pumps.

*Archimedes's screw-engine for raising water.**

In Fig. 1. of Plate XXIX, *ABCD* is a wheel, which is turned round, according to the order of the letters, by the fall of water *EF*, which need not be more than three feet. The axle *G* of the wheel is elevated so, as to make an angle of about 44 degrees with the horizon; and on the top of that axle is a wheel *H*, which turns such another wheel *I* of the same number of teeth: the axle *K* of this last wheel being parallel to the axle *G* of the two former wheels.

The axle *G* is cut into a double-threaded screw, (as in Fig. 2.) exactly resembling the screw on the axis of the fly of a common jack, which must be (what is called) a right-handed screw, like the wood-screws, if the first wheel turns in the direction *ABCD*; but must be a left-handed screw, if the stream turns that wheel the contrary way. And, whichever way the screw on the axle *G* be cut, the screw on the axle *K* must be cut the contrary way; because these axles turn in contrary directions.

The screws being thus cut, they must be covered close over with boards, like those of a cylindrical cask; and then they will be spiral tubes. Or, they may be made of tubes of stiff leather, and wrapped round the axles in shallow grooves cut therein, as in Fig. 3.

* For a complete account of the theory and operation of this engine, see *Dissertation sur la vis D'Archimède, par Hennert*, Berlin, 1767.—E. Ed.

The lower end of the axle *G* turns constantly in the stream that turns the wheel, and the lower ends of the spiral tubes are open into the water. So that, as the wheel and axle are turned round, the water rises in the spiral tubes, and runs out at *L*, through the holes *MN*, as they come about below the axle.—These holes, (of which there may be any number, as 4 or 6) are in a broad close ring on the top of the axle, into which ring the water is delivered from the upper open ends of the screw-tubes, and falls into the open box *N*.

The lower end of the axle *K* turns on a gudgeon, in the water in *N*; and the spiral tubes in that axle take up the water from *N*, and deliver it into such another box under the top of *K*; on which there may be such another wheel as *I*, to turn a third axle by such a wheel upon it. And, in this manner, water may be raised to any given height, when there is a stream sufficient for that purpose to act on the broad float-boards of the first wheel.

A quadruple pump-mill for raising water.

This engine is represented in Plate XXX, in which *ABCD* is a wheel, turned by water according to the order of the letters. On the horizontal axis are 4 small wheels, toothed almost half round: and the parts of their edges on which there are no teeth, are cut down, so as to be even with the bottoms of the teeth where they stand.

The teeth of these 4 wheels take alternately into the teeth of 4 racks, which hang by 2 chains over the pulleys *Q* and *L*; and to the

lower ends of the racks there are 4 iron rods fixed, which go down into the 4 forcing pumps, *S, R, M,* and *N*. And, as the wheels turn, the rack and pump-rods are alternately moved up and down.

Thus, suppose the wheel *G* has pulled down the rack *I*, and drawn up the rack *K* by the chain; as the last tooth of *G* just leaves the uppermost tooth of *I*, the first tooth of *H* is ready to take into the lowermost tooth of the rack *K*, and pull it down as far as the teeth go; and then the rack *I* is pulled upward through the wholse space of its teeth, and the wheel *G* is ready to take hold of it, and pull it down again, and so draw up the other. In the same manner, the wheels *E* and *F* work the racks *O* and *P*.*

These 4 wheels are fixed on the axle of the great wheel in such a manner with respect to the positions of their teeth; that while they continue turning round, there is never one instant of time in which one or other of the pump-rods is not going down, and forcing the water. So that, in this engine, there is no occasion for having a general air-vessel to all the pumps, to procure a constant stream of water flowing from the upper end of the main pipe.

The pistons of these pumps are solid plungers, the same as described in the fifth lecture of my book, to which this a supplement. See Plate XI, Fig. 4. *of that book, with the description of the figure.*

* For the proper form which must be given to the teeth of the wheels and racks, in order to produce an equable and uniform motion, see Appendix, vol. ii. This method of moving the pistons is preferable to the crank-motion employed in the engine which is represented in Plate XII.—E. Ed.

From each of these pumps, near the lowest end, in the water, there goes off a pipe, with a valve on its farther end from the pump; and these ends of the pipes all enter one close box, into which they deliver the water: and into this box, the lowef end of the main conduit pipe is fixed. So that, as the water is forced or pushed into this box, it is also pushed up the main pipe to the height that it is intended to be raised.

There is an engine of this sort, described in Ramelli's work: but I can truly say, that I never saw it till some time after I had made this model.

The said model is not above twice as big as the figure of it, here described. I turn it by a winch fixed on the gudgeon of the axle behind the water-wheel; and when it was newly made, and the pistons had valves in good order, I put tin pipes, 15 feet high, upon it, when they were joined together, to see what it could do. And I found, that in turning it moderately by the winch, it would raise a hogshead of water in an hour, to the height of 15 feet.

The universal Dialing-Cylinder.

IN Fig. 1. of Plate XXXI, *ABCD* represents a cylindrical glass tube, closed at both ends with brass plates, and having a wire or axis *EFG* fixed in the centres of the brass plates at top and bottom. This tube is fixed to a horizontal board *H*, and its axis makes an angle with the board equal to the angle of the earth's axis with the horizon of any given place, for which the cylinder is to serve as a dial. And it must be set with its axis parallel to the axis of the world in that place; the end *E* pointing to the elevated pole. Or, it may be made to move upon a joint; and then it may be elevated for any particular latitude. Plate XXXI.

There are 24 straight lines, drawn with a diamond, on the outside of the glass, equi-distant from each other, and all of them parallel to the axis. These are the hour-lines: and the hours are set to them as in the figure: the XII next *B* stands for midnight, and the opposite XII, next the board *H*, stands for mid-day or noon.

The axis being elevated to the latitude of the place, and the foot-board set truly level, with the black line along its middle in the plane of the meridian, and the end *N* toward the north; the axis *EFG* will serve as a stile or gnomon, and cast a shadow on the hour of the day, among the parallel hour-lines when the sun shines on the machine. For, as the sun's apparent diurnal motion is equable in the heavens, the shadow of

the axis will move equably in the tube; and will always fall upon *that* hour-line which is opposite to the sun, at any given time.

The brass plate *AD*, at the top, is parallel to the equator, and the axis *EFG* is perpendicular to it. If right lines be drawn from the centre of this plate, to the upper ends of the equi-distant parallel lines on the outside of the tube; these right lines will be the hour-lines on the equinoctial dial *AD*, at 15 degrees distance from each other: and the hour-letters may be set to them, as in the figure. Then, as the shadow of the axis within the tube comes on the hour-lines of that tube, it will cover the like hour-lines on the equinoctial plate *AD*.

If a thin horizontal plate *ef* be put within the tube, so that its edge may touch the tube all around; and right lines be drawn from the centre of the plate to those points of its edge which are cut by the parallel hour-lines on the tube; these right lines will be the hour-lines of a horizontal dial, for the latitude to which the tube is elevated. For, as the shadow of the axis comes successively to the hour-lines of the tube, and covers them, it will then cover the like hour-lines on the horizontal plate *ef*, to which the hours may be set, as in the figure.

If a thin vertical plate *gC* be put within the tube, so as to front the meridian or XII o'clock line thereof, and the edge of this plate touch the tube all around: and then, if right lines be drawn from the centre of the plate to those points of its edge which are cut by the parallel hour-lines on the tube; these right lines will be the hour-lines of a vertical south-dial; and the shadow of the axis will cover them at the same times that it covers those of the tube.

If a thin plate be put within the tube so as to decline, or incline, or recline, by any given number of degrees; and right-lines be drawn from its centre to the hour-lines of the tube; these right lines will be the hour-lines of a declining, inclining, or reclining dial, answering to the like number of degrees, for the latitude to which the tube is elevated. PLATE XXXI.

And thus, by this simple machine, all the principles of dialing are made very plain, and evident to the sight. And the axis of the tube, (which is parallel to the axis of the world in every latitude to which it is elevated) is the stile or gnomon for all the different kinds of sun-dials.

And, lastly, if the axis of the tube be drawn out, with the plates *AD*, *ef*, and *gC* upon it; and set up in the sun-shine, in the same position as they were in the tube; you will have an equinoctial dial *AD*, a horizontal dial *ef*, and a vertical south-dial *gC*; on all which the time of the day will be shown by the shadow of the axis or gnomon *EFG*.

Let us now suppose, that, instead of a glass tube, *ABCD* is a cylinder of wood; on which the 24 parallel hour-lines are drawn all around, at equal distances from each other; and that, from the points at top, where these lines end, right lines are drawn toward the centre, on the flat surface *AD*. These right lines will be the hour-lines on an equinoctial dial, for the latitude of the place to which the cylinder is elevated above the horizontal foot or pedestal *H*; and they are equi-distant from each other, as in Fig. 2. which is a full view of the flat surface or top *AD* of the cylinder, seen obliquely in Fig. 1.

PLATE XXXI. And the axis of the cylinder (which is a straight wire *EFG* all down its middle) is the stile or gnomon; which is perpendicular to the plane of the equinoctial dial, as the earth's axis is perpendicular to the plane of the equator.

To make a horizontal dial, by the cylinder, for any latitude to which its axis is elevated; draw out the axis and cut the cylinder quite through, as at *ehfg*, parallel to the horizontal board *H*, and take off the top part *eADfe;* and the section *ehfge* will be of an elliptical form, as in Fig. 3. Then, from the points of this section, (on the remaining part *eBCf)* where the parallel lines on the outside of the cylinder meet it, draw right lines to the centre of the section; and they will be the true hour-lines for a horizontal dial, as *abcda*, in Fig. 3. which may be included in a circle drawn on that section. Then put the wire into its place again, and it will be a stile for casting a shadow on the time of the day, on that dial. So *E* (Fig. 3.) is the stile of the horizontal dial, parallel to the axis of the cylinder.

To make a vertical south-dial by the cylinder, draw out the axis, and cut the cylinder perpendicularly to the horizontal board *H*, as at *giCkg*, beginning at the hour-line *(BgeA)* of XII, and making the section at right angles to the line *SHN* on the horizontal board. Then, take off the upper part *gADC*, and the face of the section thereon will be elliptical, as shown in Fig. 4. From the points in the edge of this section, where the parallel hour-lines on the round surface of the cylinder meet it, draw right lines to the centre of the section; and they will be the true hour-lines on a vertical

direct south-dial, for the latitude to which the cylinder was elevated; and will appear as in Fig. 4. on which the vertical dial may be made of a circular shape, or of a square shape as represented in the figure. And *F* will be its stile parallel to the axis of the cylinder. PLATE XXXI.

And thus, you may cut the cylinder any way, so as its section may either incline, or decline, or recline, by any given number of degrees; and from those points in the edge of the section; where the outside parallel hour-lines meet it, draw right lines to the centre of the section; and they will be the true hour-lines, for the like declining, reclining, or inclining dial: and the axis of the cylinder will always be the gnomon or stile of the dial. For, whichever way the plane of the dial lies, its stile (or the edge thereof, that casts the shadow on the hours of the day) must be parallel to the earth's axis, and point toward the elevated pole of the heavens.

To delineate a sun-dial on paper, which, when pasted on a cylinder of wood, shall show the time of the day, the sun's place in the ecliptic, and his altitude, at any time of observation.

Draw the right line *aAB*, parallel to the top of the paper; and with any covenient opening of the compasses set one foot in the end of the line at *a*, as a centre, and with the other foot describe the quadrantal arc *AE*, and divide it into 90 equal parts or degrees. Draw the right line *AC*, at right angles to *aAB*, and touching the quadrant *AE* at the point *A*. Then, from the centre *a*, draw right lines through as many PLATE XXXII.

degrees of the quadrant as are equal to the sun's altitude at noon, on the longest day of the year, at the place for which the dial is to serve; which altitude at London is 62 degrees; and continue these right lines till they meet the tangent line *AC*, and from these points of meeting draw straight lines across the paper, parallel to the first right line *AB*, and they will be the parallels of the sun's altitude, in whole degrees, from sun-rise till sun-set, on all the days of the year. These parallels of altitude must be drawn out to the right line *BD*, which must be parallel to *AC*, and as far from it as is equal to the intended circumference of the cylinder on which the paper is to be pasted, when the dial is drawn upon it.

Divide the space between the right lines *AC* and *BD* (at top and bottom) into 12 equal parts, for the 12 signs of the ecliptic; and, from mark to mark of these divisions at top and bottom, draw right lines parallel to *AC* and *BD*; and place the characters of the 12 signs in these 12 spaces, at the bottom, as in the figure; beginning with ♑ or Capricorn, and ending with ♓ or Pisces. The spaces including the signs should be divided by parallel lines into halves; and if the breadth will admit of it without confusion, into quarters also.

At the top of the dial, make a scale of the months and days of the year, so that the days may stand over the sun's place for each of them in the signs of the ecliptic. The sun's place, for every day of the year, may be found by any common ephemeris: and here it will be best to make use of an ephemeris for the second year after leap-year; as the nearest mean for the sun's place on the days of the leap-year, and

on those of the first, second, and third year after.

Compute the sun's altitude for every hour, (in the latitude of your place) when he is in the beginning, middle, and end, of each sign of the ecliptic; his altitude at the end of each sign being the same as at the bginning of the next. And, in the upright parallel lines, at the beginning and middle of each sign, make marks for those computed altitudes among the horizontal parallels of altitude, reckoning them downward, according to the order of the numeral figures set to them at the right hand, answering to the like division of the quadrant at the left. And, through these marks, draw the curve hour-lines, and set the hours to them, as in the figure, reckoning the forenoon-hours downwards, and the afternoon-hours upward. The sun's altitude should also be computed for the half-hours; and the quarter-lines may be drawn, very nearly in their proper places, by estimation and accuracy of the eye. Then, cut off the paper at the left hand, on which the quadrant was drawn, close by the right line *AC*, and all the paper at the right hand close by the right line *BD*; and cut it also close by the top and bottom horizontal lines; and it will be fit for pasting round the cylinder.

This cylinder is represented in miniature by Fig. 1. Plate XXXIII. It should be hollow, to hold the stile *DE* when it is not used. The crooked end of the stile is put into a hole in the top *AD* of the cylinder; and the top goes on tight, but, however, must be made to turn round on the cylinder, like the lid of a paper snuff-box. The stile must stand straight out, perpendicular

to the side of the cylinder, just over the right line *AB* in Plate XXXII, where the parallels of the sun's altitude begin: and the length of the stile, or distance of its point *e* from the cylinder, must be equal to the radius *aA* of the quadrant *AE* in Plate XXXII.

The method of using this dial is as follows:

Place the horizontal foot *BC* of the cylinder on a level table where the sun shines, and turn the top *AD* till the stile stand just over the day of the then present month. Then turn the cylinder about on the table, till the shadow of the stile falls upon it, parallel to those upright lines, which divide the signs, that is, till the shadow be parallel to a supposed axis in the middle of the cylinder: and then, the point or lowest end of the shadow, will fall upon the time of the day, as it is before or after noon, among the curve hour-lines; and will show the sun's altitude at that time, among the cross parallels of his altitude, which go round the cylinder: and, at the same time, it will show in what sign of the ecliptic the sun then is, and you may very nearly guess at the degree of the sign, by estimation of the eye.

The XXXIId Plate, on which this dial is drawn, may be cut out of the book, and passed round a cylinder whose length is 6 inches and 6 tenths of an inch below the moveable top; and its diameter 2 inches and 24 hundred parts of an inch.—Or, I suppose the copper-plate prints of it may be had of the booksellers in London. But it will only do for London, and other places of the same latitude.

When a level table cannot be had, the dial may be hung by the ring *F* at the top. And, when it is not used, the wire that serves for a stile may be drawn out, and put up within the cylinder; and the machine carried in the pocket.

To make three sun-dials upon three different planes, which shall all show the time of the day by one gnomon.

On the flat board *ABC* describe a horizontal dial, according to any of the rules laid down in the lecture on dialing; and to it fix its gnomon *FGH*, the edge of the shadow from the side *FG* being that which shows the time of the day.

To this horizontal or flat board, join the upright board *EDC*, touching the edge *GH* of the gnomon. Then, making the top of the gnomon at *G* the centre of the vertical south-dial, describe a south-dial on the board *EDC*.

Lastly, on a circular plate *IK* describe an equinoctial dial, all the hours of which dial are equi-distant from each other; and making a slit *cd* in that dial, from it edge to its centre, in the XII o'clock line; put the said dial perpendicularly on the gnomon *FG*, as far as the slit will admit; and the triple dial will be finished; the same gnomon serving all the three, and showing the same time of day on each of them.

A universal Dial on a plain cross.

PLATE XXXIV. This dial is represented by Fig. 1. of Plate XXXIV, and is moveable on a joint *C*, for elevating it to any given latitude, on the quadrant *C* 0 90, as it stands upon the horizontal board *A*. The arms of the cross stand at right angles to the middle part; and the top of it from *a* to *n*, is of equal length with either of the arms *ne* or *mk*.

Having set the middle line *tu* to the latitude of your place, on the quadrant, the board *A* level, and the point *N* northward by the needle; the plane of the cross will be parallel to the plane of the equator; and the machine will be rectified.

Then, from III o'clock in the morning, till VI, the upper edge *kl* of the arm *io* will cast a shadow on the time of the day on the side of the arm *cm*: from VI till IX the lower edge *i* of the arm *io* will cast a shadow on the hours on the side *oq*. From IX in the morning till XII at noon, the edge *ab* of the top part *an* will cast a shadow on the hours on the arm *nef*: from XII till III in the afternoon, the edge *cd* of the top part will cast a shadow on the hours on the arm *klm*: from III till VI in the evening the edge *gh* will cast a shadow on the hours on the part *ps*; and from VI till IX, the shadow of the edge *ef* will show the time on the top part *an*.

The breadth of each part *ab*, *ef*, &c. must be so great as never to let the shadow fall quite without the part or arm on which the hours are

marked, when the sun is at his greatest declination from the equator. PLATE XXXIV.

To determine the breadth of the sides of the arms which contain the hours, so as to be in just proportions to their length, make an angle *ABC* (Fig. 2.) of $23\frac{1}{2}$ degrees, which is equal to the sun's greatest declination: and suppose the length of each arm, from the side of the long middle part, and also the length of the top part above the arms, to be equal to *Bd*.

Then, as the edges of the shadow from each of the arms, will be parallel to *Bo*, making an angle of $23\frac{1}{2}$ degrees with the side *Bn* of the arm when the sun's declination is $23\frac{1}{2}$ degrees; it is plain, that if the length of the arm be *Bn*, the least breadth that it can have, to keep the edge *Bo* of the shadow *Bogd* from going off the side of the arm *no* before it comes to the end *on* thereof, must be equal to *on* or *dB*. But in order to keep the shadow within the quarter divisions of the hours, when it comes near the end of the arm, the breadth thereof should be still greater, so as to be almost doubled, on account of the distance between the tips of the arms.

To place the hours right on the arms, take the following method:

Lay down the cross *abcd* (Fig. 3.) on a sheet of paper; and, with a black lead pencil, held close to it, draw its shape and size on the paper. Then, taking the length *ae* in your compasses, and setting one foot in the corner *e*, with the other foot describe the quadrantal arc *ef*. Divide this arc into six equal parts, and through the division-marks draw right lines *ag*, *ah*, &c. continuing three of them to the arm *ce*, which are all that can fall upon it; and they will

PLATE XXXIV. meet the arm in these points through which the lines that divide the hours from each other (as in Fig. 1.) are to be drawn right across it.

Divide each arm, for the three hours it contains in the same manner; and set the hours to their proper places (on the sides of the arms) as they are marked in Fig. 3. Each of the hour-spaces should be divided into four equal parts, for the half-hours and quarters, in the quadrant *ef;* and right lines should be drawn through these division-marks in the quadrant, to the arms of the cross, in order to determine the places thereon where the sub-divisions of the hours must be marked.

This is a very simple kind of universal dial; it is very easily made, and will have a pretty, uncommon appearance in a garden.—I have seen a dial of this sort, but never saw one of the kind that follows.

A universal dial, shewing the hours of the day by a terrestrial globe, and by the shadows of several gnomons at the same time: together with all the places of the earth which are then enlightened by the sun; and those to which the sun is then rising, or on the meridian, or setting.

PLATE XXXV. This dial (See Plate XXXV,) is made of a thick square piece of wood, or hollow metal. The sides are cut into semicircular hollows, in which the hours are placed; the stile of each hollow coming out from the bottom thereof, as far as the ends of the hollows project. The corners are cut out into angles, in the insides of which, the hours are also marked: and the

edge of the end of each side of the angle serves as a stile for casting a shadow on the hours marked on the other side.

In the middle of the uppermost side or plane, there is an equinoctial dial; in the centre whereof an upright wire is fixed, for casting a shadow on the hours of that dial, and supporting a small terrestrial globe on its top.

The whole dial stands on a pillar, in the middle of a round horizontal board, in which there is a compass and magnetic needle, for placing the *meridian*-stile toward the south. The pillar has a joint with a quadrant upon it, divided into 90 degrees, (supposed to be hid from sight under the dial in the figure) for setting it to the latitude of any given place, the same way as already described in the dial on the cross.

The equator of the globe is divided into 24 equal parts, and the hours are laid down upon it at these parts. The time of the day may be known by these hours, when the sun shines upon the globe.

To rectify and use this dial, set it on a level table, or sole of a window, where the sun shines, placing the meridian-stile due south, by means of the needle: which will be, when the needle points as far from the north fleur-de-lis toward the west, as it declines westward, at your place.* Then bend the pillar in the joint, till the black line on the pillar comes to the latitude of your place in the quadrant.

* As the declination of the needle is very uncertain, and varies even at the same place, the dial should be rectified by means of a meridian-line, drawn upon the sole of the window — E. Ed.

The machine being thus rectified, the plane of its dial-part will be parallel to the equator, the wire or axis that supports the globe will be parallel to the earth's axis, and the north pole of the globe will point toward the north pole of the heavens.

The same hour will then be shown in several of the hollows, by the ends of the shadows of their respective stiles. The axis of the globe will cast a shadow on the same hour of the day, in the equinoctial dial, in the centre of which it is placed, from the 20th of March to the 22d of September; and, if the meridian of your place on the globe be set even with the meridian-stile, all the parts of the globe that the sun shines upon, will answer to those places of the real earth which are then enlightened by the sun. The places where the shade is just coming upon the globe, answer to all those places of the earth to which the sun is then setting; as the places where it is going off, and the light coming on, answer to all those places of the earth where the sun is then rising. And, lastly, if the hour of VI be marked on the equator in the meridian of your place, (as it is marked on the meridian of London in the figure) the division of the light and shade on the globe will show the time of the day.

The northern stile of the dial (opposite to the southern or meridian one) is hid from sight in the figure, by the axis of the globe. The hours in the hollow to which that stile belongs, are also supposed to be hid by the oblique view of the figure; but they are the same as the hours in the front hollow. Those also in the right and left hand semicircular hollows are mostly

hid from sight; and so also are all those on the sides next the eye of the four acute angles. Plate XXXVI.

The construction of this dial is as follows. See Plate XXXVI.

On a thick square piece of wood, or metal, draw the lines *ac* and *bd*, as far from each other as you intend for the thickness of the stile *abcd*, and, in the same manner, draw the like thickness of the other three stiles, *efgh*, *iklm*, and *nopq*, all standing outright as from the centre.

With any convenient opening of the compasses, as *aA*, (so as to leave proper strength of stuff when *KI* is equal to *aA*) set one foot in *a*, as a centre, and with the other foot describe the quadrantal arc *Ac*. Then, without altering the compasses, set one foot in *b*, as a centre, and with the other foot describe the quadrant *dB*. All the other quadrants in the figure must be described in the same manner, and with the same opening of the compasses, on their centres *ef*; *ik*; and *no*: and each quadrant divided into six equal parts, for so many hours, as in the figure; each of which parts must be subdivided into four, for the half-hours and quarters.

At equal distances from each corner, draw the right lines *Ip* and *Kp*, *Lq* and *Mq*, *Nr* and *Or*, *Ps* and *Qs*; to form the four angular hollows *IpK*, *LqM*, *FrO*, and *PsQ*: making the distances between the tips of the hollows, as *IK*, *LM*, *NO*, and *PQ*, each equal to the radius of the quadrants; and leaving sufficient room between the angular points, *p*, *q*, *r*, and *s*, for the equinoctial circle in the middle.

To divide the insides of these angles properly for the hour-spaces thereon, take the fol-

lowing method:—Set one foot of the compasses in the point *I*, as a centre; and open the other to *K*, and with that opening describe the arc *Kt*: then, without altering the compasses, set one foot in *K*, and with the other foot describe the arc *It*. Divide each of these arcs, from *I* and *K* to their intersection at *t*, into four equal parts; and from their centres *I* and *K*, through the points of division, draw the right lines *I* 3, *I* 4, *I* 5, *I* 6, *I* 7; and *K* 2, *K* 1, *K* 12, *K* 11; and they will meet the sides *Kp* and *Ip* of the angle *IpK*, where the hours thereon must be placed. And these hour-spaces in the arcs must be subdivided into four equal parts, for the half-hours and quarters. Do the like for the other three angles, and draw the dotted lines; also set the hours in the insides, where those lines meet them, as in the figure; and the like hour-lines will be parallel to each other in all the quadrants and in the angles.

Mark points for all these hours, on the upper side, and cut out all the angular hollows, and the quadrantal ones, quite through the places where their four gnomons must stand; and lay down the hours on their insides, as in Plate XXXVI, and then set in their four gnomons, which must be as broad as the dial is thick; and this breadth and thickness must be large enough to keep the shadows of the gnomons from ever falling quite out at the sides of the hollows, even when the sun's declination is at the greatest.

Lastly, draw the equinoctial dial in the middle, all the hours of which are equidistant from each other, and the dial will be finished:

As the sun goes round, the broad end of the shadow of the stile *abcd* will show the hours in the quadrant *Ac*, from sun-rise till VI in the morning; the shadow from the end *M* will show the hours on the side *Lq*, from V to IX in the morning; the shadow of the stile *efgh* in the quadrant *Dg* (in the long days) will show the hours from sun-rise till VI in the morning; and the shadow of the end *N* will show the morning hours, on the side *Or*, from III to VII.

Just as the shadow of the northern stile *abcd* goes off the quadrant *Ac*, the shadow of the southern stile *iklm* begins to fall within the quadrant *Fl*, at VI in the morning; and shows the time, in that quadrant, from VI till XII at noon; and from noon to VI in the evening in the quadrant *mE*. And the shadow of the end *O* shows the time from XI in the forenoon till III in the afternoon, on the side *rN*: as the shadow of the end *P* shows the time from IX in the morning till I o'clock in the afternoon, on the sides *Qs*.

At noon, when the shadow of the eastern stile *efgh* goes off the quadrant *hC* (in which it showed the time from VI in the morning till noon) as it did in the quadrant *gD* from sun-rise till VI in the morning, the shadow of the western stile *nopq* begins to enter the quadrant *Hp*; and shows the hours thereon from XII at noon till VI in the evening; and after *that* till sun-set, in the quadrant *qG*; and the end *Q* casts a shadow on the side *Ps* from V in the evening till IX at night, if the sun be not set before that time.

The shadow of the end *I* shows the time on the side *Kp* from III till VII in the afternoon; and the shadow of the stile *abcd* shows the time from VI in the evening till the sun sets.

The shadow of the upright central wire, that supports the globe at top, shows the time of the day, in the middle or equinoctial dial, all the summer half year, when the sun is on the north side of the equator.

In the Supplement to my Book of Lectures, all the machines that I have added to my apparatus, since that book was printed, are described, excepting two; one of which is a model of a mill for sawing timber, and the other is a model of the great engine at London-bridge, for raising water. And my reasons for leaving them out are the following.

First, I found it impossible to make such a drawing of the saw-mill as could be understood; because, in whatever view it be taken, a great many parts of it hide others from sight. And, in order to shew it in my lectures, I am obliged to turn it into all manner of positions.*

* For the plan and elevation of a saw-mill, see Gray's Experienced Mill-wright, lately published, p. 68. For the method of constructing one, see Wolfii Opera Mathematica, tom. i. p, 694, and Boecklerus's Theatrum Machinarum. An excellent saw-mill was invented by Mr. James Stanfield, in the year 1765, for which he received a reward of one hundred pounds from the Society for the encouragement of arts. The original mill which Mr. Stanfield constructed, was worked for five successive years, in consequence of succesive premiums offered and paid by the society, amounting, in all, to two hundred and twenty pounds. A description of this machine, illustrated by five folio plates, will be found in *Bailey's Designs of Machines, &c. approved and adopted by the society for the encouragement of arts*, vol. 1. p. 137.—E. Ed.

Secondly, because any person who looks on Fig. 1. of Plate XII, in the book, and reads the account of it in the Vth lecture therein, will be able to form a very good idea of the London-bridge engine, which has only two wheels and two trundles more than there are in Mr. Aldersea's engine, from which the said figure was taken.

APPENDIX

TO

FERGUSON'S

LECTURES ON MECHANICS, &c.

MECHANICS.

On the construction of undershot water-wheels for turning machinery.

ALTHOUGH no country has been more distinguished than this, by its discoveries and improvements in the mathematical sciences, yet nowhere have these improvements less effectually contributed to the advancement of the mechanical arts. The discoveries of our philosophers, particularly in the construction of machinery, have been locked up in the recesses of algebraical formulæ; and one would have imagined, that they deemed it beneath their dignity to level their speculations to the capacity of common artists. On this account, the mill-wrights of this country are still guided by their own prejudices; and, if they are furnished with some useful hints and maxims by the few practical treatises which are to be met

PLATE XXXVII. with, they are left in the dark, to be directed by their own judgment, in the most important parts of the construction. In the preceding lectures, Mr. Ferguson has given some useful directions for the construction of corn-mills; but as these are too limited to be of extensive utility, we shall endeavour to supply this defect, by treating at considerable length upon this important subject. Let us begin, then, by showing the method of constructing the mill-course, and delivering the water on the wheel.

On the construction of the mill-course.

As it is of the highest importance to have the height of the fall as great as possible, the bottom of the canal, or dam, which conducts the water from the river, should have a very small declivity; for the height of the water-fall will diminish in proportion as the declivity of the canal is increased. On this account, it will be sufficient to make *AB* (Fig. 1.) slope about one inch in 200 yards, taking care to make the declivity about half an inch for the first 48 yards, in order that the water may have a velocity sufficient to prevent it from flowing back into the river. The inclination of the fall, represented by the angle *GCR*, should be 25° 50′; or *CR*, the radius, should be to *GR* the tangent of this angle, as 100 to 48, or as 25 to 12; and since the surface of the water *Sb* is bent from *ab*, into *ac*, before it is precipitated down the fall, it will be necessary to incurvate the upper part *BCD* of the course into *BD*, that the water at the bottom may move parallel to the water at the top of the stream. For this purpose, take the points *B, D*, about 12 inches

distant from *C*, and raise the perpendiculars *BE*, *DE*: the point of intersection *E* will be the centre from which the arch *BD* is to be described; the radius being about 10⅛ inches. Now, in order that the water may act more advantageously upon the floatboards of the wheel *WW*, it must assume a horizontal direction *HK*, with the same velocity which it would have acquired when it came to the point *G*; but, in passing from *C* to *G*, the water will dash upon the horizontal part *HG*, and thus lose a great part of its velocity, it will be proper, therefore, to make it move along *FH* an arch of a circle to which *DF* and *KH* are tangents in the points *F* and *H*. For this purpose make *GF* and *GH* each equal to 3 feet, and raise the perpendiculars *HI*, *FI*, which will intersect one another in the point *I*, distant about 4 feet 9 inches and 4 tenths from the points *F*, and *H*, and the centre of the arch *FH* will be determined. The distance *HK*, through which the water runs before it acts upon the wheel, should not be less than 2 or 3 feet, in order that the different portions of the fluid may have obtained a horizontal direction: and if *HK* be much larger, the velocity of the stream would be diminished by its friction on the bottom of the course. That no water may escape between the bottom of the course *KH*, and the extremities of the floatboards, *KL* should be about 3 inches, and the extremity *o* of the floatboard *no* should be beneath the line *HKX*, sufficient room being left between *o* and *M* for the play of the wheel; or *KLM* may be formed into the arch of a circle *KM*, concentric with the wheel. The line *LMV*, called by *M. Fabre*, the *course of impulsion*, (*le*

coursier d'impulsion) should be prolonged, so as to support the water as long as it can act upon the floatboards, and should be about 9 inches distant from *OP*, a horizontal line passing through *O*, the lowest point of the fall; for if *OL* went much less than 9 inches, the water, having spent the greater part of its force in impelling the floatboards, would accumulate below the wheel and retard its motion. For the same reason, another *course*, which is called by *M. Fabre*, the *course of discharge, (le coursier de decharge)* should be connected with *LMV*, by the curve *VN*, to preserve the remaining velocity of the water, which would otherwise be destroyed by falling perpendicularly from *V* to *N*. The course of discharge is represented by *VZ*, sloping from the point *O*. It should be about 16 yards long, having an inch of declivity in every two yards. The canal which reconducts the water from the course of discharge to the river, should slope about 4 inches in the first 200 yards, 3 inches in the second 200 yards, decreasing gradually till it terminates in the river. But if the river to which the water is conveyed, should, when swoln by the rains, force the water back upon the wheel, the canal must have a greater declivity, in order to prevent this from taking place. Hence, it will be evident, that very accurate levelling is necessary for the proper formation of the mill-course.

In order to find the breadth of the course of discharge, multiply the quantity of water expended in a second,* measured in cubic feet,

* The quantity of water expended in a second may be found pretty accurately by measuring the depth of the water at *a*, (*AB*, the bottom of the canal, being nearly horizontal, and its sides

by 684, for a first number. Multiply the square root of dK, (dK being found by subtracting OK, or PR, each equal to a foot, from dO, or bR, the height of the fall) by OL, or $\frac{3}{4}$ of a foot, and also by 1000, and the product will be a second number. Divide the first number by the second, and the quotient will be nearly the last breadth of the course of discharge. If the breadth of the course, thus found, should be too great or too small, the point L has been placed too far from O, or too near it. Increase, therefore, or diminish OL; and having subtracted from dO, or bP, the quantity by which OK is greater or less than a foot, repeat the operation with this new value of dK, and a more convenient answer will be found. The preceding rule will give too great a breadth to the course, when the expense of water is great, and the height of the fall inconsiderable. But the course of discharge should always have a very considerable breadth, and greater than that of the course of impulsion, that the water, having room to spread, may have less depth; and thus procure a greater height to the fall, by making OL, and consequently OK, as small as possible; for the breadth of the course is inversely as OL, that is, it increases as OL diminishes, and diminishes as it increases. The reader may suppose that this rule still leaves us to

perpendicular,) and the breadth of the canal at the same place. Take the cube of the depth of the water in feet, and extract the square root of it. Multiply this root by the breadth of the canal, and also by 488. Divide the product by 100, and the quotient will be the expense of water in a second, measured in cubic feet.

PLATE XXXVII. guess at the breadth of the course of discharge; but, from the purposes for which it is used, it is easy to know when it is excessively large or small; and it is only when this is the case, that we have any occasion to seek for another breadth, by taking a new value of *OL*.

The section of the fluid at *K* should be rectangular, the breadth of the stream having a determinate relation to its depth. If there be very much water, the breadth should be triple the depth; if there be a moderate quantity, the breadth should be double the depth; and, if there be very little water, the breadth and depth should be equal. That this relation may be preserved, the course at the point *K* must have a certain breadth, which may be thus found:—Divide the square-root of *dK*, (found as before) by the quantity of water expended in a second, and extract the square-root of the quotient. Multiply this root by .640, when the breadth is to be triple the depth; by .523, when it is to be double; and by .37, when equal, and the product will be the breadth of the course at *K*. The depth of the water at *K* is therefore known, being either one third or one half of the breadth of the course, or equal to it, according to the quantity of the water furnished by the stream.

In Fig. 1. *bP* is called the *absolute fall*, which is found by levelling. Draw the horizontal lines *bd*, *PO*; *dO* will thus be equal to *bP*, and will likewise be the absolute fall.—The *relative fall* is the distance of the point *d* from the surface of the water at *K*, when the depth of the water is considerably less than *dK*, but is reckoned from the middle of the

water at *K*, when *dK* is very small.* The relative fall, therefore, may be determined by subtracting *OK*, which is generally a foot from the absolute fall *dO*, and by subtracting also either the whole, or one half of the natural depth of the water at *K*, according as *dK* is great or small in proportion to this depth.

The next thing to be determined is, the breadth of the course at the top of the fall *B*, and the breadth of the canal at the same place. To find this, multiply the quantity of water expended in a second by 100, for a first number; take such a quantity as you would wish, for the depth of the water, and, having cubed it, extract its square root, and multiply this root by 488, for a second number; divide the first number by the second, and the quotient will be the breadth required. The breadth, thus found, may be too great or too small in relation to the depth. If this be the case, take one half of the breadth, thus found, and add to it the number taken for the depth of the water; the sum will be the true depth, with which the operation is to be repeated, and the new result will be better proportioned than the first.

The mill-course being thus constructed, we may now find more exactly the quantity of water furnished in a second. For this purpose, subtract one half the depth of the water at *K* from *dK*, and, having multiplied the remainder by .5333, extract the square root of the pro-

* The depth of the water, here alluded to, is its natural depth, or that which it would have if it did not meet the float-boards. The effective depth is generally two and a half times the natural depth, and is occasioned by the impulse of the water on the float-boards, which forces it to swell, and increases its action upon the float-boards.

PLATE XXXVII. duct. Multiply this root by the breadth of the course at K multiplied into the depth of the water there,* and the result will be the true expense of the source in cubic feet.

In order to know whether the water will have sufficient force to move the least millstone which should be employed, namely, a millstone weighing, along with its axle and trundle, 1550 pounds avoirdupois, take the relative fall increased by one half the natural depth of the water at K, viz. dK, and multiply it by the expense of the source in cubic feet; if the product be 30.52, or above it, the machine will move without interruption. If the product be less than this number, the weight of the millstone then ought to be less than 1550 pounds, and the meal will not be ground sufficiently fine; for the resistance of the grain will bear up the millstone, and allow the meal to escape before it is completely ground.

Fig. 1.

As it is of great consequence that none of the water should escape, either below the floatboards, or at their sides, without contributing to turn the wheel, the course of impulsion KV, should be wider than the course at K, as represented in Fig. 2. where CD, the course of impulsion, corresponds with LV, in Fig. 1. AB corresponds with HK, and BC with KL. The breadth of the floatboards, therefore, should be wider than mn, and their extremities should reach a little below B, like no in Fig. 1. When this precaution is taken, no water can escape, without exerting its force upon the floatboards.

* That is, by the area of the rectangular section of the stream at K.

On the size of the water-wheel, and on the number, magnitude, and position, of its floatboards.

The diameter of the wheel should be as great as possible, unless some particular circumstances in the construction prevent it, but ought never to be less than seven times the natural depth of the stream at *K*, the bottom of the course.* It has been much disputed among philosophers, whether the wheel should be furnished with a small or a great number of floatboards. M. Pitot has shown, that when the floatboards have different degrees of obliquity, the force of impulsion upon the different surfaces will be reciprocally as their breadth: thus, in Fig. 3. the force upon *he* will be to the force upon *DO*, as *DO* to *he*.† He, therefore, concludes, that the distance between the floatboards should be equal to one half of the arch plunged in the stream; or that, when one is at the bottom of the wheel, and perpendicular to the current, as *DE*, the preceding floatboard *BC* should be leaving the stream, and the succeeding one *FG* just entering into it.‡ For if, when the three floatboard *FG*, *DE*, *BC*, have the same position as in the figure, the whole PLATE XXXVII. Fig. 3.

* The diameter here meant is double the *mean radius*, or the distance between the centre of the wheel and the middle of the natural stream, which impels it, or what is called the centre of impulsion. By adding or subtracting the half of the stream's natural depth, to or from the mean radius, we have the *exterior* and *interior* radius of the wheel.

† See Traité de l'Hydrodynamique, No. 771.

‡ Memoires de l'Academie Royale de Sciences 1729, 8vo. p. 359.

force of the current *NM* will act upon *DE*, having the most advantageous position for receiving it: whereas, if another floatboard *fg* were inserted between *FG* and *DE*, the part *ig* would cover *DO*, and, by thus substituting an oblique for a perpendicular surface, the effect would be diminished in the proportion of *DO* to *ig*. Upon this construction it is evident, that the depth of the floatboard *DE* should always be equal to the versed-sine of the arch between any two floatboards, *DE* being the versed-sine of *EG*. For the use of those who may wish to follow M. Pitot, though we are of opinion that he recommends too small a number of floats, we have upon the above principles, calculated the following table, which exhibits the proper diameter of water-wheels, the number of floatboards they should contain, and the size of the floatboards, any two of these quantities being given. According to M. Pitot, the proper relation between these is of so great importance, that if a water-wheel, 16 feet diameter, with its floatboards 3 feet deep, should have nine instead of seven, one-twelfth of the whole force of impulsion would be lost.*

* Desaguliers has adopted the rule given by Pitot. See his Experimental Philosophy, v. 2, p. 424.

TABLE,

Of the number of floatboards to undershot wheels.

Diameter of the wheel in feet.	Depth of the floatboards in feet.						
	1	1.5	2	2.5	3	3.5	4
10	10	8	7	6	5	5	5
11	10	8	7	6	5	5	5
12	11	9	8	7	6	6	5
13	11	9	8	7	6	6	5
14	12	9	8	7	7	6	6
15	12	9	8	7	7	6	6
16	12	10	9	8	7	7	6
17	12	10	9	8	7	7	6
18	13	11	9	8	8	7	6
19	13	11	10	9	8	7	7
20	14	11.	10	9	8	7	7
21	14	12	10	9	8	7	7
22	15	12	10	9	8	8	7
23	15	12	10	9	8	8	7
24	15	12	11	10	9	8	8
25	16	13	11	10	9	8	8
26	16	13	11	10	9	8	8
27	16	13	11	10	9	8	8
28	17	13	12	10	9	9	8
29	17	14	12	11	10	9	8
30	17	14	12	11	10	9	9
32	18	14	12	11	10	9	9

PLATE XXXVII. In order to find from the preceding table the number of floatboards for a wheel 20 feet in diameter, (the diameter of the wheel being reckoned from the extremity of the floatboards) their depth being two feet;—enter the left hand column with the number 20, and the top of the table with the number 2, and in a line with these numbers will be found 10, the number of floatboards which such a wheel would require.

As the numbers representing the depths of the floatboards, and the diameter of the wheel, increase more rapidly than the numbers in the other columns, the preceding table will not shew us with accuracy the diameter of the wheel when the number and depth of the floatboards are given; 10 floatboards, for example, 2 feet deep, answering to a wheel either 19, 20, 21, 22, or 23 feet diameter. This defect, however, may be supplied by the following method.—Divide 360 degrees by the number of floatboards, and the quotient will be the arch between each. Find the natural versed-sine of this arch, and say, as 1000 is to this versed-sine, so is the wheel's radius to the depth of the floatboards; and to find the diameter of the wheel, say, as the above versed-sine is to 1000, so is the depth of the floatboards to the wheel's radius.

We have already said, that the number of floatboards found by the preceding table is too small. Let us attend to this point, as it is of considerable importance. It is evident from
Fig. 3. Fig. 3, that when one of the floats, as *DE*, is perpendicular to the stream, it receives the whole impulse of the water in the most advantageous manner; but when it arrives at the position *de*, and the succeeding one *FG* into the position *fg*, so that the angle *eAg* may be bi-

sected by the perpendicular *AE*, they will have the most disadvantageous situation; for a great part of the water will escape below the extremities *g* and *e*, of the floatboards, without having any effect upon the wheel; and the p[illegible]ig of the floatboard, which is really impelled, is less than *DE*, and oblique to the current. The wheel, therefore, must move irregularly, sometimes quick, and sometimes slow, according to the position of the floats with respect to the stream; and this inequality will increase with the arch plunged in the water. M. Pitot proceeds upon the supposition, that if another float *fg*, were placed between *FG* and *DE*, it would destroy the force of the water that impels it, and cover the corresponding part *DO* of the preceding floatboard. But this is not the case. The water, after acting upon *fg*, still retains a part of its motion, and bending round the extremity *g*, strikes *DE* with its remaining force. Considerable advantage, therefore, must be gained by using more floatboards than M. Pitot recommends.*

M. Bossut† has shown, that when the wheel has an uniform velocity, the most advantageous number of floats is determined; and he ob-

* In Mr. Smeaton's experiments, the water-wheel, which was 25 inches in diameter, had 24 floats; and he observes, 'that, 'when the number was reduced to 12, it caused a diminution of 'the effect, on account of a greater quantity of water escaping 'between the floats and the floor: but a circular sweep being a-'dapted thereto, of such a length, that one float entered the 'course before the preceding one quitted it, the effect came so 'near to the former, as not to give hopes of advancing it by in-'creasing the number of floats beyond 24 in this particular 'wheel.' Smeaton's Experimental enquiry, p. 24: or, Phil. Tran. 1759. v. 51.

† Traite de l' Hydrodynamique, notes on chap. x; also No. 778.

PLATE XXXVII. serves, that when the velocity of the stream is thrice that of the wheel, and when 72 degrees of the circumference are immersed, the number of floatboards should be 36. When a greater arch is plunged in the stream, the velocity continuing the same, the number should be increased, and *vice versa.*

This rule, however, is too difficult to be of use to the practical mechanic. From what has been said, it is evident, that in order to remove any inequality of motion in the wheel, and prevent the water from escaping beneath the tips of the floatboards, the wheel should be furnished with the greatest number of floatboards possible, without loading it, or weakening the rim on which they are placed.* This is the rule given by Mr. Fabre,† and it is not difficult to see, that if the millwright should err in furnishing the wheel with too many floatboards, the error will be perfectly trifling, and that he would lose much more by erring on the other side. The floatboards should not be rectangular, like *abnc* in Fig. 3. but should be levelled like *abmc.* For, if they were rectangular, the extremity *bn* would interrupt a portion of the water, which would otherwise fall on the corresponding part of the preceding floatboard.—The angle *abm* may be found thus:—Subtract from 180° the number of degrees contained in the immersed arch *CEG*, and the half of the remainder will be the angle required. It has

Fig. 3.

* Brisson (Traite Elementaire de Physique) observes, that there should be 48 floats instead of 40, as generally used in a wheel 20 feet in diameter.

† Sur les Machines Hydrauliques, p 55. No. 103. See Traite de l'Hydrodynamique, par Bossut.

been already observed, that the effective depth of the water at *K*, (Fig. 1.) is generally two and a half times greater than the natural depth. The height *DE*, therefore, of the floatboards, should be two and a half times the natural depth of the current at *K*. The breadth of the floatboards should always be a little greater than the breadth of the course at *K*, the method of finding which has been already pointed out. Plate XXXVII. Fig. 1.

M. Pitot has shown,* that the floatboards should be perpendicular to the rim, or, in other words, a continuation of the radius. This, indeed, is true in theory, but it appears from the most unquestionable experiments, that they should be inclined to the radius. This was discovered by Deparcieux, in 1753, (not in 1759, as Fabre asserts) who shows, that the water will thus heap up on the floatboards, and act, not only by its impulse, but also by its weight.† This discovery has been confirmed also by the Abbe Bossut,‡ who found, that when the velocity of the water is about $\frac{300}{27}$ of a foot, or 11 feet per second, the inclination of the floatboard to the radius should be between 15 and 30 degrees. M. Fabre, however, is of opinion, that when the velocity of the steam is 11 feet per second, or above this, the inclination should never be less than 30 degrees; that when this veloci y diminishes, the inclination should diminish in proportion; and that when it is four feet, or under, the inclination should be nothing. In

* Mem. Acad. Royale, 1729, 8vo. p. 350.

† Mem. de l'Acad. 1754, 4to. p. 614, 8vo. p. 944.

‡ Traits de l'Hydrodynamique, Nos. 814 and 817.

PLATE XXXVII. Fig. 3.

order to find the inclination for wheels of different radii, let AH (Fig. 3.) be the radius, bisect PH, the height of the floatboard, in i, and having drawn PK perpendicular to PA, set off PK equal to Pi, and join HK; HK will be the position of the floatboard inclined to the radius AH by the angle KHP. This construction supposes the greatest value of the angle KHP to be 26 degrees 34 minutes.

On the formation of the spur-wheel and trundle.

The radius of the spur-wheel is found by multiplying the mean radius of the water-wheel by that of the lantern, which may be of any size, and also by the number of turns which the spindle or axis of the lantern performs in a second,* and then by the number 2.151. This product being divided by the square-root of the relative fall, the quotient will be the radius required. The number of teeth in the wheel should be to the number of staves in the trundle as their respective radii. In order to find the exact number, take the proper diameter of the teeth and the staves, which ought to be two and a half inches each in common machines, and determine also how much is to be allowed for the play of the teeth, which should be about two and a half tenths of an inch; add these three numbers, and divide by this sum, the mean cir-

* The method of determining the velocity of the spindle or the millstone will be afterwards pointed out. The axis of the lantern should, in general, make about 90 turns in a minute.

cumference of the spur-wheel,* the quotient will be nearly the number of teeth in the wheel.—Let us call this quotient x, to avoid circumlocution. Multiply x by the mean radius of the trundle, and divide the product by the radius of the spur-wheel. If the quotient be a whole number, it will be the exact number of staves in the trundle, and x, if an integer, will be the exact number of teeth in the wheel. But should the quotient be a mixed number, diminish the integer, which may still be called x, by the numbers 1, 2, 3, &c. successively, and at every diminution multiply x, thus diminished, by the radius of the trundle, and divide the product by the radius of the wheel. If any of these operations give a quotient without a remainder, this quotient will be the number of staves in the trundle, and x, diminished by one or more units, will be the number of teeth in the wheel. Thus let the radius of the trundle be one foot, that of the wheel four feet, the thickness of the teeth and the staves 2 and a half inches, or $\frac{30}{144}$ of a foot, and the space for the play of the teeth two and a half tenths of an inch, or $\frac{3}{144}$; the sum of the three quantities will be $\frac{63}{144}$ or $\frac{7}{16}$ of a foot; and 25 feet, or $\frac{176}{7}$ of a foot, the circumference of the wheel, divided by $\frac{7}{16}$ will give $\frac{2816}{49}$ or $57\frac{23}{49}$ feet.—Multiply the integer x, or 57 by 1, the radius of the lantern, but as the product 57 will not divide by 4, the radius of the wheel, let us diminish x, or 57, by unity, and the remainder 56 being multiplied by 1, the radius of the trundle, and divided by four, the radius of the

* The mean radius is reckoned from the centre of the wheel to the centre of the teeth.

wheel, gives 14 without a remainder, which will therefore be the number of staves, while 56, or x, diminished by unity, is the number of teeth in the spur-wheel.

Had it been possible to make the number of teeth equal to $57\frac{23}{49}$, $2\frac{1}{2}$ inches would have been the proper thickness for the teeth and the staves; but, as the number must be diminished to 56, there will be an interval left, which must be distributed among the teeth and staves, so that a small addition must be made to each. To do this, divide the circumference of the wheel $\frac{176}{7}$ of a foot by the number of teeth 56, and, from the quotient $\frac{450}{1000}$ subtract the interval for the play of the teeth $\frac{3}{144}$ or $\frac{20}{1000}$, the remainder $\frac{430}{1000}$ being halved, will give $\frac{215}{1000}$ of a foot, or 2 inches and 5.8 tenths for the thickness of every tooth and stave, $\frac{8}{100}$ of an inch being added to each tooth and stave to fill up the interval.

It may, however, sometimes happen, that, in diminishing x successively by unity, a quotient will never be found without a remainder. When this is the case, seek out the mixed number which approaches nearest an integer, and take the integer to which it approximates for the number of staves in the lantern. Thus, when the radius of the wheel is $4\frac{1}{13}$ feet, the different quotients obtained, after diminishing x by 1, 2, 3, 4, will be $14\frac{226}{1000}$, $13\frac{981}{1000}$, $13\frac{736}{1000}$, $13\frac{490}{1000}$, and $13\frac{245}{1000}$. The nearest of these to an integer is $13\frac{981}{1000}$, being only $\frac{19}{1000}$ less than 14, which will therefore be the number of staves in the trundle.*

* See Fabre sur les Machines Hydrauliques, p. 304. § 546.

In a succeeding article on the teeth of wheels, we have shown what form must be given them, in order to produce a uniformity of action.—The following method, however, will be pretty accurate for common works. Take *EB*, equal to the radius of the trundle,* and describe the acting part *BA*, and with the same radius describe *CD*. When the teeth of the wheel are perpendicular to its plane, as in the spur-wheels of corn-mills, we must bisect *CD* in *n*, and drawing *mn* perpendicular to *BD*, make the plane *BACD* move round upon *mn* as an axis; the figure thus generated like *abc*, will be the proper shape for the teeth. PLATE XI. Fig. 7. Fig. 8.

The pivots, or gudgeons, on which vertical axes move, should be conical; and those which are attached to horizontal arbors, should be cylindrical, and as small and short as possible. A gudgeon 2 inches in diameter will support a weight of 3239 pounds avoirdupois, though we often meet with gudgeons 3 or 4 inches in diameter, when the weight to be supported is considerably less. By attending to this, the friction of the gudgeons will be much diminished, and the machine greatly improved. Particular care, too, should be taken, that the axis of the gudgeons be exactly in a line with the axis of the arbor which they support,† otherwise the action or the motion of the wheels which they carry will be affected with periodical inequalities.

* The staves of the trundle should be as short as possible.

† The diameter of the gudgeon must be proportional to the square-root of the weight which it supports.

Of the formation, size and velocity of the millstone, &c.

PLATE XXXVII. In the IVth lecture* Mr. Ferguson has given several useful directions for the formation of the grinding surfaces of the millstones; to which we have only to add, that when the furrows are worn shallow, and, consequently, to be new dressed with the chisel, the same quantity of stone must be taken from every part of the grinding surface, that it may have the same convexity or concavity as before. As the upper millstone should always have the same weight when its velocity remains unchanged, it will be necessary to add to it as much weight as it has lost in the dressing. This will be most conveniently done by covering its top with a layer of plaster, of the same diameter as the layer of stone taken from its grinding surface, and as much thicker than the layer of stone, as the specific gravity of the stone exceeds that of the plaster. That the reader may have some idea of the manner in which the furrows, or channels, are arranged, we have represented, in Fig. 4. the grinding surface of the upper millstone, upon the supposition that it moves from east to west, or for what is called a right-handed mill. When the millstone moves in the opposite direction, the position of the furrows must be reversed.

In Fig. 5. we have a section of the millstone's spindle and lantern. The under mill-

* Vol. 1.

stone *MPHG*, which never moves, may be of any thickness. Its grinding surface must be of a conical form, the point *b* being about an inch above the horizontal line *PR*, and *Ma* and *Pb* being straight lines. The upper millstone *EFPM*, which is fixed to the spindle *CD* at *C*, and is carried round with it, should be so hollowed that the angle *OMa*, formed by the grinding surfaces, may be of such a size that *On* being taken equal to *nM*, *ns* may be equal to the thickness of a grain of corn.* The diameter *ON* of the mill-eye *mC* should be between 8 and 14 inches; and the weight of the upper millstone *EP* joined to the weight of the spindle *CD* and the trundle *X*, (the sum of which three numbers is called the *equipage* of the turning millstone) should never be less than 1550 pounds avoirdupois, otherwise the resistance of the grain would bear up the millstone, and the meal be ground too coarse.

In order to find the weight of the equipage:— Divide the third of the radius of the gudgeon by the radius of the water-wheel which it supports, and having taken the quotient from 2.25, multiply the remainder by the expense of the source, by the relative fall, and by the number 19914, and you will have a first quantity, which may be regarded as pounds. Multiply the square-root of the relative fall by the weight of the arbor of the water-wheel, by the radius of its

* In note 3, p. 88, vol. 1, we have said that the corn does not begin to be ground till it has insinuated itself as far as two-thirds of the radius, but for reasons, which may be seen in *Fabre sur les Machines Hydrauliques*, p. 238, the grinding should commence at the point *n* equidistant from *O* and *M*.

PLATE XXXVII. gudgeon, and by the number 1617, and a second quantity will be had, which will also represent pounds. Divide the third part of the radius of the gudgeon by the radius of the water-wheel, and having augmented the quotient by unity, multiply the sum by 1005, and a third quantity will be obtained. Subtract the second quantity from the first, divide the remainder by the third, and the quotient will express the number of pounds in the equipage of the millstone.

The weight of the equipage being thus found, extract its square root, expressed in pounds, which multiply by .039, and the product will be the radius of the millstone in feet.*

In order to find the weight and thickness of the upper millstone, the following rules must be observed:

Fig. 5. 1. To find the weight of a quantity of stone equal to the mill-eye;—Take any quantity which seems most proper for the weight of the spindle *CD* and the lantern *X*, and subtract this quantity from the weight of the millstone's equipage, for a first quantity. Find the area of the mill-eye, and multiply it by the weight of a cubic foot of stone of the same kind as the millstone, and a second quantity will be had. Multiply the area of the millstone by the weight of a cubic foot of the same stone, for a third quantity. Multiply the first quantity by the second, and divide the product by the third, and the quotient will be the weight required.

* This rule supposes, that when the diameter of the mill-stone is 5 feet the weight of the equipage should be 4307 avordupois pounds

2. To find the number of cubic feet in the turning millstone, supposing it to have no eye:—From the weight of the spindle and lantern, subtract the quantity found by the preceding rule, for the first number. Subtract this first number from the weight of the equipage, and a second number will be obtained. Divide this second quantity by the weight of a cubic foot of stone of the same quality as the millstone, and the quotient will be the number of cubic feet in *EMPF*, *mC* being supposed to be filled up.

3. To find the quantities *mN* and *EM*, i. e. the thickness of the millstone at its centre and circumference:—Divide the solid content of the millstone, as found by the preceding rule, by its area, and you will have a first quantity. Add *bR*, which is generally about an inch, to twice the diameter of a grain of corn, for a second quantity. Add the first quantity to one third of the second, and the sum will be the thickness of the millstone at the circumference.—Subtract one-third of the second quantity from the first quantity, and the remainder will be its thickness at the centre.*

The size of the millstone being thus found, its velocity is next to be determined. M. Fabre observes, that the flour is the best possible when a millstone five feet in diameter makes from 48 to 61 revolutions in a minute. Mr. Ferguson allows 60 turns to a millstone six feet in diameter, and Mr. Imison 120 to a millstone $4\frac{1}{2}$ feet in diameter. In mills upon Mr. Imison's

* These rules are founded upon formulæ, which may be seen in *Fabre sur les Machines Hydrauliques*, p. 172, 229.

construction, the great heat that must be generated by such a rapid motion of the millstone, must render the meal of a very inferior quality: much time, on the contrary, will be lost, when such a slow motion is employed as is recommended by M. Fabre and Mr. Ferguson. In the best corn-mills in this country, a millstone five feet in diameter revolves, at an average, 90 times in a minute.* The number of revolutions in a second, therefore, which must be assigned to a millstone of a different size may be found by dividing 150 by the diameter of the millstone in feet.

The spindle *cD*, which is commonly 6 feet long, may be made either of iron or wood.—When it is of iron, and the weight of the millstone 7558 pounds avoirdupois, it is generally 3 inches in diameter; and when made of wood, it is 10 or 11 inches in diameter. For millstones of a different weight, the thickness of the spindle may be found by proportioning it to the square root of the millstone's weight, or, which is nearly the same thing, to the weight of the millstone's equipage.

The greatest diameter of the pivot *D*, upon which the millstone rests, should be proportional to the square root of the equipage, a pivot half an inch diameter being able to support an equipage of 5398 pounds. In most machines, the diameter of the pivots is by far too large, being capable of supporting a much greater weight than they are obliged to bear.

* Mr. Fenwick, of Newcastle, an excellent practical mechanic, observes, that, in the best corn-mills in England, millstones from 4½ to 5 feet in diameter revolve from 90 to 100 times in a minute

The friction is, therefore, increased, and the performance of the machine diminished. PLATE XXXVII.

The bridge-tree *AB* is generally from 8 to 10 feet long, and should always be elastic, that it may yield to the oscillatory motion of the millstone.* When its length is 9 feet, and the weight of the equipage 5182 pounds, it should be six inches square; and when the length remains unchanged, and the equipage varies, the thickness of the bridge-tree should be proportional to the square-root of the equipage. Fig. 5.

On the performance of undershot mills.

The performance of any machine may be properly represented by the number of pounds which it will elevate, in a given time, by means of a rope *KL*, wound upon the spindle *CD*, and passing over the pulley *L*.† In order to find the weight which a given machine will raise:— Divide the third part of the radius of the gudgeon of the water-wheel, by the mean radius of the wheel itself, and, having subtracted the quotient from 2.25, multiply the remainder by the expense of water in a second in cubic feet, by the height of the relative fall, and by the number 19911, for a first quantity. Multiply the weight of the arbor of the water-wheel, and its appendages, (viz. the water-wheel itself and the spur-wheel) by the radius of the gudgeon in Fig. 5.

* See *Belidor, Architecture, Hydraulique*, 638; or Desagulier's Exper. Philos. vol. 2, p. 429.

† It was in this way that Smeaton measured the performance of his models.

decimals of a foot, by the square-root of the relative fall, and the number 1617, and divide the product by the mean radius of the water-wheel, and a second quantity will be bad. Divide the third part of the gudgeon's radius by the mean radius of the water-wheel, augment the quotient by unity, and multiply the sum by the radius of the spindle *CD* for a third quantity. Subtract the second quantity from the first, and divide the remainder by the third quantity, the quotient will be the number of pounds which the machine will raise. Multiply the diameter of the spindle *CD* by 3.1416, and you will have a quantity equal to the height which *W* will rise by one turn of the spindle; this quantity, therefore, being multiplied by the number of turns which the spindle performs in a minute, will give the height through which the weight *W* will rise in the space of a minute.

Mr. Fenwick* found, by a variety of accurate experiments made upon good corn-mills, whose upper millstone, being from 4½ to 5 feet in diameter, revolved from 90 to 100 times in a minute, that a mill, or any power capable of raising 300 pounds avoirdupois with a velocity of 210 feet per minute, will grind *one* boll of good corn in an hour; and that 2, 3, 4, or 5 bolls will be ground in an hour, when a weight of 300 pounds is raised with a velocity of 350, 506, 677, or 865 feet respectively, in a minute.†

* Four Essays on Practical Mechanics, 2d. edit. 1802, p. 60.

† As the differences of these numbers increase nearly by 16, they may be continued by always augmenting the difference between the two last numbers by 16, and adding the difference

Or, to arrange the numbers more properly:

Number of bolls ground in an hour	1	2	3	4	5	6
Number of feet through which 300lb. is raised in a minute	210	350	506	677	865	1069

Supposing it, therefore, to be found, by the preceding rules, that a mill would raise 600 pounds through 253 feet in a minute of time, we have $300:600=253:506$; that is, the same power that can raise 600 pounds through 253 feet, will raise 300 pounds through 506 feet, consequently, such a mill will be able to grind three bolls of corn in an hour.*

According to M. Fabre, the quantity of meal ground in an hour may be determined by multiplying 62.4 Paris pounds by the square of the radius of the millstone, and the product will be the number of pounds of meal. But, as this rule is founded upon the erroneous supposition, that the quality of the flour is best when a millstone, 5 feet in diameter, performs 48 revolutions in a minute, we have made the calculation anew, upon the supposition, that the velocity

thus augmented to the last number, for the number required. Thus, by adding 16 to 188, the difference between 677 and 865, we have 204, which being added to 865, gives 1069 for the number of feet, nearly, through which the power must be able to raise a weight of three hundred pounds in a minute, in order to grind six bolls of corn in an hour.

* The proper result of Mr. Fenwick's experiment was, that a power requisite to raise a weight of 300 pounds avoirdupois, with a velocity of 190 *feet per minute*, would grind one boll of good corn in an hour: but, in order to make the above numbers accurate in practice, he increased the velocity one tenth, and made it 210 feet per minute.

of a millstone, 5 feet diameter, should be 90 revolutions in a minute, and have found, that, when mills are constructed upon this principle, the quantity of flour ground in an hour, in pounds avoirdupois, will be equal to the product of the square of the millstone's radius, and the number 125.

The following important maxims have been deduced from Mr. Smeaton's accurate experiments on undershot mills, and merit the attention of every practical mechanic.

Maxim 1.—That the virtual or effective head of water being the same,* the effect will be nearly as the quantity of water expended. That is, if a mill, driven by a fall of water whose virtual head is 10 feet, and which discharges 30 cubic feet of water in a second, grinds four bolls in an hour; another mill, having the same virtual head, but which discharges 60 cubic feet of water, will grind eight bolls of corn in an hour.

Maxim 2.—That the expense of water being the same, the effect will be nearly as the height of the virtual or effective head.

* The *virtual*, or *effective head* of water moving with a certain velocity, is equal to the height from which a heavy body must fall in order to acquire the same velocity. The height of the virtual head, therefore, may be easily determined from the water's velocity, for the heighis are as squares of the velocities, consequently, as the square roots of the heights. Mr. Smeaton observes, that, in the large opening of mills and sluices, where great quantities of water are discharged from moderate heads, the real head of water, and the virtual head, as determined from the velocity, will nearly agree. See his Experiments on Mills, p. 23.

Maxim 3.—That the quantity of water expended being the same, the effect is nearly as the square of its velocity.—That is, if a mill, driven by a certain quantity of water, moving with the velocity of 4 feet per second, grinds three bolls of corn in an hour; another mill, driven by the same quantity of water, moving with the velocity of five feet per second, will grind nearly $4\frac{7}{10}$ bolls of corn in an hour, because $3:4.7=4^2:5^2$ nearly, that is, as 16 to 25, the squares of the respective velocities of the water.

Maxim 4.—The aperture being the same, the effect will be nearly as the cube of the velocity of the water.—That is, if a mill driven by water, moving through a certain aperture, with the velocity of 4 feet per second, grind 3 bolls of corn in an hour; another mill driven with water, moving through the same aperture with the velocity of 5 feet per second, will grind $5\frac{43}{50}$ bolls nearly in an hour, for $3:5\frac{43}{50}=4^3\ 5^3$ nearly, that is, as 64 to 125, the cubes of the waters respective velocity.

On the method of constructing a mill-wright's table, on new principles.

Although a mill-wright's table has been constructed by Mr. Ferguson,* and afterwards altered a little by Mr. Imison, so far as concerns the velocity of the millstone; yet, as we

* See Vol. 1, p. 160.

shall now show, the principles upon which it is computed, are far from being correct. Following Desaguliers* and Maclaurin,† Mr. Ferguson has adopted the determination of Parent and Pitot, respecting the relative velocity of the water and the wheel.‡ But Mr. Smeaton has shown, that instead of the wheel moving with $\frac{1}{3}$ of the velocity of the water, when the effect is a maximum, as Parent imagined, the greatest effect is produced when the velocity of the wheel is between $\frac{1}{3}$ and $\frac{1}{2}$, the maximum being much nearer to $\frac{1}{2}$ than $\frac{1}{3}$. He observes, also, that $\frac{1}{2}$ would be the true maximum, 'if 'nothing were lost by the resistance of the air, 'the scattering of the water carried up by the 'wheel, and thrown off by the centrifugal force, '&c. all which tend to diminish the effect more 'at what would be the maximum, if these did 'not take place, than they do when the motion 'is a little slower.'§ But, in making this alteration, we are warranted not merely by the results of Mr. Smeaton's experiments, but by the more accurate deductions of theory. In the investigations from which Parent and Pitot concluded that the velocity of the wheel should be $\frac{1}{3}$ of the velocity of the water in order to produce a maximum effect, they considered

* Desagulier's Experimental Philosophy, v. 2, p. 424. Lect. 12.

† Maclaurin's Fluxion's Art. 907, p. 728.

‡ M. Lambert has also adopted the determination of Parent, in his Memoir on Undershot Mills in the *Nouv. Ment. de l'Acad. de Berlin*, 1775, *p.* 63.

§ Smeaton on Mills, p. 77. M. Bossut and M. Fabre, along with Smeaton, makes the velocity of the wheel two-fifths of the velocity of the water. See Traite d'Hydrodynamique, par Bossut, § 808.9, Fabre, § 66.

the impulse of the stream upon one floatboard only, and, therefore, made the force of impulsion proportional to the square of the difference between the velocities of the stream and the floatboard. The action of the current, however, is not confined to one floatboard, but is exerted on several at the same time, so that the floatboards which are accurately fitted to the mill-course, abstract from the water its excess of velocity, and the force of impulsion becomes proportional only to the difference between the velocities of the stream and the floatboards. The Chevalier de Borda, in his *Memoire sur les Roues Hydrauliques,** has taken advantage of this circumstance, and has shown, that in theory the velocity of the wheel is $\frac{1}{2}$ that of the current, and that in practice it is never more than three-eighths of the stream's velocity.

Mr. Waring, a gentleman in America, has also pointed out some circumstances to which Mr. Smeaton, and other philosophers, did not attend in their deductions,† and shows that the greatest effect is produced when the velocity of the wheel is *one half* the velocity of the water.

The constant number, too, which is used by Mr. Ferguson for finding the velocity of the water from the heights of the fall, viz. 64,2882, is not correct. From the recent experiments of Mr. Whitehurst on pendulums, it appears,

* Published in the Memoires de L'Acad. Royale Paris, 1767, p. 285, 4to.

† Transactions of the American Philosophical Society, v. 3, p. 144.

PLATE XXXVII. that a heavy body falls 16.087 feet in a second of time; consequently, the constant number should be 64,348.

In Mr. Ferguson's table, the velocity of the millstone is too small, and Mr. Imison, in correcting this mistake, has erred farther on the other side. From this circumstance, the mill-wright's table, as hitherto published, is fundamentally erroneous, and is more calculated to mislead than to direct the practical mechanic. Proceeding, therefore, upon the accurate deductions of Smeaton, as corrected by Waring, and employing a more correct constant number, and a more suitable velocity for the millstone, we may construct a new mill-wright's table by the following rules.

1. Find the perpendicular height of the fall of water in feet above the bottom of the mill-course at *K*; and having diminished this number by one half of the natural depth of the water at *K*, call that the height of the fall.* (Fig. 1.)

2. Since bodies acquire a velocity of 32.174 feet in a second, by falling through 16.087 feet, and, since the velocities of falling bodies are as the square-roots of the heights through which they fall, the square-root of 16.087 will

* The height of the fall here meant is the relative or virtual height, and it is supposed that the mill course is so accurately constructed, that the water will have the same velocity at *K* as it would have at *R* by falling perpendicularly through *CR*. This will be nearly the case when the mill-course is formed according to the directions formerly given; though in general a few inches should be taken from the fall, in order to obtain accurately its relative or virtual height.

be to the height of the fall as 32.174 to a fourth number, which will be the velocity of the water. Therefore the velocity of the water may be always found by multiplying 32.174, by the square-root of the height of the fall, and dividing that product by the square-root of 16.087; or it may be found more easily by multiplying the height of the fall by the constant number 64.348, and extracting the square-root of the product, which will be the velocity of the water required.*

3. Take one half of the velocity of the water, and it will be the velocity which must be given to the floatboards or the number of feet they must move through in a second in order that the greatest effect may be produced.

4. Divide the circumference of the wheel by the velocity of its floatboards per second, and the quotient will be the number of seconds in which the wheel revolves.

* That the velocity of the water is equal to the square root of the product of the height of the fall, and the constant number 64.348 may be shown in the following manner.—Let x be the velocity of the water, m the height of the fall, $a=16.087$, and consequently $2a=32.174$. Then by the first part of the second rule $\sqrt{a}:\sqrt{m}=2a:x$ therefore $x=\frac{2a\sqrt{m}}{\sqrt{a}}$; multiplying by $\sqrt{a}$ we have $x\sqrt{a}=2a\sqrt{m}$; putting all the quantities under the radical sign there comes out $\sqrt{xxa}=\sqrt{4aam}$; squaring both sides of the equation we have $xxa=4aam$ dividing by a gives $xx=4am$ or $x=\sqrt{4am}$ But since the constant number 64.348 is double of 32.174, it will be equal to $4a$, then by the latter part of rule second we have $x=\sqrt{4am}$, which is the same value of x as was found from the first part of the rule.

5. Divide 60 by this last number, and the quotient will be the number of revolutions which the wheel performs in a minute.

6. Divide 90 (the number of revolutions which a millstone 5 feet diameter should perform in a minute) by the number of revolutions made by the wheel in a minute, and the quotient will be the number of turns which the millstone ought to make for one revolution of the wheel.

7. Then, as the number of revolutions of the wheel in a minute is to the number of revolutions of the millstone in a minute, so must the number of staves in the trundle be to the number of teeth in the wheel in the nearest whole numbers that can be found.*

8. Multiply the number of revolutions performed by the wheel in a minute by the number of revolutions made by the millstone for one of the wheel, and the product will be the number of revolutions performed by the millstone in a minute.

In this manner the following table has been calculated for a water-wheel 15 feet in diameter, which is a good medium size, the millstone being 5 feet in diameter, and revolving 90 times in a minute.

N. B.—The table is used in the same way as Mr. Ferguson directs in Vol. I.

* We have filled up the *sixth* column of the table in the common way; but for the proper method of finding the relation between the radius of the spur-wheel and trundle, and the exact number of teeth in the one, and staves in the other, we must refer the reader to p. 161 of this volume.

THE MILL-WRIGHT'S TABLE,

Calculated upon new principles.

Height of the effective fall of water.	Velocity of the water per second.	Velocity of the wheel per second.	Revolutions of the wheel per minute.	Revolutions of the millstone for one of the wheel.	Teeth in the wheel, and staves in the trundle.		Revolutions of the millstone per minute by these staves and teeth.
Feet.	Feet. 100 parts of a foot.	Feet. 100 parts of a foot.	Revol. 100 parts of a rev.	Revol. 100 parts of a rev.	Teeth.	Staves.	Revol. 100 parts of a rev.
1	8.02	4.01	5.10	17.65	106	6	90.01
2	11.34	5.67	7.22	12.47	87	7	90.03
3	13.89	6.95	8.85	10.17	81	8	90.00
4	16.04	8.02	10.20	8.82	79	9	89.96
5	17.94	8.97	11.43	7.87	71	9	89.95
6	19.65	9.82	12.50	7.20	65	9	90.00
7	21.22	10.61	13.51	6.66	60	9	89.98
8	22.69	11.34	14.45	6.23	56	9	90.02
9	24.06	12.03	15.31	5.88	53	9	90.02
10	25.37	12.69	16.17	5.57	56	10	90.06
11	26.60	13.30	16.95	5.31	53	10	90.00
12	27.79	13.90	17.70	5.08	51	10	89.91
13	28.92	14.46	18.41	4.89	49	10	90.02
14	30.01	15.01	19.11	4.71	47	10	90.00
15	31.07	15.53	19.80	4.55	48	11	90.09
16	32.09	16.04	20.40	4.45	44	10	89.96
17	33.07	16.54	21. 5	4.28	47	11	90.09
18	34.03	17. 2	21.66	4.16	50	12	90.10
19	34.97	17.48	22.26	4.04	44	11	89.93
20	35.97	17.99	22.86	3.94	48	12	90.07
1	2	3	4	5	6		7

On horizontal mills.

PLATE XXXVII. Although horizontal water-wheels are very common on the Continent of Europe, and are strongly recommended to our notice by the simplicity of their construction, yet they have almost never been erected in this country,* and therefore are not described in any of our treatises on practical mechanics. In order to supply this defect, and recommend them to the attention of the mill-wright, we shall give a brief account of their theory and construction. In Fig. 6. we have a representation of one of these mills. *AB* is the large water-wheel which moves horizontally upon its arbor *CD*. This arbor passes through the immoveable millstone *EF* at *D*, and, being fixed to the upper one *GH*, carries it once round for every revolution of the great wheel. *N* is the hopper, and *I* the mill-shoe, the rest of the construction being the same as in vertical corn-mills.

The mill-course is constructed in the same manner for horizontal as for vertical wheels, with this difference only, that the part *mBnC*, Fig. 2. of which *KL* in Fig. 1. is a section, instead of being rectilineal like *mn*, must be circular like *mP*, and concentric with the rim of the wheel, sufficient room being left between it and the tips of the floatboards for the play of the wheel.

The equipage of the millstone of a horizontal mill* may be found by multiplying the product of the 100th part of the expense of the

* Great Britain.

† The equipage comprehends the millstone, the water-wheel, and its arbor.

water in cubic feet, and the relative fall, by 5078, and the product will be the weight of the equipage in pounds avoirdupois.

The mean radius of the wheel *AB* is to be determined by multiplying the product of the relative fall, and the square-root of the expense of water in a second by 0.062.

What has been said respecting the number, position, and form of the floatboards, of vertical wheels, may be applied also to horizontal ones. In the latter, however, the floatboards must be inclined, not only to the radius, but also to the plane of the wheel, in the same angle in which they are inclined to the radius, so that the lower and the outermost sides of the floatboards may be farthest up the stream.

Since the millstone of horizontal mills performs the same number of revolutions as the water-wheel; and since a millstone five feet in diameter should never make less than 48 turns in a minute, the wheel must perform the same number of revolutions in the same time; and in order that the effect may be a *maximum*, or the greatest possible, the velocity of the current must be double that of the wheel. Suppose the millstone, for example, to be five feet diameter, and the water-wheel six feet, it is evident that the millstone and wheel must at least revolve 48 times in a minute; and since the circumference of the wheel is 18.8 feet, the floatboards will move through that space in the 48th part of a minute, that is, nearly at the rate of 15 feet per second, which, being doubled, makes the velocity of the water 30 feet, answering, as appears from the preceding table, to a fall of 14 feet. But, if the given fall of water be less than 14 feet, we may procure the

PLATE XXXVII. same velocity to the millstone, by diminishing the diameter of the wheel. If the wheel, for instance, be only five feet diameter, its circumference will be 15.7 feet, and its floats will move at the rate of 12.56 feet in a second, the double of which is 25.12 feet per second, which answers to a head of water less than 10 feet high. As the diameter of the water-wheel, however, should never be less than seven times the breadth of the mill-course at *K*, there will be a certain height of the fall beneath which we cannot employ horizontal wheels, without making the millstone revolve too slowly. This height will be found by the following table:

Fig. 1.

When the natural depth of the water at the bottom of the fall is to the breadth of the mill-course at the same place, as	3 to 1	2 to 1	1 to 1 Equal.	$\frac{1}{2}$ to 1	$\frac{1}{3}$ to 1
The relative fall beneath which we cannot employ horizontal mills, will be	7.314	8.602	11.350	14.976	17.6[illegible]
	Ft. Dec.	Ft. Dec.	Ft. Dec.	Ft. Dec.	Ft. Dec.

Thus, if the natural depth of the water at *K*, be three times the width of the mill-course at the same place, the relative fall beneath which we cannot employ a horizontal wheel, will be 7.314 feet. Since the depth of the water is so great in this case, a great quantity of it will be discharged in a second, and, therefore, it requires a less velocity, or a less height of the fall to impel the wheel, whereas, if the depth of the water had been only one third of the

width of the mill-course, such a small quantity would be discharged in a second, that we must make up for the want of water, by giving a greater velocity to what we have, or by making the height of the fall 17.613 feet.

In order to find the radius of the millstone in horizontal mills, multiply the expense of water in cubic feet in a second, by the relative fall; extract the square-root of the product, and multiply this root by 0.267, the product will be the radius of the millstone in feet.

The quantity of meal ground in an hour may be found by the rules already given for vertical mills, or by multiplying the product of the expense of water and the relative fall, by 456 lb. and the result will be the quantity required.

The thickness of the millstone at the centre and circumference, and the thickness of the arbor and pivots may be determined by the rules already laid down for vertical mills.

In horizontal wheels, the mill-course is sometimes differently constructed. Instead of the water assuming a horizontal direction before it strikes the wheel, as in the case of undershot mills, the floatboard is so inclined as to receive the impulse perpendicularly, and in the direction of the declivity of the waterfall. When this construction is adopted, the greatest effect will be produced when the velocity of the floatboards is not less than $\frac{\sqrt{2aD}}{2. \text{Sin.} x}$; in which a represents the accelerating force of gravity = 16.087 feet, D the height of the waterfall, and x the angle which the direction of the fall makes with a vertical line. But since this quantity increases as the sign of x decreases,

it follows that without taking from the effect of these wheels, we may diminish the angle *A*, and thus augment considerably the velocity of the floatboards, according to the nature of the machinery employed; whereas, in vertical wheels, there is only one determinate velocity, which produces a maximum effect.*

In the southern provinces of France, where horizontal wheels are very generally employed, the floatboards are made of a curvilineal form, so as to be concave towards the stream. The Chevalier de Borda observes, that in theory a double effect is produced when the floatboards are concave, but that this effect is diminished in practice, from the difficulty of making the fluid enter and leave the curve in a proper direction. Notwithstanding this difficulty, however, and other defects which might be pointed out, horizontal wheels with concave floatboards are always superior to those in which the floatboards are plane, and also to vertical wheels, when there is a sufficient head of water. If the fall of water be five or six feet, a horizontal wheel with concave floatboards may be erected, whose maximum effect will be to that of ordinary vertical wheels, as 3 to 2.

* See *Memoire sur les Roues Hydrauliques*, *Mem. de l'Acad. Royal, Par.* 1767, p. 285.

On double corn-mills.

It frequently happens that one water-wheel drives two millstones, in which case the mill is said to be double; and when there is a copious discharge of water from a high fall, the same water-wheel may give sufficient velocity to three, or four, or five millstones. Mr. Ferguson has given a brief description of a double mill in Vol. I. p. 96. and a drawing of one in Plate VII. Fig. 4. but has laid down no maxim of construction for the use of the practical mechanic. In supplying this defect, let us first attend to double horizontal mills, in which the axis *CD* is furnished with a wheel which gives motion to two trundles, the arbors of which carry the millstones. PLATE XXXVII. Fig. 6.

In order to find the weight of the equipage for each millstone, multiply the product of the expense of water, and the relative fall, by 48116 lb. and divide the product by 2000, if there be two millstones, by 3000 if three, and so on; the quotient will be the weight of the equipage of each millstone.

To determine the radius of the wheel that drives the trundles, find first the radius of the millstones by the rules already given, and having added it to half the distance between the two neighbouring millstones,* subtract from this *sum* the radius of the lantern, which may be taken at pleasure, and the remainder will

* This quantity may be taken at pleasure, and should never be less than 2 feet, however great be the number of the mill-stones.

be the radius required when there are two millstones. But when there are three millstones, or four, or five, or six, before subtracting the radius of the lantern, divide the sum by 0.864, 0.705, 0.587, or 0.5, respectively.

The mean radius of the water-wheel may be found by multiplying the square-root of the relative fall by the radius of the millstone, by the radius of the wheel that drives the trundles, and by 231, and then dividing the product by the radius of the lantern multiplied by 1000, the quotient will be the wheel's radius. It may happen, however, that the diameter of the wheel found in this way will be too great. When this is the case, we may be certain that the radius of the lantern has been taken too small. In order then to get a less value for the wheel's radius, increase a little the radius of the lantern, and find new numbers both for the water-wheel, and that which drives the trundles, by the preceding rule. It may happen, also, that in giving an arbitrary value to the radius of the lantern, the diameter of the wheel found by the rule may be too small, that is, less than seven times the breadth of the mill-course at the bottom of the fall. When this takes place, make the diameter of the water-wheel seven times the width of the mill-course, and you may find the radius of the other wheel and lanterns by the following rules.

1. To find the radius of the wheel that impels the trundles; add the radius of the millstone to half the distance between any two adjoining millstones for a first quantity. Multiply the square-root of the relative fall by the radius of the millstone and by .231, and hav-

ing divided the product by the radius of the water-wheel, add unity to the quotient, and multiply the sum by 1. if there be two millstones, by .864 if three, by .705 if four, by .587 if five, and by .5 if six, and the result will be a second quantity. Divide the first by the second quantity, and the quotient will be the radius of the wheel that drives the trundles.

2. To find the radius of the lantern, multiply the radius of the wheel as found by the preceding rule, by the square-root of the relative fall, and by .231, and divide the product by the radius of the water-wheel, the quotient will be the lantern's radius.

By the rules formerly given find the quantity of meal ground by one millstone, and having multiplied this by the number of millstones, the result will be the quantity ground by the compound mill.

If the equipage of the millstone of a vertical mill, as found in p. 165, should be too great, that is, if it should require too large a millstone, then we must employ a double mill, like that which is represented in Plate VII. Fig. 4. Vol. I. or one which has more than two millstones.

In order to know the equipage of each millstone, find it by the rule for a single mill, and having multiplied the quantity by .947, divide the product by the number of millstones, and the quotient will be the equipage of each millstone.

The radius of the wheel *D*, Plate VII. Fig. 4. Vol. I. will be found by the same rule which was given for horizontal mills; but it must be attended to, that the lantern, whose radius is there employed, is not *BB*, but *FG* or *EH*.

To determine the mean radius of the large spur-wheel *AA*, which is fixed upon the arber of the water-wheel, multiplying the square of the radius of the lanterns *FG* or *EH*, by the radius of the water-wheel, and also by 4308, and a first quantity will be had. Multiply the square-root of the relative fall by the radius of one of the millstones, by the radius of the wheel *D*, and by 1000, and a second quantity will be obtained. Divide the first quantity by the second, and the quotient will be the mean radius of the wheel *AA*.

The quantity of meal ground by a compound mill of this kind is found by the same rule that was employed for compound mills driven by a horizontal water-wheel.

On breast-mills.

Plate XXXVIII. A breast water-wheel partakes of the nature both of an overshot and of an undershot wheel; it is driven partly by the impulse, but chiefly by the weight of the water. Fig. 1. represents a water-wheel of this description, where *MO* is the stream of water falling upon the float-board *o*, with a velocity corresponding to the height *mn*, and afterwards acting by its weight upon the floatboards between *o* and *B*. The mill-course *oB* is concentric with the wheel, which is fitted to it in such a manner that very little water is permitted to escape at the sides and extremities of the float-boards. The effect of a mill driven in this manner, is equal, ac-

cording to Mr. Smeaton, 'to the effect of an 'undershot mill, whose head is equal to the 'difference of level between the surface of wa-'ter in the reservoir and the point where it 'strikes the wheel, added to that of an over-'shot, whose height is equal to the difference 'of level between the point where it strikes 'the wheel and the level of the tail-water.'* That is, the effect of the wheel *A* is equal to that of an undershot wheel driven by a fall of water equal to *mn*, added to that of an over-shot wheel whose height is equal to *nD*. M. Lambert, of the Academy of Sciences at Berlin,† has shown, that when the floatboards arrive at the position *op*, they should be horizontal, or the point *p* should be lower than *o*, in order that the whole space between any two adjacent floatboards may be filled with water; and that *Cm* should be equal to the depth of the floatboards. He observes also, that a breast-wheel should be used when the fall of water is above four feet, and below ten, provided the discharge of water be sufficiently copious; that an undershot wheel should be preferred when the fall is below four feet, and an overshot wheel when the fall exceeds ten feet; and that, when the fall exceeds 12 feet, it should be divided into two, and two breast-mills erected. This, however, is only a general rule, which many circumstances may render it necessary to overlook. The following table, which may be of essential utility to the practical mechanic, is calculated from the formulæ of Lambert, and exhibits at one view the result of his investigations.

* Smeaton on Mills, *Scholium*, p. 36.
† Nouveaux Memoires de L'Academie de Berlin, 1775, p. 71.

TABLE FOR BREAST-MILLS.

Height of the fall in feet = *CD*, Fig. 1.	Breadth of the float-boards.	Depth of the float-boards.	Radius of the water-wheel reckoned from the extremity of the floats.	Velocity of the wheel per second.	Time in which the wheel performs one revolution.	Turns of the mill-stone for one of the wheel.	Force of the water upon the float-boards.	The length of *mn* in Fig. 1.	The length of *no* in Fig. 1.	Water required per second to turn the wheel.
	Ft. Dec.	Ft. Dec.	Ft. Dec.	Ft. Dec.	Secs. Dec.		lbs. Avoir.	Ft. Dec.	Ft. Dec.	Cub. Ft.
1	0.17	198. 6	0.75	2.18	1.92	4.80	1536	0.08	0.23	74.30
2	0.34	35. 1	1.50	3.09	2.72	6.80	1084	0.15	0.46	37.15
3	0.51	12. 7	2.26	3.78	3.33	8.32	886	0.23	0.68	24.77
4	0.69	6. 2	3.01	4.36	3.84	9.60	768	0.30	0.91	18.57
5	0.86	3.57	3.76	4.88	4.28	10.70	686	0.38	1.14	14.86
6	1.03	2.25	4.51	5.35	4.70	11.76	626	0.46	1.37	12.38
7	1.20	1.53	5.26	5.77	5.08	12.70	581	0.53	1.60	10.61
8	1.37	1.10	6.02	6.17	5.43	13.58	543	0.60	1.83	9.29
9	1.54	0.81	6.77	6.55	5.76	14.40	512	0.68	2.05	8.26
10	1.71	0.77	7.52	6.90	6.07	15.18	486	0.76	2.28	7.43
1	2	3	4	5	6	7	8	9	10	11

PLATE XXXVIII.

It is evident from the preceding table, that, when the height of the fall is less than three feet, the depth of the floatboards is so great, and their breadth so small, that the breast-mill cannot well be employed; and, on the contrary, when the height of the fall approaches to 10 feet, the depth of the floatboards is too small in proportion to their breadth. These two extremes, therefore, must be avoided in practice. The last column contains the quantity of water necessary for impelling the wheel, but the total expense of water should always exceed this, by the quantity, at least, which escapes between the mill-course and the sides and extremities of the floatboards. PLATE XXXVIII.

Mr. Siebike, son of the inspector of mills at Berlin, has given the following dimensions of an excellent breast water-wheel, differing very little from that which is represented in Fig. 1. The water, however, instead of falling through the height *Cn*, which is 16 inches* Rhynland measure, is delivered on the floatboard *op*, through an adjutage $6\frac{1}{2}$ inches high. The height *nD* is 4 feet 2 inches; consequently, the whole height *CD* must be $5\frac{1}{2}$ feet. The radius of the wheel *AB* is $6\frac{1}{2}$ feet, the breadth of each floatboard $6\frac{1}{2}$ inches, and its depth 28 inches. The point *P* of the wheel moves at the rate of 7,588 feet in a second. The expense of water in a second is 5,266 cubic feet, and the force upon the floatboards 356 pounds avoirdupois. The turning millstone weighed 1976 pounds avoirdupois; its diameter was 3 feet $8\frac{1}{2}$ inches, and it performed $2\frac{1}{2}$ revolutions in a second.

* A Rhynland foot is to an English foot as 1033 to 1000, or one Rhynland foot is equal to 12 inches and 4 lines English.

MECHANICS.

Practical Remarks on the Performance and Construction of Overshot Water-wheels.

On the method of computing the effective power of overshot wheels in turning machinery.

IN overshot mills, where the wheel is moved by the weight of the water in the buckets, each bucket has a different power to turn the wheel; and this power is proportioned to the distance of the bucket from the top or bottom of the wheel; or, more accurately, to the sine of the arch contained between the centre of the bucket and the top or bottom of the wheel, according as the bucket is above or below its centre. The bucket, for instance, placed upon the top of the wheel, has no power to turn it; the bucket next to this contributes but a little to turn the wheel, because it is virtually placed at the extremity of a very short lever; whereas, the bucket, which is equally distant from the top and bottom of the wheel, and which is level with the centre, has the greatest power to turn it, because it acts at the extremity of a lever equal to the wheel's semidiameter. If we suppose, then, that each bucket contains *one* gallon of water, equal in weight

to 10.2 pounds avoirdupois; we may, by the simplest operations in trigonometry, compute, in pounds avoirdupois, the power which each bucket exerts in turning the wheel; and, by taking the sum of these, we will have the effective weight of the water* in the buckets, and, consequently, its proportion to the real weight of the water, with which the semi-circumference of the wheel is loaded. Those who choose to make this calculation, will find, that the total weight of water upon the semi-circumference of an overshot wheel is to the effective weight as 1 to .637; but, as two or three of the buckets at the bottom of the arch are always empty, the proportion will rather be as 1 to .75 nearly. From these principles, we may deduce the following method, simpler than any hitherto given, of computing the effective weight of water upon overshot wheels of any diameter.

Rule.—Multiply the constant number 6.12 by half the number of buckets, and this product by the number of gallons in each bucket, and the result will be the effective weight of the water upon the wheel, three buckets being supposed empty at the bottom. This rule is pretty accurate for wheels from 20 to 32 feet in diameter. But when the diameter of the wheel is less than 20 feet, the answer given by

* This phrase which is used by practical mechanics, is very exceptionable; as every drop of water in the buckets, except that in the vertical bucket is *effective*. By the effective weight of the water, therefore, we must understand that weight which, if suspended at the opposite extremity of the wheel, would keep it in equilibrio, or balance the loaded arch.

the rule must be diminished one pound avoirdupois for every foot which the wheel is less than 20.

Suppose that it is required to find the effective weight of water upon a wheel of 18 feet in diameter, having 40 buckets, each containing two gallons ale-measure. Then $6.12 \times 20 \times 2 = 244.8$. But, as the diameter of the wheel is 2 feet below 20, we must deduct 2 pounds from the preceding answer, and the result will be 242.8 pounds avoirdupois.

On the performance of overshot and undershot mills.

From a number of accurate experiments made by the ingenious Mr. Fenwick, upon a variety of excellent overshot mills, it appears, that when the water-wheel is 20 feet in diameter, 392 gallons of water per minute (ale-measure) will grind one boll of corn per hour (Winchester-measure); 675 gallons per minute will grind 2 bolls; 945 gallons will grind 3 bolls; 1270 gallons will grind 4 bolls, and 1623 gallons will grind 5 bolls. From these data it will be easy to compute the performance of an overshot mill, whatever be the diameter of the wheel and the supply of water.

EXAMPLE 1.—Let it be required to find how many bolls of corn will be ground by an overshot mill, driven by a wheel 25 feet in diameter, upon which 1150 gallons of water are dis-

charged in a minute. Say as the nearest num-
Galls. Bolls. Galls. Bolls.
ber 1270 : 4 = 1150 : 3.62, the quantity of corn ground by a wheel 20 feet in diameter. Then to find the quantity which a 25 feet wheel
Feet. Bolls.
will grind: say, as 20 : 3.62=25 : 4.52 bolls, the answer required.

EXAMPLE 2.—If it be required to grind $3\frac{1}{2}$ bolls of corn, where the stream discharges 2220 gallons in a minute, what must be the diameter of the wheel? Find the number of gallons which a 20 feet wheel will require for grinding the given quantity of corn by the following pro-
Bolls. Galls. Bolls. Galls.
portion. As 4 : 1270=3.5 : 1111. Then, by
Galls. Feet. Galls.
inverse proportion, 1111 : 20=2220 : 10 feet, the diameter of the wheel required.

In order to find the proportionate quantity of corn ground by an undershot mill, the wheel and quantity of water being the same as in an overshot mill;—divide the quantity ground in the overshot mill by 2.4, and the quotient will be the answer. If it be required to know what size of wheel is necessary for making an undershot mill grind a certain quantity of corn, the supply of water being given; find the size of an overshot wheel necessary for producing the same effect, and multiply this by 2.4: the product will be the required diameter of the undershot wheel.

On the formation of buckets, and the proper velocity of overshot wheels.

PLATE XXXIX. Fig. 4.

Let *AM* be part of the shrouding or ring of buckets of an overshot wheel; *GOFABCD* is the form of one of these buckets. The shoulder *AB* of the bucket should be one half of *AE*, the depth of the shrouding; *AF* should be $\frac{1}{3}$ more than *AE*. The arm *BC* of the bucket must be so inclined to *AB*, that *HC* may be $\frac{5}{6}$ of *AE*; and *CD*, the wrist of the bucket, must make such an angle with *BCn*, the direction of the arm, that *Dn* may be $\frac{1}{3}$ of *En*.

A very considerable improvement in the construction of the bucket has been made by Mr. Robert Burns, at Cartside, Renfrewshire. He divides the bucket by a partition *mB*, of such a height, that the portions of the bucket on each side of it may be of equal capacity.—Dr. Robison observes, that this principle is susceptible of considerable extension, and recommends two or more partitions, particularly when the wheel is made of iron. By this means the fluid is retained longer in the lower buckets, and when there is a small supply of water, it may be delivered into the outer portion of the bucket, which, being at a greater distance from the centre of motion, increases the power of the water to turn the wheel. Dr. Robison advises, that the rim of the wheel, and, consequently, the breadth of the buckets, should be pretty large, in order that the quantity of water which they receive from the spout may not

nearly fill the bucket. The spout which conveys the water should be considerably narrower than the breadth of the bucket; and the shoulder *AB* should be perforated with a few holes, in order to prevent the water from being lifted up by the ascending buckets. The distance of the spout from the receiving bucket, should, in general, be two or three inches, that the water may be delivered with a velocity a little greater than that of the rim of the wheel; otherwise the wheel will be retarded by the impulse of the buckets against the stream, and much power would be lost by the water dashing over them.*

The proper velocity of an overshot wheel is a point upon which some celebrated mechanicians have entertained different sentiments.—From a variety of experiments, Mr. Smeaton infers, in general, that the circumference of the wheel should move with the velocity of a little more than 3 *feet per second.*'—'Experience,' says he, 'confirms, that this velocity of three 'feet in a second is applicable to the highest 'overshot wheels, as well as the lowest; and, 'all other parts of the work being properly 'adapted thereto, will produce very nearly the 'greatest effect possible; however, this also is 'certain, from experience, that high wheels 'may deviate farther from this rule before they

* If the spout be *one inch and seven-tenths* above the receiving bucket, it will deliver the water with the velocity of the wheel, that is, about three feet per second. In order, therefore to make the velocity of the water exceed a little that of the wheel, the height of the spout should be 2½ inches, and the water will move at the rate of 3 feet seven inches per second. Dr. Robison recommends 3 or 4 inches; but this is evidently too great, as 4 inches gives a velocity of 4 feet 7 inches per second.

'will lose their power, by a given aliquot part 'of the whole, than low ones can be admitted 'to do; for a wheel of 24 feet high may move 'at the rate of six feet per second, without 'losing any considerable part of its power; 'and, on the other hand, I have seen a wheel 'of 33 feet high, that has moved very steadily 'and well with a velocity but little exceeding '2 feet.'* M. Deparcieux† shows, that most work is performed by an overshot mill when it moves slowly, and that the more we retard its motion by increasing the work to be performed, the greater will be the performance of the wheel.

This important conclusion was deduced by Deparcieux, from experiments made upon a small wheel 20 inches in diameter, furnished with 48 buckets which received the water like a breast-wheel. On the axis of this wheel were placed cylinders of different sizes, the smallest being one inch, and the largest four inches, in diameter, around which was wrapped a cord, with a weight attached to it.‡ When the one inch cylinder was used, a weight of 12 ounces was elevated to the height of 69 inches and 9 lines; and a weight of 24 ounces was elevated 40 inches. When the four inch cylinder was employed, a weight of 12 ounces was raised to the altitude of 87 inches and 9 lines, and a weight of 24 ounces

* Smeaton on Mills, p. 33.

† Memoires de l'Academie des Sciences, 1754, p. 603, 671, 4to. p. 928, 1033, 8vo.

‡ The model employed in Mr. Smeaton's experiments resembles very much that of Deparcieux, though their experiments were made about the same time.

to the height of 45 inches and 3 lines. From these results, it is evident, that, with the four inch cylinder, when the motion was slowest, the effect was greatest; and that, when a double weight was used, which diminished the wheel's velocity, the effect was increased, the weight being raised to more than half its former height.*

This increase of performance, by diminishing the wheel's velocity, has been ascribed to different causes. Deparcieux and Brisson account for it by saying, that when the motion of the wheel is slow, the same portion of water acts more efficaciously. Dr. Robison and Mr. Smeaton ascribe it to a greater quantity of water pressing on the wheel; for, when the wheel's motion is slow, the buckets receive more water as they pass the spout. One of the most powerful causes, however, is a diminution of the centrifugal force of the water in

* Mr. Jean Albert Euler, whose memoir on the best method of employing the force of water and other fluids, gained the prize proposed by the Royal Society of Gottingen in 1754; has also shown, that the more slowly an overshot or breast-wheel with buckets moves, the greater will be its performance. See the *Journal Etranger*, Decembre 1756. Mr. Smeaton, too, deduces from his experiments this general rule, that, *cæteris paribus*, the less the velocity of the wheel, the greater will be its effect. But he observes, on the other hand, that when the wheel of his model made about twenty turns in a minute, the effect was nearly the greatest. When it made thirty turns the effect was diminished about one twentieth part; and that when it made forty, it was diminished about one quarter. When it made less than eighteen and a quarter turns, its motion was irregular; and when it was loaded so as not to admit its making eighteen turns, the wheel was overpowered by its load. Smeaton on Mills, p. 33. For further information on this subject, we must refer the reader to the original memoirs quoted above; in the first of which Deparcieux proves his point by reasoning, and in the second by experiment; or to Brisson's *Traité de Physique*, Tom. 1. p. 306. Edit. 3. where there is a general view of Deparcieux's experiments.

PLATE XXXIX. the buckets; for when the velocity of the wheel is great, the water, receding from the centre, is thrown out of the buckets, and they are emptied sooner than they would have been, had the wheel moved with less velocity.

In the memoirs of the academy of Berlin for 1775, M. Lambert has published a dissertation on the theory of overshot mills; but does not seem to be in the least degree acquainted with the improvements which have been made upon them in this country. He supposes the buckets to have the form *GFfg*, so that about one quarter only of the circumference of the wheel is filled with water. Notwithstanding these circumstances, however, the following table, computed from his formulæ, may be of considerable advantage to the mill-wright.

Fig. 4.

PLATE XXXVIII.

TABLE FOR OVERSHOT MILLS.

Height of the fall, reckoning from the surface of the stream.	Radius of the wheel, reckoning from the extremity of the buckets.	Width of the buckets.	Depth of the buckets.	Velocity of the wheel per second.	Time in which the wheel performs one revolution.	Turns of the mill-stone for one of the wheel.	Force of the water upon the buckets.	The length of *mn* in Fig. 1.	The length of *no* in Fig. 1.	Quantity of water required per second to turn the wheel.
Feet.	Ft. Dec.	Ft. Dec.	Ft. Dec.	Ft. Dec.	Sec. Dec.		lbs. Avoir.	Ft. Dec.	Ft. Dec.	Cub. Ft.
7	2.83	1.00	2.02	5.27	3.38	8.45	636	0.33	1.15	10.55
8	3.22	1.14	1.44	5.63	3.61	9.02	595	0.38	1.32	9.23
9	3.63	1.27	1.07	5.94	3.83	9.57	565	0.42	1.48	8.21
10	4.04	0.43	0.82	6.30	4.04	10.10	531	0.48	1.65	7.38
11	4.45	0.57	0.65	6.60	4.23	10.57	511	0.52	1.81	6.71
12	4.86	0.71	0.52	6.89	4.42	11.05	486	0.57	1.98	6.15
1	2	3	4	5	6	7	8	9	10	11

M. Le Chevalier de Borda, in his excellent memoir on water-wheels,* has shown, that overshot wheels will produce a maximum effect, when their diameter is equal to the greatest height of the fall, when the water enters the buckets on a level with the surface of the reservoir or canal, and when the velocity of the wheel is infinitely small. But, though the greatest possible effect can be produced, only when these conditions are observed, yet a small deviation from them, which is absolutely necessary in practice, does not greatly diminish their performance. If, for example, we suppose the waterfall to be 12 feet, and the diameter of the wheel only 11 feet, so that the water falls through the space of one foot before it enters the buckets; and, if we suppose, also, that 25 degrees of the semi-circumference of the wheel are unloaded, while the remaining 155 degrees are filled with water; then, when the wheel has a velocity of four feet per second, the maximum effect is diminished only $\frac{1}{12}$th; and, if the velocity be augmented to six feet per second, the diminution amounts only to one-tenth of the greatest possible effect.

In practice, however, a fall of two or three inches is sufficient; so that, if the wheel, in the preceding example, had been made 11 feet 9 inches, the diminution of effect would have been still more inconsiderable.

In comparing the relative effects of water-wheels, the Chevalier de Borda observes, that overshot wheels will raise through the height of the fall, a quantity of water equal to that by

* Memoires de l'Acad. Roy. Par. 1767, 4to. p. 286.

which they are driven; that undershot vertical wheels will produce only $\frac{3}{8}$ of this effect; that horizontal wheels will produce a little less than one half of it when the floatboards are plane, and a little more than one half when the floatboards are of a curvilineal form.

MECHANICS.

Account of an Improvement in Flour-Mills.

IN the generality of flour-mills in Scotland and England, a considerable quantity of manual labour is necessary before the wheat is converted into flour. When the grain is ground and conveyed into the trough from the millstones, it is afterwards put into bags and raised to the top of the mill-house, to be laid into the cooling boxes or beaches, from which it is conveyed into the bolting machine, to be separated from the bran or husk. This manual labour may be saved, by adopting an improvement for which we are indebted to the ingenuity of the American mill-wrights. A large screw is placed horizontally in the trough which receives the flour from the millstones. The thread or spiral line of the screw is composed of pieces of wood about 2 inches broad, and 3 long, fixed into a wooden cylinder 7 or 8 feet in length, which forms the axis of the screw. When the screw is turned round this axis, it forces the meal from one end of the trough to the other, where it falls into another trough, from which it is raised to the top of the mill-house by means of elevators, a piece of machinery similar to the chain-pump. These elevators consist of a chain of buckets or concave vessels like large teacups, fixed at proper distances upon a leathern band, which goes round two wheels, one of which is placed at the top of the mill-house, and the

other at the bottom, in the meal-trough. When the wheels are put in motion, the band revolves, and the buckets, dipping into the meal-trough, convey the flour to the upper story, where they discharge their contents. The band of buckets is inclosed in two square boxes, in order to keep them clean, preserve them from injury, and especially to prevent any waste of flour.*

* The inventor of the above patent improvement is the ingenious *Oliver Evans*, of Philadelphia; from whom the editor of this edition has obtained the following brief description.

"The grain is emptied from the waggon into a spout projecting through the wall of the mill-house, by which it descends into a *hopper* hung to a scale-beam: and, when weighed, a gate is drawn to let it descend into the *grain-elevator*, which raises it to the upper story, and deposits it in a garner over the rolling screen (or in any other store-room), from this it descends through the machinery adapted to clean it, and is deposited in a garner over the millstones, from which it descends, is ground, and the meal falls into the *conveyor*; whence it is conveyed from the several pairs of millstones, as ground, into the *meal-elevator*, which raises it to the *cooling-floor* and deposits it within the reach of the *hopper-boy*; this spreads it over the surface of a large circle, gathering it at the same time towards the centre, in a gradual spiral manner, stirring it continually, exposing it to the air to cool and dry, and supplying the *bolting-hopper* very regularly. That part of the flour which requires to be bolted over again, to improve its quality, is raised by another elevator, and likewise deposited within reach of the hopper-boy, which mixes it with the meal coming from the millstones: and thus the bolting is completed at one operation; and the flour being brought to its best quality, is deposited in the *packing-chest*. The whole is done with the machinery moved by the water, and without manual labour, from the time the grain is emptied from bags in the waggon, until it is completely manufactured into superfine flour, of the best quality, that can be made of the grain; and (if the machinery be properly constructed) without waste of either grain or meal. A mill furnished with these improvements, can be attended by less than half the usual number of millers, and is much better prepared for carrying on large and extensive business.

This machinery, if properly constructed, will be long found to be more durable, and less subject to want repair than the other moving parts of a flour-mill. For a full description of the whole, with directions for constructing every part thereof, see the Millwright's and Miller's Guide, a work published by the inventor —A. Ed.

MECHANICS.

On the Formation of the Teeth of Wheels and the Leaves of Pinions.

THOUGH nothing is more essential to the perfection of machinery than the proper formation of the teeth of wheels, and those parts of engines by which their force and velocity are conveyed to other parts; yet no branch of mechanical science has been more overlooked by the speculative and practical mechanics of this country. In vain do we search our systems of experimental philosophy for information on this point. Their authors seem either to be unacquainted with the subject, or to reckon it beneath their notice; though it has engaged the attention of De la Hire, Camus, and other foreign academicians, who have written very ingenious dissertations on the formation of the teeth of wheels. It is in the memoirs, indeed, of these philosophers, that all our knowledge upon this subject is contained, if we except a few general remarks by the learned Dr. Robison.*

* Two ingenious memoirs have also been written upon this subject by A. G. Kaestner, entitled, *De Dentibus Rotarum*, and published in the Commentationes Reg. Soc. Gottingensis, Vol. iv. and v. 1781, &c. The celebrated Euler has also treated this subject with great ability in the *Nov. Comment. Petropol.* 1754, 1755, tom. v.

It would be easy to shew, did the nature of this work permit it, that when one wheel drives another, it is not driven with a uniform force and velocity; or, in other words, the one wheel will act sometimes with greater, and sometimes with less force, and the other will move sometimes with a greater, and sometimes with a less velocity, unless the teeth of one or of both the wheels be parts of a curve, generated after the manner of an epicycloid,* by the revolution of another curve along the convex or concave side of a circle. It will be sufficient for our present purpose to show, that, when one wheel impels another, by the action of epicycloidal teeth, the movements of these wheels will be equal. Let the wheel *B* drive the wheel *A*, by the action of the epicycloidal teeth *mn*, *mn*, &c. upon the infinitely small pins or spindles *a*, *b*, *c*, and let the epicycloids *mn*, &c. be generated by the circumference *cba*, moving over the circumference m'' m' m. It is evident from the formation of the epicycloid, that the arch *ab* is equal to the arch m m', and the arch *ac* to m m''; for, when the part b m' of the epicycloid m' n' is forming, every point of the arch *ab* is applied to every point of the arch m m'; and the same may be said of the arch *ac*. Since, then, the wheels *B* and *A*, that is, the power and the weight, move through equal spaces in equal times, equal weights acting in opposite

PLATE XXXVII. Fig. 7.

* Under curves of this description are comprehended those which are formed by evolving the circumferences of circles, for it is demonstrable that these involutes are epicycloids, the centres of whose generating circles are infinitely distant.

directions at the points *a* and *m*, will be in equilibrio: but, as the power of the wheel *B* must always be greater than the resistance of the wheel *A*, which is put in motion, this power will, during the whole of the action, have the same relation to the resistance which it overcomes, and the one wheel will impel the other with a uniform force and velocity.

For the discovery of this property of the epicycloid, which Dr. Robison erroneously ascribes to De la Hire, or Dr. Hook, we are indebted to the Danish astronomer *Olaus Roemer*, the discoverer of the progressive motion of light; and Wolfius, upon whose authority this fact is stated,* laments that the mechanics of his time did not avail themselves of the discovery.

In order to ensure a uniformity of pressure and velocity in the action of one wheel upon another, it is not necessary that the teeth either of one or both wheels should be exactly epicycloids. If the teeth of one of them be either circular or triangular, with plane sides, or like a triangle, with its sides converging to the wheel's centre, or, in short, of any other form, this uniformity of force and motion will be attained, provided that the teeth of the other wheel have a figure which is compounded of that of an epicycloid, and the figure of the

* Ex eodem fonte *Olaus Roemerus*, cum Parisiis commoraretur, quamvis non sine subsidio Geometriæ sublimioris, deduxit figuram dentium in rotis epicycloidalem esse debere: id quod post eum quoque ostendit *Philippus de la Hire;* sed quod dolendum hactenus in praxin recepta non est. Wolfii Opera Mathematica, Tom. 1. p. 684. The same fact is stated by Libnitz in the Miscellan. Berolinens. 1710, p. 315.

teeth of the first wheel.* But, as it is often difficult to describe this compound curve, and sometimes impossible to discover its nature, we shall endeavour to select such a form for the teeth as may be easily described by the practical mechanic, while it insures a uniformity of pressure and velocity. But, in order to avoid circumlocution and obscurity, we shall call the small wheel (which is supposed always to be driven by a greater one) the *pinion*, and its teeth the *leaves* of the pinion. The line which joins the centres of the wheel and pinion is called the *line of centres.* Now there are three different ways in which the teeth of one wheel may act upon the teeth of another; and each of these modes of action requires a different form for the teeth.

I. When the teeth of the wheel begin to act upon the leaves of the pinion just as they arrive at the line of centres; and when their mutual action is carried on after they have passed this line.

II. When the teeth of the wheel begin to act upon the leaves of the pinion, before they arrive at the line of centres, and conduct them either to this line, or a very little beyond it.

III. When the teeth of the wheel begin to act upon the leaves of the pinion, before they arrive at the line of the centres, and continue to act after they have passed this line.

* M. de la Hire has shown, in a variety of cases, how to find this compound curve.

PLATE XXXVIII. I. The first of these modes of action is recommended by Camus and De la Hire, the latter of whom has investigated the form of the teeth solely for this particular case. It is represented in Fig. 2. where *B* is the centre of the wheel, *A* the centre of the pinion, and *AB* the line of centres. It is evident from the figure, that the part *b* of the tooth *ab* of the wheel, does not begin to act on the leaf *m* of the pinion, till they arrive at the line of centres *AB*; and that all the action is carried on after they have passed this line, and is completed when the leaf *m* comes into the situation *n*. When this mode of action is adopted, the acting faces of the leaves of the pinion should be parts of an *interior epicycloid* generated by a circle of any diameter rolling upon the concave superficies of the pinion, or within the circle *adh*; and the acting faces *ab* of the teeth of the wheel should be portions of an *exterior epicycloid* formed by the *same* generating circle rolling upon the convex superficies *odp* of the wheel.

Fig. 2.

Now, it is demonstrable, that when one circle rolls within another, whose diameter is double that of the rolling circle, the line generated by any point of the latter will be a *straight line*, tending to the centre of the larger circle. If the generating circle, therefore, mentioned above, should be taken with its diameter equal to the radius of the pinion, and be made to roll upon the concave superficies *adh* of the pinion, it will generate a straight line tending to the pinion's centre, which will be the form of the acting faces of its leaves; and the teeth of the wheel will, in this case, be exterior epicycloids, formed by a generating circle, whose diameter is equal

to the radius of the pinion, rolling upon the convex superficies *odp* of the wheel. This form of the teeth, viz. when the acting faces of the pinion's leaves are right lines tending to its centre, is exhibited in Fig. 3. and is, perhaps, the most advantageous, as it requires less trouble, and may be executed with greater accuracy than if the epicycloidal form had been employed: it is recommended both by De la Hire and Camus as particularly advantageous in clock and watch work. PLATE XXXVIII. Fig. 3.

The attentive reader will perceive from Fig. 2. that in order to prevent the teeth of the wheel from acting upon the leaves of the pinion, before they reach the line of centres *AB*; and that one tooth of the wheel may not quit the leaf of the pinion till the succeeding tooth begins to act upon the succeeding leaf, there must be a certain proportion between the number of leaves in the pinion, and the number of teeth in the wheel, or between the radius of the pinion and the radius of the wheel, when the distance of the leaves *AB* is given. But in machinery the number of leaves and teeth are always known from the velocity which is required at the working point of the machine: it becomes a matter, therefore, of great importance, to determine with accuracy the relative radii of the wheel and pinion.*

For this purpose, let *A*, Fig. 3. be the pinion having the acting faces of its leaves straight lines tending to the centre, and *B* the centre of the wheel, *AB* will be the distance of their

* A very ingenious Proportion-Compass has been invented by M. le Cerf, watchmaker at Geneva, for finding the relative diameters of wheels and pinions. It is described at length in the Phil. Trans. v. 68, p. 950.

centres. Then, as the tooth C is supposed not to act upon the leaf Am till it arrives at the line AB, it ought not to quit Am till the following tooth F has reached the line AB. But, since the tooth always acts in the direction of a line drawn perpendicular to the face of the leaf Am, from the point of contact, the line CH, drawn at right angles to the face of the leaf Am, will determine the extremity of the tooth CD, or the last part of it, which should act upon the leaf Am, and will also mark out CD for the depth of the tooth. Now, in order to find AH, HB, and CB, put a for the number of teeth in the wheel, b for the number of leaves in the pinion, c for the distance of the pivots A and B, and let x represent the radius of the wheel, and y that of the pinion. Then, since the circumference of the wheel is to the circumference of the pinion, as the number of teeth in the one to the number of leaves in the other, and, as the circumferences of circles are proportional to their radii, we will have $a : b = x : y$, then, by composition, (Eucl. 5. 18.) $a+b : b = c : y$ (c being equal to $x+y$), and, consequently, the radius of the pinion, viz. $y = \frac{cb}{a+b}$; then, by inverting the first analogy, we have $b : a = y : x$, and, consequently, the radius of the wheel, viz. $x = \frac{ay}{b}$; y being now a known number.

Now, in the triangle AHC, right angled at C, the side AH is known, and, likewise, all the angles (HAC being equal to $\frac{360}{b}$) the side AC,

therefore, can be easily found by plane trigonometry. Then, in the oblique-angled triangle *ACB*, the angle *CAB*, equal to *HAC*, is known, and also the two sides *AB*, *AC*, which contain it; the third side, therefore, viz. *CB*, may be determined; from which *DB*, equal to *HB*, already found, being subtracted, there will remain *CD* for the depth of the teeth. When the action is carried on after the line of centres, it often happens that the teeth will not work in the hollows of the leaves. In order to remedy this, the angle *CBH* must always be greater than half the angle *HBP*. The angle *HBP* is equal to 360 degrees, divided by the number of teeth in the wheel, and *CBH* is easily found by plane trigonometry.

Instead of pinions or small wheels, the millwrights in this country frequently substitute lanterns or trundles, which consist of cylindrical staves, fixed at both ends into two round pieces of board. From the use of trundles, however, Dr. Robison discourages the practical mechanic, when he observes, 'that De la Hire 'justly condemned the common practice of 'making the small wheel or pinion in the form 'of a lantern,' and that, when 'the teeth of the 'large wheel take a deep hold of the cylindri- 'cal pins of the trundle, the line of action is so 'disadvantageously placed, that the one wheel 'has scarcely any tendency to turn the other.'* It is with the greatest deference to such an able philosopher as Dr. Robison, that we presume to contradict this statement both with respect to the fact which is asserted, and the principle which is maintained. In no part of De la Hire's dis-

* The same observation is made in Imison's Elements of Science.

PLATE XXXVIII. sertation upon this subject does he condemn the use of lanterns. On the contrary, he actually *demonstrates*, that when the teeth of the great wheel are formed in a particular manner, and drive a small wheel whose teeth are cylindrical pins, the pressure and angular velocity of the one wheel will be equal to the pressure and angular velocity of the other; or, in other words, their action will be uniform. To this form of the teeth of the great wheel, when those of the small wheel are cylindrical, we shall now direct the reader's attention, and we earnestly recommend it to the notice of the practical mechanic, because it furnishes us with a method of removing, or, at least, of greatly diminishing, the friction which arises from the mutual action of the teeth.

Fig. 4. Let *A* be the centre of the pinion or small wheel *TCH*, whose teeth are circular like *ICR*, having their centres in the circle *PDE*. Upon *B*, the centre of the large wheel, at the distances *BC*, *BD*, describe the circles *FCK*, *GDO;* and with *PDE*, as a generating circle, form the exterior epicycloid *DNM*, by rolling it upon the convex superficies of the circle *GDO*. The epicycloid *DNM* thus formed, would have been the proper form for the teeth of the large wheel *GDO*, had the circular teeth of the small wheel been infinitely small; but as their diameter must be considerable, the teeth of the wheel should have another form. In order to determine their proper figure, divide the epicycloid *DNM* into a number of equal parts, 1, 2, 3, 4, &c. as shown in the figure, and let these divisions be as small as possible. Then, upon the points 1, 2, 3, &c. as centres, with the distance *DC*, equal to the radius of the circular tooth, de-

scribe portions of circles similar to those in the figure; and the curve *OPT*, which touches these circles, and is parallel to the epicycloid *DNM*, will be the proper form for the teeth of the large wheel. PLATE XXXVIII.

In order that the teeth may not act upon each other till they reach the line of centres *AB*, the curve *OP* should not touch the circular tooth *ICR*, till the point *O* has arrived at *D*. The tooth *OP*, therefore, will commence its action upon the circular tooth at the point *I*, where it is cut by the circle *DRE*. On this account, the part *ICR* of the cylindrical pin being superfluous, may be cut off, and the teeth of the small wheel will be segments of circles similar to the shaded parts of the figure.

If the teeth of wheels and the leaves of pinions be formed according to the directions already given, they will act upon each other, not only with uniform force, but also without friction. The one tooth rolls upon the other, and neither slides nor rubs to such a degree as to retard the wheels, or wear their teeth. But as it is impossible in practice to give that perfect curvature to the acting faces of the teeth which theory requires, a certain quantity of friction will remain after every precaution has been taken in the formation of the communicating parts. This friction may be removed, or, at least, greatly diminished, in the following manner:

If, instead of fixing the circular teeth, as in Fig. 4. to the wheel *DRE*, they may be made to move upon axles or spindles fixed in the circumference of the wheel, all the friction will be taken away except that which arises from the motion of the cylindrical tooth upon its axis.

PLATE XXXIX. The advantages which attend this mode of construction are many and obvious. The cylindrical teeth may be formed by a turning lathe with the greatest accuracy; the curve required for the teeth of the large wheel is easily traced; the pressure and motion of the wheels will be uniform; and the teeth are not subject to wear, because whatever friction remains is almost wholly removed by the revolution of the cylindrical spokes about their axes. The reader will also observe, that this improvement may be most easily introduced, when the small wheel has the form of a trundle or lantern; and that it may be adopted in cases where lanterns could not be conveniently used. Fig. 3. In Fig. 3. is represented the manner by which cylindrical teeth, moveable upon their axes, may be inserted in the circumference of wheels, *B* is the part of the wheel on which the tooth is to be fixed; *A* is the cylindrical tooth which moves upon its axis *bc* made of iron, whose extremities run in bushes of brass, fixed in the projecting pieces of wood *b*, *c*. This improvement, however, can only be adopted where the machinery is large. For small works, the teeth of the pinion or small wheel should be rectilineal, and those of the large wheel epicycloidal.

II. Having hitherto supposed, that the mutual action of the teeth does not commence till they arrive at the line of centres, let us now attend to the form which must be given them, when the whole of the action is carried on before they reach the line of centres, or when it is completed a very little below this line. This mode of action is not so advantageous as that which we have been considering, and should,

if possible, always be avoided. It is represented in Fig. 1. where *A* is the centre of the pinion, *B* that of the wheel, and *AB* the line of centres. It is evident from the figure, that the tooth *C* of the wheel acts upon the leaf *D* of the pinion before they arrive at the line *BA*; that it quits the leaf when they reach this line, and have assumed the position of *E* and *F*; and that the tooth *C* works deeper and deeper between the leaves of the pinion the nearer it comes to the line of centres. From this last circumstance a considerable quantity of friction arises, because the tooth *C* does not, as before, roll upon the leaf *D*, but slides upon it; and, from the same cause, the pinion soon becomes foul, as the dust which lies upon the acting faces of the leaves is pushed into the hollows between them. One advantage, however, attends this mode of action, for it allows us to make the teeth of the large wheel rectilineal, and thus renders the labour of the mechanic less, and the accuracy of his work greater, than if they had been of a curvilineal form. If the teeth *C*, *E*, therefore, of the wheel *BC*, be made rectilineal, having their surfaces directed to the wheel's centre, the acting faces of the leaves *D*, *F*, &c. must be epicycloids formed by a generating circle, whose diameter is equal to the radius *Bo* of the circle *op*, rolling upon the circumference *mn* of the pinion *A*. But if the teeth of the wheel and the leaves of the pinion be made curvilineal as in the figure, the acting faces of the teeth of the wheel must be portions of an interior epicycloid formed by any generating circle rolling within the concave superficies of the circle *op*, and the acting faces of the pinion's leaves must be portions of an

PLATE XXXIX. Fig. 1.

PLATE XXXVIII. exterior epicycloid produced by rolling the same generating circle upon the convex circumference *mn* of the pinion.

When the teeth of the large wheel are cylindrical spindles, either fixed or moveable upon their axes, an exterior epicycloid must be formed like *DNM* in Fig. 4. by a generating circle whose radius is *AC*, rolling upon the convex circumference *FCK*, *AC* being in this case the diameter of the wheel, and *FCK* the circumference of the pinion. By means of this epicycloid a curve *OPT* must be formed as before described, which will be the proper curvature for the acting faces of the leaves of the pinion, when the teeth of the wheel are cylindrical. The relative diameter of the wheel and pinion, when the number of teeth in each is known, may be found by the same formulæ which were given for the first mode of action, with this difference only, that in this case the radius of the wheel is reckoned from its centre to the extremity of its teeth, and the radius of the pinion from its centre to the bottom of its leaves.

III. The third way in which one wheel may drive another, is when the action is partly carried on before the acting teeth arrive at the line of centres, and partly after they have passed this line.

PLATE XXXIX. Fig. 2. This mode of action, which is represented in Fig. 2. is a combination of the two first modes, and consequently partakes of the advantages and disadvantages of each. It is evident from the figure that the portion *eh* of the tooth acts upon the part *bc* of the leaf till they reach the line of centres *AB*, and that the part *ed* of the tooth acts upon the portion *ba* of the leaf after they have passed this line. It follows, therefore, that

the acting parts *eh* and *bc* must be formed according to the directions given for the first mode of action, and that the remaining parts *ed*, *ba*, must have that curvature which the second mode of action requires; consequently, *eh* should be part of an interior epicycloid formed by any generating circle rolling on the concave circumference *mn* of the wheel, and the corresponding part *bc* of the leaf should be part of an exterior epicycloid formed by the same generating circle rolling upon *bEO*, the convex circumference of the pinion: the remaining part *cd* of the tooth should be a portion of an exterior epicycloid, formed by any generating circle rolling upon *eL*, the concave superficies of the wheel; and the corresponding part *ba* of the leaf should be part of an interior epicycloid described by the same generating circle, rolling along the concave side *bEO* of the pinion. As it would be extremely troublesome, however, to give this double curvature to the acting faces of the teeth, it will be proper to use a generating circle, whose diameter is equal to the radius of the wheel *BC*, for describing the interior epicycloid *eh* and the exterior one *bc*, and a generating circle, whose diameter is equal to *AC*, the radius of the pinion, for describing the interior epicycloid *ba*, and the exterior one *ed*. In this case the two interior epicycloids *eh*, *ba*, will be straight lines tending to the centres *B* and *A*,* and the labour of the mechanic will by this means be greatly abridged.

In order to find the relative diameters of the wheel and pinion, when the number of teeth in

* Traite des Epicycloides, par M D. De La Hire. Prop. V.

the one and the number of leaves in the other are given, and when the distance of their centres is also given, and the ratio of ES to CS, let a be the number of teeth in the wheel, b the number of leaves in the pinion, c the distance of the pivots A, B, and let m be to n as ES to CS, then the arch ES, or the angle SAE, will be equal to $\frac{360^\circ}{b}$, and LD, or the angle LBD, will be equal to $\frac{360^\circ}{a}$. But as $ES : CS = m : n$; consequently, $LD : LC = m : n$, therefore, (Euc. 6. 16.) $LC \times m = LD \times n$, and $LC = \frac{LD \times n}{m}$; but LD is equal to $\frac{360}{a}$, therefore, by substituting this in its stead, we have $LC = \frac{360 \times n}{am}$.

Now, in the triangle APB, AB is known, and also PB, which is the cosine of the angle ABD, PC being perpendicular to DB, AP, therefore, which is the radius of the pinion, may be found by plane trigonometry. The reader will observe, that the point P marks out the parts of the tooth D and the leaf SP, where they commence their action; and the point I marks out the parts where their mutual action ceases;* AP, therefore, is the proper

* The letter L marks the intersection of the line BL with the arch em, and the letter E the intersection of the arch bO with the upper surface of the leaf m. The letters D and S correspond with L and E respectively, and P with I.

radius of the pinion, and BI the proper radius of the wheel, the parts of the tooth L without the point I, and of the leaf SP without the point P being superfluous. Now, to find BI, we have $ES : CS = m : n$, consequently, (Euc. 6, 16.) $CS \times m = ES \times n$, and $CS = \frac{ES \times n}{m}$, but ES was formerly shown to be equal to $\frac{360}{b}$, therefore, by substitution, $CS = \frac{360 \times n}{bm}$. Now the arch ES, or angle EAS, being equal to $\frac{360}{b}$, and CS, or the angle CAS, being equal to $\frac{360 \times n}{bm}$, their difference EC, or the angle EAC, will be equal to $\frac{360}{b} - \frac{360 \times n}{bm}$. By subtracting we have $\frac{360mb - 360bn}{bbm}$, and dividing by b, gives $\frac{360m - 360n}{bm}$, or $\frac{360° \times \overline{m - n}}{bm}$. The angle EAC being thus found, the triangle EAB. or IAB, which is almost equal to it, is known. because AB is given, and likewise AI, which is equal to the cosine of the angle IAB, AC being radius, and AIC being a right angle, consequently IB the radius of the wheel may be found by trigonometry. It was formerly shown,* that AC, the radius of what is called the primitive opinion, was equal to $\frac{cb}{a+b}$, and that BC, the radius of the primitive wheel, was equal to $\frac{AC \times a}{b}$. If, then, we subtract AC or

* See page 208.

the one and the number of leaves in the other are given, and when the distance of their centres is also given, and the ratio of *ES* to *CS*, let *a* be the number of teeth in the wheel, *b* the number of leaves in the pinion, *c* the distance of the pivots *A*, *B*, and let *m* be to *n* as *ES* to *CS*, then the arch *ES*, or the angle *SAE*, will be equal to $\frac{360°}{b}$, and *LD*, or the angle *LBD*, will be equal to $\frac{360°}{a}$. But as $ES : CS = m : n$; consequently, $LD : LC = m : n$, therefore, (Euc. 6. 16.) $LC \times m = LD \times n$, and $LC = \frac{LD \times n}{m}$; but *LD* is equal to $\frac{360}{a}$, therefore, by substituting this in its stead, we have $LC = \frac{360 \times n}{am}$.

Now, in the triangle *APB*, *AB* is known, and also *PB*, which is the cosine of the angle *ABD*, *FC* being perpendicular to *DB*, *AP*, therefore, which is the radius of the pinion, may be found by plane trigonometry. The reader will observe, that the point *P* marks out the parts of the tooth *D* and the leaf *SP*, where they commence their action; and the point *I* marks out the parts where their mutual action ceases;* *AP*, therefore, is the proper

* The letter *L* marks the intersection of the line *BL* with the arch *cm*, and the letter *E* the intersection of the arch *bO* with the upper surface of the leaf *m*. The letters *D* and *S* correspond with *L* and *E* respectively, and *P* with *I*.

radius of the pinion, and BI the proper radius of the wheel, the parts of the tooth L without the point I, and of the leaf SP without the point P being superfluous. Now, to find BI, we have $ES : CS = m : n$, consequently, (Euc. 6, 16.) $CS \times m = ES \times n$, and $CS = \frac{ES \times n}{m}$, but ES was formerly shown to be equal to $\frac{360}{b}$, therefore, by substitution, $CS = \frac{360 \times n}{bm}$. Now the arch ES, or angle EAS, being equal to $\frac{360}{b}$, and CS, or the angle CAS, being equal to $\frac{360 \times n}{bm}$, their difference EC, or the angle EAC, will be equal to $\frac{360}{b} - \frac{360 \times n}{bm}$. By subtracting we have $\frac{360mb - 360bn}{bbm}$, and dividing by b, gives $\frac{360m - 360n}{bm}$, or $\frac{360^\circ \times m - n}{bm}$. The angle EAC being thus found, the triangle EAB, or IAB, which is almost equal to it, is known, because AB is given, and likewise AI, which is equal to the cosine of the angle IAB, AC being radius, and AIC being a right angle, consequently IB the radius of the wheel may be found by trigonometry. It was formerly shown,* that AC, the radius of what is called the primitive opinion, was equal to $\frac{cb}{a+b}$, and that BC, the radius of the primitive wheel, was equal to $\frac{AC \times a}{b}$. If, then, we subtract AC or

* See page 208.

rate method of generating involutes than by unlapping a thread from the circumference of the *evolute* or circle to be evolved.

Thus have we endeavoured to lay before our readers all the information which we have upon this important subject; and we trust it will be candidly received, as it is the only essay on the subject which has appeared in our language.* The demonstrations of the propositions have been purposely left out, as being rather foreign to the object of a practical work. To the mechanic, they are of no consequence; and the mathematician can either demonstrate it himself, or have recourse to the original memoirs of Camus and De la Hire. Before leaving this subject, however, it may be proper to show how interior and exterior epicycloids may be formed, and how the teeth should be disposed on the circumference of the wheel.

* In a book entitled, *Imison's Elements of Science and Art*, which professes to be a second edition of *Imison's School of Arts*, there are some practical directions for the formation of the teeth of wheels, but they are so defective in principle that they cannot be trusted. The author seems merely to have heard that the acting faces of the teeth should be epicycloidal, but to have been totally ignorant whether the epicycloids should be exterior or interior, and what should be their bases and generating circles. The directions which this author gives for forming the teeth of a rack, and the lifting cogs or cams, of forge-hammers, are equally destitute of scientific principle.

On the formation of exterior and interior epicycloids, and on the disposition of the teeth on the wheel's circumference.

Nothing can be of greater importance to the practical mechanic, than to have a method of drawing epicycloids with facility and accuracy; the following, we trust, is the most simple mechanical method that has yet been devised.— Take a piece of plain wood *GH*, and fix upon it another piece of wood *E*, having its circumference *mb* of the same curvature as the circular base upon which the generating circle *AB* is to roll. When the generating circle is large, the shaded segment *B* will be sufficient: in any part of the circumference of this segment, fix a sharp-pointed nail *a*, sloping in such a manner that the distance of its point from the centre of the circle may be exactly equal to its radius; and fasten to the board *GH* a piece of thin brass, or copper, or tin-plate *ab*, distinguished by the dotted lines. Place the segment *B* in such a position that the point of the nail *a* may be upon the point *b*, and roll the segment towards *G*, so that the nail *a* may rise gradually, and the point of contact between the two circular segments may advance towards *m*; the curve *ab*, described upon the brass plate, will be an accurate exterior epicycloid. Remove, with a file, the part of the brass on the left hand of the epicycloid, and the remaining concave arch or gage *ab* will be a pattern-tooth, by means of which all the rest may be easily formed. When an *interior epicycloid* is wanted, the concave side of its circular base must be used. The method of describing it is represented in Fig. 3. where *CD* is the generating circle, *F* the concave circular base, *MN* PLATE XI. Fig. 2.

the piece of wood on which this base is fixed, and *cd* the interior epicycloid formed upon the plate of brass, by rolling the generating circle *C*, or the generating segment *D*, towards the right hand. The *cycloid*, which is useful in forming the teeth of *rack-work*, is generated precisely in the same manner, with this difference only, that the base on which the generating circle rolls must be a straight line.

Although, in general, it is necessary to give the proper curvature only to one side of the teeth, yet it may be proper to form both sides with equal care, that the wheels may be able to move in a retrograde direction. This is particularly necessary when a reciprocating power is employed. In the case of a mill moving by the force of a single stroke steam-engine, the direction of the pressure on the communicating parts of the machinery is changed twice every stroke. During the working stroke, the wheels which convey the motion from the beam to the machinery, are acting with one side of their teeth; but, during the returning stroke, the wheels act with the other side of their teeth.*

In order that the teeth may not embarrass one another before their action commences, and that one tooth may begin to act upon its corresponding leaf of the pinion, before the preceding tooth has ceased to act upon the preceding leaf, the height, breadth, and distance of the teeth must be properly proportioned. For this purpose, the pitch-line or circumference of the wheel, which is represented in Plate XXXIX. Fig. 2 & 3. by the dotted arches, must be divided into as many equal spaces as the number of teeth which the

* See Dr. Robison's Treatise on *Machinery* in the Supplement to the Encyclopædia Britannica, v. ii, p. 104. § 38.

wheel is to carry. Divide each of these spaces into 16 equal parts; allow seven of these for the greatest breadth of the tooth, and 9 for the distance between them. When the wheel drives a trundle, each space should be divided into seven equal parts, three of these being allotted for the thickness of the tooth, and $3\frac{2}{3}$ for the diameter of the cylindrical stave of the trundle.* If each of the spaces already mentioned, or if the distance between the centres of each tooth, be divided into 3 equal parts, the height of the teeth must be equal to two of these.† These distances and heights, however, vary according to the mode of action which is employed.‡

On the formation of cycloids and epicycloids geometrically by means of points, and the method of drawing lines parallel to them.

As the preceding mechanical method of forming epicycloidal curves, may be regarded by some as too difficult in practice and too liable to error, we shall point out a geometrical method of describing epicycloids by means of points, and a more accurate way of drawing lines parallel to them than that which is described in the preceding pages, and represented in Fig. 4. of Plate XXXVIII.

Let the radius *AB*, Fig. 5. Plate XXXIX, of the large wheel be called a, and the radius *BC* of the lesser one, or generating circle, be called b, and let the variable quantity x be equal to PLATE XXXIX. Fig. 5.

* Imison allows 3 parts for the thickness of the tooth, and 4 for the diameter of the stave. But it is evident that in this case the staves of the wallower would stick between the teeth of the wheel.

† Some make the height of the teeth almost equal to the distance between the centres of each, but this can be determined only by the method formerly stated. See p. 121.

‡ See Wolfii Opera Mathematica, tom. i, p. 696-7.

PLATE XXXIX. Fig. 5.

$\frac{a}{b}\times z$, z being any number of degrees taken at pleasure, and equal to the variable angle *BAO*. Then, having drawn the chord *BO*, we will have *ABO*, or $AOB=90°-\frac{z}{2}$; the chord $BO=2a\times$ sin. $\frac{z}{2}$; and $AOD=90°\times\frac{x}{2}$. Whence $BOD=\frac{x+z}{2}$; and $OD=2b+\text{sin.}\frac{x}{2}$. The line *OD* being thus determined, we have one point *D* of the epicycloid *BD*. If the angle *BAO*, or the variable quantity z be gradually diminished, and *OD* determined anew, we will have other points of the epicycloid between *D* and *B*: or, if z be increased, other points of the epicycloid beyond *D* will be determined. Since a very small arch of any curve may be represented by the arch of a circle equicurve to it in the same point, we may describe a small portion of the epicycloid at *D*, with a radius equal to $\frac{a+b}{a+2b}\times 2OD$. This radius being reckoned from *D*, or the line *DO*, which is perpendicular to the epicycloid at *D*, will give the centre from which the elementary arc at *D* may be described. In finding the different points *D*, *d*, of the epicycloid *BD*, we determine at the same time the lines *DO*, *dO*, perpendicular to the epicycloid in the respective points *D*, *d*; hence it will be an easy matter to draw a curve parallel to the epicycloid *BD* at any given distance. Thus, let *M* be the given distance, then take the line *M* in the compasses, and set it from *D* to *F* on the perpendicular *DF*, and also from *d* to *f*, on *df*, and so on for the other points. A number of points *F*, *f*, &c. will, therefore, be determined, through

which we can describe the curve *EfFG*, which will be parallel to the epicycloid *BD*, and distant from it by the given quantity *M*.

In order to illustrate this method by an example, let *AB*, the radius of the large wheel, be 42.991 inches, and $BC=25.7946=AB\times 0.6$; then $a:b$ as $10:6$. Let us suppose $z=12°$. Then $x=\frac{10}{6}\times 12$, or $x=20°$; consequently $\frac{z}{2}=6°$; $BOD=16°$. Since BO is equal to $2a\times\sin.\frac{z}{2}$ we will have

$$\text{Logarithm } 2\,a = 1.9344123$$
$$\text{Log. Sine } \frac{z}{2} \text{ or } 6°=9.0192346$$

Therefore, $BD=8.9876$ 0.9536469 Log.

In order to find $OD=2b\times\sin.\frac{x}{2}$ we have

$$\text{Logarithm } 2\,b = 1.7135636$$
$$\text{Log. Sine } \frac{x}{2} \text{ or } 10°=9.2396702$$

Therefore, $OD=8.9584$ 0.9522338

The radius of curvature at the point *D*, consequently, will be $=\frac{16}{11}\times OD$ when $z=12°$* that is, the radius of curvature will be 13.030.

* The radius of curvature being always $\frac{a+b}{a+2b}\times 2OD$, it will be equal, in the present example, to $\frac{10+6}{10+12}\times 2OD$, or $\frac{16}{22}\times 2OD$, or $\frac{16}{11}\times OD$.

PLATE XXXIX. If z be successively diminished to 6°, 4°, and 2°, we will have the results contained in the following table, which are found in the same way as when $z=12°$.

z	x	EBO	BOD	BO	OD	Radius of Curvature.
2°	3° 20′	1°	2° 40′	1.5006	1.5004	2.1824
4	6 40	2	5 20	3.0007	2.9996	4.3631
6	10 0	3	8 0	4.5000	4.4962	6.5400
12	12 0	6	16 0	8.9876	8.9584	13.0300

By means of this table, 4 points of the epicycloid may be found. Make the angle $BAO=12°$, $BOD=16$, and $OD=8.9584$, which will determine the point D; and so on with the rest.

As it would be extremely difficult to project the wheels C and A upon paper, when they are very large, we shall show how to describe the epicycloid without using the centres C and A. Draw BE perpendicular to the line CA that joins the centres of the wheels, and make the angle EBO equal to one half of z, viz. 6 degrees. Make BO, as before found, equal to 8.9876; the angle $BOD=16°$, and $OD=8.9584$, and the point D will be determined when the line CA is only given in position.

Fig. 6. In the cycloid let the line Bo, Fig. 6, be equal to $b\times z$, b being the radius of the generating circle c, and z any number of degrees taken at pleasure. Then $DO=2b\times\text{Sine}\,\frac{z}{2}$ and $DOB=\frac{z}{2}$. From D let fall the perpendicular DK, and let $DK=y$ and $BK=x$; then $Do\times\text{sine}\,\frac{z}{2}=DK$ or $y=2b\times\text{sine}\,\frac{z}{2}^2=b\times\text{versed sine}\,z=b\times\overline{1-\cos. z}$. Likewise we have $KO=2b\times\cos.\frac{z}{2}\times\text{sine}\,\frac{z}{2}=b\times\text{sine}\,z$.

Whence *BK'* or $x=b\times\overline{z\text{—sine } z}$. Wherefore *BK'* and *DK'* being thus found, the point *D* in the cycloid will be determined; and, by diminishing z continually, we will have other points of the cycloid between *D* and *B*, and, by increasing it, we will have points beyond *D*.

To illustrate this by an example, let $b=1$ and $z=120°=180°—60°$, then, since $b=1$ we will have $y=$versed sine $120°=2—\overline{\text{versed sine } 60°}$ $=1.500$. To find x, which is $b\times\overline{z\text{—sine } z}$, or, in the present case$=z$—sine z, since b is equal to 1. The arch z or 120° being $\frac{1}{3}$ of the circumference of a circle whose radius is 1, and whose circumference is 3.1415927×2, or 6.2831854 will be equal to 2.0943951, and the sine of 120° or its supplement 60° is 0.8660254. Therefore,

$$\begin{array}{r} 120°=2.0943951 \\ \text{sine } 120°=0.8660254 \\ \hline x=1.2283697 \end{array}$$

If z be made 5°, x will be$=0.0001108$, and $y=0.0038053$. The numbers x and y being thus determined, we have only to make *BK'* equal to x and *K'D* to y in order to determine the point *D*. It may be proper to observe, that the variable number z should be taken pretty small both for the cycloid and the epicycloid, as it is only a little portion of these curves that is required for the teeth of wheels; and when several points of the curve are determined, the intervening space may be made arches of a circle equicurve to the epicycloid at the same point.*

* See Kæstner's Memoir *de Dentibus Rotarum*, in the Commentationes Societatis Regiæ Gottingensis 1782, vol. v, pp. 9, 24.

greater than the number of teeth in the wheel fixed upon *CD*, and their radii must have the same proportion. Draw *cd* parallel to *CD* at any convenient distance, and draw *ab* parallel to *AB* at four times that distance, then the lines *im* and *in* drawn perpendicular to *AB* and *CD* respectively, will mark the situation and size of the wheels required. In this case the cones are *Oni* and *Omi*; and *srni*, *rpmi*, are the portions of them that are employed. Plate XLVIII.

The formation of the teeth of bevelled wheels is more difficult than one would at first imagine, and no author, so far as I know, has attempted to direct the labours of the mechanic on this point. The teeth of such wheels, indeed, must be formed by the same rules which we have given for other wheels; but since different parts of the same tooth are at different distances from the axis, these parts must have the curvature of their acting surfaces proportioned to that distance. Thus, in Fig. 10, the part of the tooth at *r* must be more incurvated than the part at *i*, as is evident from the inspection of Fig. 8. and the epicycloid for the part *i* must be formed by means of circles whose diameters are *ni* and *im*, while the epicycloid for the part *r* must be generated by circles, whose diameters are *sr* and *rp*.

Let us suppose a plane to pass through the points *OBD*; the lines *OB*, *OD*, will evidently be in this plane, which may be called the *plane of centres*. Now, when the teeth of the wheel *pi*, which is supposed to drive *si*, the smallest of the two, commence their action on the teeth of *si*, when they arrive at the plane of centres, and continue their action after they have passed this plane, the curve given to the teeth of *si* at *i*,

On the nature of bevelled wheels, and the method of giving an epicycloidal form to their teeth.

Plate XLVIII. The principle of bevelled wheels was pointed out by De la Hire, so long ago as the end of the 17th century.* It consists in one fluted or toothed cone acting upon another, as is represented in Fig. 8. where the cone *OD* drives the cone *OC*, conveying its motion in the direction *OC*. If these cones be cut parallel to their bases, as at *A* and *B*, and if the two small cones between *AB* and *O* be removed, the remaining parts *AC* and *BD* may be considered as two bevelled wheels, and *BD* will act upon *AC* in the very same manner, and with the same effect that the whole cone *OD* acted upon the whole cone *OC*; and if the section be made nearer the bases of the cones, the same effect will be produced. This is the case in Fig. 9, where *CD* and *DE* are but very small portions of the imaginary cones *ACD* and *ADE*.

In order to convey motion in any given direction, and determine the relative size and situation of the wheels for this purpose, let *AB*, Fig. 10. be the axis of a wheel, and *CD* the given direction in which it is required to convey the motion by means of a wheel fixed upon the axis *AB*, and acting upon another wheel fixed on the axis *CD*, and let us suppose that the axis *CD* must have four times the velocity of *AB*, or must perform four revolutions while *AB* performs one. Then the number of teeth in the wheel fixed upon *AB* must be four times

* Traite de Mecanique prop. 66, published in the Memoires de l'Academie Royale, &c. depuis 1666 jusquà 1699, tom. ix.

greater than the number of teeth in the wheel fixed upon *CD*, and their radii must have the same proportion. Draw *cd* parallel to *CD* at any convenient distance, and draw *ab* parallel to *AB* at four times that distance, then the lines *im* and *in* drawn perpendicular to *AB* and *CD* respectively, will mark the situation and size of the wheels required. In this case the cones are *Oni* and *Omi;* and *srni*, *rpmi*, are the portions of them that are employed. PLATE XLVIII.

The formation of the teeth of bevelled wheels is more difficult than one would at first imagine, and no author, so far as I know, has attempted to direct the labours of the mechanic on this point. The teeth of such wheels, indeed, must be formed by the same rules which we have given for other wheels; but since different parts of the same tooth are at different distances from the axis, these parts must have the curvature of their acting surfaces proportioned to that distance. Thus, in Fig. 10, the part of the tooth at *r* must be more incurvated than the part at *i*, as is evident from the inspection of Fig. 8. and the epicycloid for the part *i* must be formed by means of circles whose diameters are *ni* and *im*, while the epicycloid for the part *r* must be generated by circles, whose diameters are *sr* and *rp*.

Let us suppose a plane to pass through the points *OBD;* the lines *OB*, *OD*, will evidently be in this plane, which may be called the *plane of centres*. Now, when the teeth of the wheel *pi*, which is supposed to drive *si*, the smallest of the two, commence their action on the teeth of *si*, when they arrive at the plane of centres, and continue their action after they have passed this plane, the curve given to the teeth of *si* at *i*,

PLATE XLVIII.

should be a portion of an interior epicycloid formed by any generating circle rolling on the concave superficies of a circle whose diameter is *ni*, and the curvature of the teeth at *r* should be part of a similar epicycloid, formed upon a circle, whose diameter is *sr*. The curvature of the teeth of the wheel *pi* at *i*, should be part of an exterior epicycloid formed by the same generating circle rolling upon the concave circumference of a circle whose diameter is *mi*; and the epicycloid for the teeth at *r* is formed in the same way, only instead of *mi*, the diameter of the circle must be *pr*. When any other mode of action is adopted, the teeth are to be formed in the same manner that we have pointed out for common wheels, with this difference only, that different epicycloids are necessary for the parts *i* and *r*. It may be sufficient, however, to find the form of the teeth at *i*, as the remaining part of the tooth may be shaped by directing a straight rule from different points of the epicycloid at *i* to the centre *O*, and filing the tooth till every part of its acting surface coincides with the side of the rule. The reason of this operation will be obvious, by attending to the shape of the tooth in Fig. 8. When the small wheel *si* impels the large one *pi*, the epicycloids which were formerly given to *si* must be given to *pi*, and those which were given to *pi* must be transferred to *si*.*

* A method of bevelling wheels with a simple instrument invented by Mr. James Kelly of New Lanark cotton mills, may be seen in the Repertory of Arts, vol. vi, p. 106. This instrument, founded on the equality of vertical angles, is so very simple that it must have occurred to mechanics of common ingenuity.

MECHANICS.

On the formation of the Teeth of a Rackwork, the Arms of Levers, the Wipers of Stampers, and the lifting Cogs or Cams of Forge-Hammers.

THE teeth of a wheel may act upon those of a rack, according to the three different ways in which one wheel acts upon another; and each of these modes of action requires a different form for the communicating parts. PLATE XL.

From what has been said in the preceding dissertation, it would be easy to deduce the proper form for the teeth of a rackwork, merely by considering the rack as part of a wheel whose centre is infinitely distant. But, as the epicycloids are in this case converted into other curves, which have different names, and are generated in a different manner, it may be proper, for the sake of the practical mechanic, to add a few observations illustrative of the subject.*

In Fig. 4. let *AB* be the wheel which is employed to elevate the rack *C*, and let their mutual action not commence till the acting teeth Fig. 4.

* Since this article was written, I have seen a paper by Kæstner on the same subject, in the *Novi Commentarii Sac. Reg. Gotting.* 1771, *tom. 2*, p 117; but it contains nothing new, excepting a method of describing involutes by means of points.

PLATE XL. have reached the line of centres *AC*. In this case, *C* becomes as it were the pinion or wheel driven, and the acting faces of its teeth must be *interior epicycloids*, formed by any generating circle rolling within the circumference *pq*; but as *pq* is a straight line, these interior epicycloids will be *cycloids*, or *trochoids*, as they are sometimes called. which are curves generated by a point in the circumference of a circle, rolling upon a straight line or plane surface. The acting face *op*, therefore, will be part of a *cycloid* formed by any generating circle, and *mn*, the acting face of the teeth of the wheel, must be an *exterior* epicycloid produced by the same generating circle rolling on *mr* the convex surface of the wheel.* If it be required to make *op* a straight line, as in the figure, then *mn* must be an *involute* of the circle *mr*, formed in the manner represented in Fig. 1.

Fig. 4. likewise represents a wheel depressing the rack *c* when the third mode of action is used, viz. when the action commences above the line of centres and is carried on below this line. In this case also *c* becomes the pinion, and *DE* the wheel; *eh*, therefore, must be part of an interior epicycloid formed by any generating circle rolling on the concave side *ex* of the wheel, and *bc* must be an exterior epicycloid produced by the same generating circle rolling upon the circumference of the rack.—The remaining part *cd* of the teeth of the wheel, must be an exterior epicycloid described by any generating circle moving upon the convex side *ex*, and *ba* must be an interior epicy-

* See page 206.

cloid formed by the same generating circle rolling within the circumference of the rack. But as the circumference of the rack is in this case a straight line, the exterior epicycloid *bc* and the interior one *ba* will be cycloids formed by the same generating circles which are employed in describing the other epicycloids. Since it would be difficult, however, as has already been remarked, to give this compound curvature to the teeth of the wheel and rack, we may use a generating circle whose diameter is equal to *Dx*, the radius of the wheel for describing the interior epicycloid *eh*, and the exterior one *bc*; and a generating circle whose diameter is equal to the radius of the rack for describing the interior epicycloid *ab*, and the exterior one *de*; *ab* and *eh*, therefore, will be straight lines, *bc* will be a cycloid, and *de* an involute of the circle *ex*, the radius of the rack being infinitely great.

In the same manner may the form of the teeth of rackwork be determined, when the second mode of action is employed, and when the teeth of the wheel or rack are circular or rectilineal. But if the rack be part of a circle, it must have the same form for its teeth as that of a wheel of the same diameter with the circle of which it is a part.

In machinery, where large weights are to be raised, such as fulling-mills, mills for pounding, &c. or where large pistons are to be elevated by the arms of levers, it is of the greatest consequence that the power should raise the weight with a uniform force and velocity; and this can be effected only by giving a proper form to the wipers or communicating parts. A certain class of mechanics generally excuse themselves for

PLATE XL. not attending to the proper form of the teeth of wheels, by alleging that the scientific form differs but little from their's, and that teeth, however badly formed, will, in the course of time, work into their proper shape. This excuse, however, will not apologize for their negligence in the present case. The scientific form of the wipers of stampers, and the arms of levers, are so widely different from the form which is generally assigned them, as to increase very much the performance of the machine, and preserve its parts from that injury which is always occasioned by the want of a uniform motion.

Now there are two cases in which this uniformity of motion may be required, and each of these demands a different form for the communicating parts. 1. When the weight is to be raised perpendicularly, as the piston of a pump, &c. 2. When the weight to be raised or depressed moves upon a centre, and rises or falls in the arch of a circle, such as the sledge-hammer in a forge, the stampers in a fulling-mill, &c.

Fig. 5. 1. In Fig. 5. of Plate XL, let *AB* be an axis driven by a water-wheel or any other power, at right angles, to which is fixed the bar *mm*, on whose extremities the wipers *mn mn* are fastened. The wiper *mn* acting upon the arm *PE*, raises the piston or weight *EF* to the required height. The piston then falls, and is again raised by the lower wiper. We have represented in the figure only one piston, but it often happens that two or three are to be employed, and in this case the axis *AB* must carry four or six wipers, which should be so distributed upon its circumference, that when one piston is about to fall, the other may begin to rise. Now, in

order that these pistons may be raised with a uniform motion, the form of the wiper *mn* must be the evolute of a circle whose diameter is *mm;* or, in other words, it must be an epicycloid, formed by a generating circle, whose centre is infinitely distant, rolling upon the convex circumference of another circle whose diameter is *mm.* But as a small roller *P,* is frequently fixed to the extremity of the arm *E,* to diminish the friction of the working parts, we must draw a curve within the above-mentioned involute, and parallel to it, the distance between them being equal to the radius of the roller;* and this new curve will be the proper form for the wiper *mn* when a roller is employed.

The piston *EF* may also be raised or depressed uniformly, by giving a proper curvature to the arm *PE,* and fixing the roller upon the extremities of the bar *mm.* Thus, in Fig. 5. let *CD* be an axis, moved by any power, in which are fixed the arms *DH, MR,* having rollers *HR* at their extremities, which act upon the curved arm *op.* When the piston *EF* is raised to the proper height by the action of the roller *H* upon *op,* it then falls, and is again elevated by the arm *M.* In order that its motion may be uniform, the arm *op* must be part of a cycloid, the radius of whose generating circle is equal to the length of the arm *DH,* reckoning from its extremity *H,* or the centre of the roller, to the centre of the axle *DC.* But when a roller is fixed upon the extremity *H,* we must draw a curve parallel to the cycloid, and without it, at the distance of

* The method of doing this is shown in Plate XXXVIII, Fig. 4. See pages 210, 211.

PLATE XL. the roller's semi-diameter; and this curve will be the proper form for the arm *op*. It is evident that when this mode of raising the piston is adopted, the arm *DH* must be bent as in the figure, otherwise the extremity *p* would prevent the roller *H* from acting upon the arm *op*.

Fig. 6. In Fig. 6. we have another method of raising a weight perpendicularly with a uniform motion. Let *AH* be a wheel moved by any power which is sufficient to raise the weight *MN* by its extremity *O*, from *O* to *e*, in the same time that the wheel moves round one fourth of its circumference, it is required to fix upon its rim a wing *OBCDEH*, which shall produce this effect with a uniform effort. Divide the quadrant *OH* into any number of equal parts *Om mn*, &c. the more the better, and *oe* into the same number *ob*, *bc*, *cd*, &c. and through the points *m*, *n*, *p*, *H*, draw the indefinite lines *AB*, *AC*, *AD*, *AE*, and make *AB* equal to *Ab*, *AC* to *Ac*, *AD* to *Ad*, and *AE* to *Ae*; then, through the points *O*, *B*, *C*, *D*, *E*, draw the curve *OBCDE*, which is a portion of the spiral of Archimedes, and will be the proper form for the wiper or wing *OHE*.* It is evident, that, when the point *m* has arrived at *O*, the extremity of the weight will have arrived at *b*; because *AB* is equal to *Ab*, and for the same reason when the points *n*, *p*, *H*, have successively arrived at *O*, the extremity of the weight will have arrived at the corresponding points *c*, *d*, *e*. The motion, therefore, will be uniform, because the space described by the weight is proportional to the space described by the moving power, *Ob* being to *Oc*, as *Om* to *On*. If it be required

* For a different way of forming this spiral, see Wolfii Opera Mathematica, tom. i, p. 399.

to raise the weight *MN* with an accelerated or retarded motion, we have only to divide the line *Oe*, according to the law of acceleration or retardation, and divide the curve *OBCDE* as before. Plate XLI.

2. When the lever moves upon a centre, the weight will rise in the arch of a circle, and consequently a new form must be given to the wipers or wings. The celebrated Deparcieux, of the Academy of Sciences of Paris, has given an ingenious and simple method of tracing mechanically the curves which are necessary for this purpose. Though this method was published about 50 years ago in the Memoirs of the Academy, it does not seem to be at all known to the mechanics of this country. We shall, therefore, lay it before the reader in as abridged and simplied a form as the nature of the subject will permit. Let *AB* be a lever lying horizontally, which it is required to raise uniformly through the arch *BC* into the position *AC*, by means of the wheel *BFH* furnished with the wing *BNOP*, which acts upon the extremity *C* of the lever; and let it be required to raise it through *BC* in the same time that the wheel *BFH* moves through one half of its circumference; that is, while the point *M* moves to *B* in the direction *MFB*. Divide the chord *CB* into any number of equal parts, the more the better, in the points 1, 2, 3, and draw the lines 1*a*, 2*b*, 3*c*, parallel to *AB*, or a horizontal line passing through the point *B*, and meeting the arch *CB* in the points *a*, *b*, *c*. Draw the lines *CD*, *aD*, *bD*, *cD*, and *BD*, cutting the circle *BFH* in the points *m*, *n*, *o*, *p*. Fig. 1.

Having drawn the diameter *BM*, divide the semicircle *BFM* into as many equal parts as

the chord *CB*, in the points *q*, *s*, *u*. Take *Bm* and set it from *q* to *r*: take *Bn* and set it from *s* to *t*: take *Bo* and set it from *u* to *v*: and, lastly, set *Bp* from *M* to *E*. Through the points *r*, *t*, *v*, *E*, draw the indefinite lines *DN*, *DO*, *DP*, *DQ*, and make *DN* equal to *Dc*; *DO* equal to *Db*; *DP* equal to *Da*; and *DQ* equal to *DC*. Then, through the points *Q*,*P*,*O*,*N*,*B*, draw the spiral *B*,*N*,*O*,*P*,*Q*, which will be the proper form for the wing of the wheel, when it moves in the direction *EMB*.

That the spiral *BNO*, will raise the lever *AC*, with a uniform motion, by acting upon its extremity *c*, will appear from the slightest attention to the construction of the figure. It is evident, that when the point *q* arrives at *B*, the point *r* will be in *m*, because *Bm* is equal to *qr*, and the point *N* will be at *c*, because *DN* is equal to *Dc*; the extremity of the lever, therefore, will be found in the point *c*, having moved through *Bc*. In like manner, when the point *s* has arrived at *B*, the point *t* will be at *n*, and the point *O* in *b*, where the extremity of the lever will now be found; and so on with the rest, till the point *M* has arrived at *B*.— The point *E* will then be in *p*, and the point *Q* in *C*; so that the lever will now have the position *AC*, having moved through the equal heights *Bc*, *cb*, *ba*, *ac*,* in the same time that the power has moved through the equal spaces *qB*, *sq*, *us*, *Mu*. The lever, therefore, has been raised uniformly, the ratio between the velocity

* The arches *Bc*, *cb*, &c. are not equal; but the perpendiculars let fall from the points, *c*, *a*, *b*, &c. upon the horizontal lines, passing through *a*, *b*, &c. are equal, being proportional to the equal lines, *c*1, 1, 2. Eucl. VI, 2.

of the power, and that of the weight, remaining always the same.

If the wheel *D* turns in a contrary direction, according to the letters *MHB*, we must divide the semicircle *BHEM*, into as many equal parts as the chord *cB*, viz. in the points *e*, *g*, *h*. Then, having set the arch *Bm*, from *e* to *d*, the arch *Bn*, from *g* to *f*, and the rest in a similar manner, draw through the points *d*, *f*, *h*, *E*, the indefinite lines *DR*, *DS*, *DT*, *DQ*; make *DR* equal to *Dc*; *DS* equal to *Db*; *DT* equal to *Da*; and *DQ* equal to *DC*; and, through the points *B*, *R*, *S*, *T*, *Q*, describe the spiral *BRSTQ*, which will be the proper form for the wing, when the wheel turns in the direction *MEB*. For, when the point *e* arrives at *B*, the point *d* will be in *m*, and *R* in *c*, where the extremity of the lever will now be found, having moved through *Bc* in the same time that the power, or wheel, has moved through the division *eB*. In the same manner it may be shown, that the lever will rise through the equal heights *cb*, *ba*, *aC*, in the same time that the power moves through the corresponding spaces *eg*, *gi*, *iM*. The motion of the lever, therefore, and also that of the power, are always uniform. Of all the positions that can be given to the point *B*, the most disadvantageous are those which are nearest the points *F*, *H*; and the most advantageous position is when the chord *Bc* is vertical, and passes, when prolonged, through *D*, the centre of the circle.* In this particular case, the two curves have equal bases, though they differ a little in point

* In the figure we have taken the point *B* in a disadvantageous position, because the intersections are in this case more distinct.

of curvature. The farther that the centre *A* is distant, the nearer do these curves resemble each other; and if it were infinitely distant, they would be exactly similar, and would be the spirals of Archimedes, as the extremity *c*, would, in this case, rise perpendicularly.

The intelligent reader will easily perceive, that 4, 6, or 8 wings may be placed upon the circumference of the circle, and may be formed by dividing into the same number of equal parts as the chord *BC*, $\frac{1}{4}$, $\frac{1}{6}$, or $\frac{1}{8}$ of the circumference, instead of the semicircle *BFM*.

That the wing *BNO* may not act upon any part of the lever between *A* and *C*, the arm *AC* should be bent; and that the friction may be diminished as much as possible, a roller should be fixed upon its extremity *C*. When a roller is used, however, a curve must always be drawn parallel to the spiral described according to the preceding method, the distance between it and the spiral being every where equal to the radius of the roller.

When two or more wings are placed upon the circumference of the wheel, it has been the custom of practical mechanics to make them portions of an ellipse, whose semi-transverse axis is equal to *QD*, the greatest distance of the curve from the centre of the circle. But it will appear, from a comparison of the elliptical arch, with the spiral *N*, that it will not produce a uniform motion.—If it should be required to raise the lever with an accelerated or retarded motion, we have only to divide the chord *BC*, according to the degree of retardation or acceleration required, and the circle into the same number of equal

parts as before, and then describe the curve according to the method already laid down. PLATE XLI.

As it is frequently more convenient to raise or depress weights by the extremity of a constant radius, furnished with a roller instead of wings fixed upon the periphery of a wheel, we shall now proceed to determine the curve which must be given to the arm of the lever, which is to be raised or depressed, in order that this elevation or depression may be effected with a uniform motion.

Let *AB* be a lever, which it is required to raise uniformly through the arch *BC*, into the position *AC*, by means of the arm or constant radius *DE*, moving upon *D* as a centre, in the same time that the extremity *E* describes the arch *EeF*. From the point *C* draw *CH* at right angles to *AB*, and divide it into any number of equal parts, suppose three, in the points 1, 2; and through the points 1, 2, draw 1*a*, 2*b*, parallel to the horizontal line *AB*, cutting the arch *CB* in the points *a*, *b*, through which draw *aA*, *bA*. Upon *D* as a centre, with the distance *DE*, describe the arch *EieF*, and upon *A* as a centre, with the distance *AD*, describe the arch *eOD*, cutting the arch *EieF* in the point *e*. Divide the arches *Eie* and *Fse*, each into the same number of equal parts as the perpendicular *cH*, in the points *k*, *i*, *s*, *m*, and through these points, about the centre *A*, describe the arches *kz*, *ig*, *qr*, *mn*. Take *zx* and set it from *k* to *l*, and take *gf* and set it from *i* to *h*. Take *rq* also and set it from *s* to *t*, and set *nm* from *o* to *p*, and *dc* from *e* to *O*. Then through the points *E*, *l*, *h*, *O*, and *O*, *t*, *p*, *F*, draw the two curves *ElhO*, and *OtpF*, which Fig. 2.

will be the proper form that must be given to the arm of the lever. If the handle *DE* move from *E* towards *F*, the curve *EO* must be used, but if in the contrary direction, we must employ the curve *OF*.

It is evident that when the extremity *E* of the handle *DE*, has run through the arch *Ek*, or rather *El*, the point *l* will be in *k*, and the point *z* in *x*, because *xz* is equal to *kl*, and the lever will have the position *Ab*. For the same reason, when the extremity *E* of the handle has arrived at *i*, the point *h* will be in *i*, and the point *g* in *f*, and the lever will be raised to the position *Aa*. Thus, it appears, that the motion of the power and the weight are always proportional. When a roller is fixed at *E*, a curve parallel to *EO*, or *OF*, must be drawn as formerly.

It is upon these principles that the detent levers of clocks, and those connected with the striking part, should be formed. In every machine, indeed, where weights are to be raised or depressed, either by variable or constant levers, its performance depends much on the proper form of the communicating parts.

Hitherto we have supposed, that the wheel which carries the wipers or wings, moves in the same plane with the lever or weight to be raised. Circumstances, however, often occur, which render it necessary to elevate the lever by means of a wheel moving at right angles to the plane in which the lever moves; and when this method is adopted, a different form must be given to the wipers. As no writer on mechanics, so far as I know, has treated upon this subject, it becomes the more necessary to supply the defect by a few observations.

Let ABC (Fig. 3.) be the lever which is to be raised round the axis AD, by the action of the wing mn of the wheel D, upon the roller C, fixed at the extremity of the lever;—it is required to find the form which must be given to the wiper mn. It is evident from Fig. 4. where CB is a section of the lever and roller, and BA the arch through which it is to be raised, that the breadth of the wiper must always be equal to mn or rB, the versed sine of the arch BA, through which the roller moves, so that the extremity n of the wiper may act upon the roller B at the commencement of the motion, and the other extremity m may act upon the roller A, when the lever arrives at the required position CA. It is easy to perceive, however, that if the acting surface mn of the wiper be always parallel to the horizon, or perpendicular to the radii of the wheel, or the plane in which it moves, it will act disadvantageously, except at the commencement of the motion, when mn is parallel to CB. For when mn has arrived at the position op, the extremity o will act upon the roller A, but in such an oblique and disadvantageous manner, that it will scarcely have any power to turn it upon its axis, or move the lever round the fulcrum C. The friction of the roller upon its axis, therefore, will increase, and the power of the wiper to turn the lever will diminish, in proportion to the length of the arch BA; and if CA arrive at a vertical position, the power of the wing will be solely employed in wrenching it from its fulcrum. Plate XLI. Fig. 3. Fig. 4.

In order to avoid this inconvenience, we must endeavour to give such a form to the wiper, that its acting surface may always be parallel to the lever, or axis of the roller, having the

PLATE XLII. position *mn* when the roller is at *B*, and the position *ob* when the roller is at *A*.

Having stated the peculiarities of this construction, let us now attend to the method by which the acting surface of the wiper must be formed. Since the lever *CB* is to be raised perpendicularly through the equal spaces *rc*, *ca*, *aA*, in equal times, the acting surface of the wiper must evidently be part of the spiral of Archimedes,* the method of describing which is shewn in Fig. 6. of Plate XL; but the difficulty lies in giving different degrees of inclination to the acting surface, in order that the part in contact with the roller may be parallel to the

Fig. 6. direction of the lever. Let *A*, Fig. 6. be the wheel which is to be furnished with wings, and let *Cb*, the perpendicular height through which the lever is to rise, be equal to *Ar* in Fi . 4. Divide the quadrant *Db* into any number of equal parts, the more the better, suppose three, in the points *c* and *r*, and describe the spiral of Archimedes, *DinC*, as formerly directed. Divide *Ar*, (Fig. 4.) the sine of the arch *BA*, into the same number of equal parts in the points *c*, *a*, and draw *af*, *cg*, parallel to *cB*, and cutting the circle in the points *d*, *e*, and the tangent *Bh* in the points *f*, *g*; and through the points *C* and *d* draw *Cki*. The line *df* is equal to the difference between radius and the cosine of the arch *d* ; *fi* is equal to the difference between the tangent and the sine of the same arch; *iB* being the tangent, and *fB* the sine of the arch *dB*, or angle *dcB*; *ad* is equal to *af*—*df*, or to the difference between *df* and the versed sine of the whole arch *AB*; and *ak* is equal to

* See page 236.

$\dfrac{fi \times da}{df}$, for, on account of the similar triangles, PLATE XLI.
dfi, dak, we have $df : fi = da : ak$*; and, consequently, $ak = \dfrac{fi \times da}{df}$.

Since then the points *r, c, a, A,* (in Fig. 4.) correspond respectively with the points *D, i, n, C,* of the spiral in Fig. 6. take *fi* and set it from *n* to *m,* and *ak* from *n* to *o;* take also *gh,* and set it from *i* to *h* (Fig. 6.); set *cq* from *i* to *k,* and make *cB* in Fig. 6. equal to *pb* in Fig. 4. or the difference between the tangent and sine of the arch *AB;* and, through the points *D, k, o, C,* and *D, h, m, B,* draw the curves *DoC, DmB,* which will be the proper form for the sides *ON, MP,* of the spiral wiper *MONP* (Fig. 5.); the acting surface *MONP* must then Fig. 5. be wrought in such a manner as to consist of a variety of planes, differently inclined to the plane *BON* of the wiper, the inclination being nothing at *O* and *M,* but increasing gradually till the inclination at *N, P,* becomes equal to the angle *DCE,* or *ACB,* in Fig. 4.† From the construction of Fig. 4. it is evident that the arches *Be, ed, dA,* are not equal, nor are they aliquot parts of *AB.* But since the arch *AB,* and its sine *Ar* are known, and since the sines of the other arches are known, viz. *bc, ba,* the arches themselves may be easily found by a table of natural sines.

* The lines *df, fi, ad, ak,* may also be found geometrically by making *AC* equal to the real length of the lever.

† The curves which must be employed in practice, should be curves drawn parallel to those formed by the preceding method, at the distance of the semi-diameter of the roller.

PLATE XLI. In Fig. 5. we have a perspective view of a wheel furnished with two wipers formed in this manner. *FC* and *LN* correspond with *bC* and *rA*, in Figs. 6. and 4. The curves *AnmC*, and *OP*, correspond with *DkoC* in Fig. 6. and *MP* with *DhmB*. The diagonal curve *MN* corresponds with the diagonal curve *DinC*, and *OM*, the breadth of the wiper, with *mn* or *rB*, the versed sine of the arch *AB*, in Fig. 4. The breadth *OM*, however, should always be a little greater than the versed sine of the arch through which the lever is to be raised, since *MN* is the path of the roller over the wiper's surface.

Having thus described the different methods of raising weights, whether perpendicularly or round a centre, with a uniform velocity and force: it would be unnecessary to apply the principles of construction to those machines which are formed for the elevation of weights. The practical mechanic can easily do this for himself. There is one case, however, which deserves peculiar attention, because the wipers, formed scientifically, will not produce the intended effect. This happens in the large sledge-hammer which is employed in forges. In Fig.
Fig. 7. 7. *BC* is a large hammer moved round *A* as a centre, by means of the wiper *MW* acting upon its extremity *AC*, or upon the roller *R*. The hammer must be tossed up with a sudden motion, so as to strike the elastic oaken spring *E*, which, being compressed, drives back the hammer with great force upon the anvil *D*. Now, if spiral wipers, constructed according to the directions already given, are employed, the hammer will indeed be raised equably without the least jolting, but it will rise no higher than the

wiper lifts it, and will therefore fall merely with its own weight. But if the wipers are constructed in the common way, and the hammer elevated with a motion greatly accelerated, it will rise much higher than the wiper lifts it—it will impinge against the oaken beam *E*, and be repelled with great effect against the iron on the anvil *D*. In any of the preceding constructions, this accelerated motion may be produced, merely by dividing *BL* (Plate XLI, Fig. 1 and 2.) according to the law of acceleration, and proceeding as already directed.

MECHANICS.

On the nature and construction of Wind-mills.

Description of a wind-mill.

PTALE XLII. THE limited and imperfect manner in which Mr. Ferguson has treated of wind-mills in the preceding volume, renders it necessary that the subject should now be prosecuted at greater length. The few observations which he has made upon these machines, presuppose that the reader is acquainted with their nature and construction; a species of knowledge which is not to be expected in the readers of a popular and elementary work. For the purpose of supplying this defect, and enabling the reader to understand the observations which may be made on the form and position of the sails, and on the relative advantages of horizontal and vertical wind-mills, we shall give a description of a wind-mill invented by Mr. James Verrier, containing several improvements on the common construction, for which the author was liberally rewarded by the Society of Arts.

Fig. 1. This machine is represented in Fig. 1. where *AAA* are the three principal posts, 27 feet $7\frac{1}{2}$ inches long, 22 inches broad at their lower extremities, 18 inches at their upper ends, and 17 inches thick. The column *B* is 12 feet

$2\frac{1}{2}$ inches long, 19 inches in diameter at its lower, and 16 inches at its upper extremity: it is fixed in the centre of the mill, passes through the first floor *E*, having its upper extremity secured by the bars *GG*. *EEE* are the girders of the first floor, one of which only is seen, being 8 feet 3 inches long, 11 inches broad, and 9 thick: they are mortised into the posts *AAA* and the column *B*, and are about 8 feet 3 inches distant from the ground floor. *DDD* are three posts 6 feet 4 inches long, 9 inches broad, and 6 inches thick: they are mortised into the girders *EF* of the first and second floor, at the distance of 2 feet 4 inches from the posts *A*, &c. *FFF* are the girders of the second floor, 6 feet long, 11 inches broad, and 9 thick: they are mortised into the posts *A*, &c. and rest upon the upper extremities of the post *D*, &c. The three bars *GGG* are 3 feet $1\frac{1}{2}$ inches long, 7 inches broad, and 3 thick: they are mortised into the posts *D* and the upper end of the column *B*, 4 feet 3 inches above the floor. *P* is one of the beams which support the extremities of the bray-trees or brayers: its length is 2 feet 4 inches, its breadth 8 inches, and its thickness 6 inches. *I* is one of the bray-trees into which the extremity of one of the bridge-trees *K* is mortised. Each bray-tree is 4 feet $9\frac{1}{2}$ inches long, $9\frac{1}{2}$ inches broad, and 7 thick; and each bridge-tree is 4 feet 6 inches long, 9 inches broad, and 7 thick, being furnished with a piece of brass on their upper surface to receive the under pivot of the millstones. *LL* are two iron screw-bolts, which raise or depress the extremities of the bray-trees. *MMM* are the three millstones, and *NNN* the iron spindles or arbors on which the turning millstones are fixed. *O* is one of three wheels

or trundles which are fixed on the upper ends of the spindles *NNN*: they are 16 inches in diameter, and each is furnished with 14 staves. *f* is one of the carriage-rails on which the upper pivot of the spindle turns, and is 4 feet 2 inches long, 7 inches broad, and 4 thick. It turns on an iron bolt at one end, and the other end slides in a bracket fixed to one of the joists, and forms a mortise in which a wedge is driven to set the rail and trundle in or out of work: *t* is the horizontal spur-wheel that impels the trundles; it is 5 feet 6 inches diameter, is fixed to the perpendicular shaft *T*, and is furnished with 42 teeth. The perpendicular shaft *T* is 9 feet 1 inch long, and 14 inches in diameter, having an iron spindle at each of its extremities, the under spindle turns in a brass block fixed into the higher end of the column *B*; and the upper spindle moves in a brass plate inserted into the lower surface of the carriage-rail *C*. The spur-wheel *r* is fixed on the upper end of the shaft *Z*, and is turned by the crown-wheel *v* on the windshaft *c*, it is 3 feet 2 inches in diameter, and is furnished with 15 cogs. The carriage-rail *C*, which is fixed on the sliding kerb *Z*, is 17 feet 2 inches long, 1 foot broad, and 9 inches thick. *YYQ* is the fixed kerb, 17 feet 3 inches diameter, 14 inches broad, and 10 thick, and is mortised into the posts *AAA*, and fastened with screw-bolts. The sliding kerb *Z* is of the same diameter and breadth as the fixed kerb, but its thickness is only 7½ inches. It revolves on 12 friction-rollers fixed on the upper surface of the kerb *YYQ*, and has 4 iron half-staples *Y*, *Y*, &c. fastened on its outer edge, whose perpendicular arms are 10 inches long, 2 inches broad, and 1 inch thick, and embrace the outer edge of the fixed kerb, to prevent the sliding one

from being blown off. The capsills *X*, *V*, are 13 feet 9 inches long, 14 inches broad, and 1 foot thick: they are fixed at each end with strong iron screw-bolts to the sliding kerb, and to the carriage-rail *C*. On the right hand of *w* is seen the extremity of a cross rail, which is fixed into the capsills *X* and *V* by strong iron bolts: *e* is a bracket 5 feet long, 16 inches broad, and 10 inches thick; it is bushed with a strong brass collar, in which the inferior spindle of the windshaft turns, and is fixed to the cross rail *w*: *b* is another bracket 7 feet long, 4 feet broad, and 10 inches thick; it is fixed into the fore ends of the capsills, and in order to embrace the collar of the windshaft, it is divided into two parts, which are fixed together with screw-bolts. The windshaft *c* is 15 feet long, 2 feet in diameter at the fore end, and 18 inches at the other: its pivot at the back end is 6 inches diameter; and the shaft is perforated to admit an iron rod to pass easily through it. The vertical crown-wheel *v* is 6 feet in diameter, and is furnished with 54 cogs, which drive the spur-wheel *r*. The bolster *d*, which is 6 feet 3 inches long, 13 inches broad, and half a foot thick, is fastened into the cross rail *w*, directly under the centre of the windshaft, having a brass pulley fixed at its fore end. On the upper surface of this bolster is a groove, in which the sliding bolt *R* moves, having a brass stud at its fore end. This sliding bolt is not distinctly seen in the figure, but the round top of the brass stud is visible below the letter *h*: the iron rod that passes through the windshaft bears against this brass stud. The sliding bolt is 4 feet 9 inches long, 9 inches broad, and $\frac{1}{3}$ of a foot thick. At its fore end is fixed a line which passes over the brass pulley in the bolster, and appears at *a* with

PLATE XLII. a weight attached to its extremity, sufficient to make the sails face the wind, that is strong enough for the number of stones employed; and, when the pressure of the wind is more than sufficient, the sails turn on an edge, and press back the sliding bolt, which prevents them from moving with too great velocity; and as soon as the wind abates, the sails, by the weight *a*, are pressed up to the wind till its force is sufficient to give the mill a proper degree of velocity. By this apparatus the wind is regulated and justly proportioned to the resistance or work to be performed; an uniformity of motion is also obtained, and the mill is less liable to be destroyed by the rapidity of its motion.

Fig. 2. That the reader may understand how these effects are produced, we have represented, in Fig. 2. the iron rod, and the arms which bear against the vanes; *ah* is the iron rod which passes through the windshaft *c* in Fig. 1.; *h* is the extremity which moves in the brass stud that is fixed upon the sliding bolt; *ai*, *ai*, &c. are the cross arms at right angles to *ah*, whose extremities *ii*, similarly marked in Fig. 1, bear upon the edges of the vanes. The arms *ai* are $6\frac{1}{2}$ feet long, reckoning from the centre *a*, 1 foot broad at the centre, and 5 inches thick; the arms *n*, *n*, &c. that carry the vanes or sails, are $18\frac{1}{2}$ feet long, their greatest breadth is 1 foot, and their thickness is 9 inches, gradually diminishing to their extremities, where they are only 3 inches in diameter. The 4 cardinal sails, *m*, *m*, *m*, *m*, are each 13 feet long, 8 feet broad at their outer ends, and 3 feet at their lower extremities; *p*, *p*, &c. are the 4 assistant sails, which have the same dimensions as the cardinal ones to which they are joined by the line *SSSS*.—

The angle of the sail's inclination, when first opposed to the wind, is 45 degrees, and regularly the same from end to end.

It is evident, from the preceding description of this machine, that the windshaft *c* moves along with the sails: the vertical crown-wheel *v* impels the spur-wheel *r*, fixed upon the axis *T*, which carries also the spur-wheel *t*. This wheel drives the three trundles *H*, one of which only is seen in the figure, which, being fixed upon the spindles *N*, &c. communicate motion to the turning millstones.

That the wind may act with the greatest efficacy upon the sails, the windshaft or principal axis must always have the same direction as the wind. But as this direction is perpetually changing, some apparatus is necessary for bringing the windshaft and sails into their proper position. This is sometimes effected by supporting the machinery on a strong vertical axis, whose pivot moves in a brass socket firmly fixed into the ground, so that the whole machine, by means of a lever, may be made to revolve upon this axis, and be properly adjusted to the direction of the wind. Most wind-mills, however, are furnished with a moveable roof, which revolves upon friction-rollers inserted in the fixed kerb of the mill; and the adjustment is effected by the assistance of a simple lever. As both these methods of adjusting the windshaft require human assistance, it would be very desirable that the same effect should be produced solely by the action of the wind. This may be done by fixing a large wooden vane or weathercock at the extremity of a long horizontal arm which lies in the same vertical plane with the windshaft. By this means, when the surface of the vane, and its

distance from the centre of motion are sufficiently great, a very gentle breeze will exert a sufficient force upon the vane to turn the machinery, and will always bring the sails and windshaft to their proper position. This weathercock, it is evident, may be applied either to machines which have a moveable roof, or which revolve upon a vertical arbor.

Prior to the French revolution, wind-mills were more numerous in Holland and the Netherlands than in any other part of the world, and there they seem to have been brought to a very high state of perfection. This is evident, not only from the experiments of Mr. Smeaton, from which it appears, that sails weathered in the Dutch manner, produced nearly a maximum effect, but also from the observations of the celebrated Coulomb. This philosopher examined above 50 wind-mills in the neighbourhood of Lisle, and found that each of them performed nearly the same quantity of work when the wind moved with the velocity of 18 or 20 feet per second, though there were some trifling differences in the inclination of their windshafts, and in the disposition of their sails. From this fact, Coulomb justly concluded that the parts of the machine must have been so disposed as to produce nearly a maximum effect.

In the wind-mills on which Coulomb's experiments were made, the distance from the extremity of each sail, to the centre of the windshaft or principal axis, was 33 feet. The sails were rectangular, and their width was a little more than 6 feet, five of which were formed with cloth stretched upon a frame, and the remaining foot consisted of a very light-board. The line which joined the board and the cloth formed, on the

side which faced the wind, an angle sensibly concave at the commencement of the sail, which diminished gradually till it vanished at its extremity. Though the surface of the cloth was curved, it may be regarded as composed of right lines perpendicular to the arm or whip which carries the frame, the extremities of these lines corresponding with the concave angle formed by the junction of the cloth and the board. Upon this supposition, these right lines at the commencement of the sail, which was distant about 6 feet from the centre of the windshaft, formed an angle of 60 degrees with the axis or windshaft, and the lines at the extremity of the wing formed an angle increasing from 78 to 84 degrees, according as the inclination of the axis of rotation to the horizon increased from 8 to 15 degrees; or, in other words, the greatest angle of weather was 30 degrees, and the least varied from 12 to 6 degrees, as the inclination of the windshaft varied from 8 to 15 degrees.* A pretty distinct idea of the surface of wind-mill sails may be conveyed by conceiving a number of triangles standing perpendicular to the horizon, in which the angle contained between the hypothenuse and the base is constantly diminishing: the hypothenuse of each triangle will then be in the superficies of the vane, and they would form that superficies, if their number were infinite.

* The *weather* of the sails is the angle which the surface of the sails forms with the plane of their motion, and is always equal to the complement of the angle which that surface forms with the axis.

On the form and position of wind-mill sails.

M. Parent seems to have been the first mathematician who considered the subject of wind-mill sails in a scientific manner. The philosophers of his time entertained such erroneous opinions upon this point, as to suppose that the surface of the sails should be equally inclined to the direction of the wind and the plane of their motion; or, what is the same thing, that the angle of weather should be 45 degrees.* But it appears from the investigations of Parent, that a maximum effect will be produced when the sails are inclined $54\frac{2}{3}$ degrees to the axis of rotation, or when the angle of weather is $35\frac{1}{3}$ degrees. In obtaining this conclusion, however, M. Parent has assumed data which are inadmissible, and has neglected several circumstances which must materially affect the result of his investigations. The angle of inclination assigned by Parent, is certainly the most efficacious for giving motion to the sails from a state of rest, and for preventing them from stopping when in motion; but he has not considered that the action of the wind upon a sail at rest, is different from its action upon a sail in motion: for, since the extremities of the sails move with greater rapidity than the parts nearer the centre, the angle of weather should be greater towards the centre than at the extremity, and should vary with the velocity of each part of the sail. The reason of this is very

* See Wolfii Opera Mathematica, tom. 1, p. 680, where this angle is recommended.

obvious. It has been demonstrated by Bossut,* and, after him, by Fabre,† and has been sufficiently established by experience, that, when any fluid acts upon a plane surface, the force of impulsion is always exerted most advantageously when the impelled surface is in a state of rest, and that this force diminishes as the velocity of the surface increases. Now, let us suppose, with Parent, that the most disadvantageous angle of weather for the sails of wind-mills is $35\frac{1}{3}$ degrees for that part of the sail which is nearest the centre of rotation, and that the sail has every where this angle of weather; then, since the extremity of the sail moves with the greatest velocity, it will, in a manner, withdraw itself from the action of the wind; or, to speak more properly, it will not receive the impulse of the wind so advantageously as those parts of the sail which have a less degree of velocity. In order, therefore, to make up for this diminution of force, we must make the wind act more perpendicularly upon the sail, by diminishing its obliquity; that is, we must increase its inclination to the axis or direction of the wind; or, what is the same thing, we must diminish its angle of weather. But, since the velocity of every part of the sail is proportional to its distance from the centre of motion, every elementary portion of it must have a different angle of weather diminishing from the centre to the extremity of the sail. The law or rate of diminution, however, is still to be discovered, and we are fortunately in possession of a theorem of M'Laurins, which determines this law of variation. Let *a* represent the velocity

* Traite d'Hydrodynamique, § 772.

† Sur les Machines Hydrauliques, p. 1, § 94, 95.

PLATE XLII. of the wind, and c the velocity of any given part of the sail, then the effort of the wind upon that part of the sail will be greatest when the tangent of the angle of the wind's incidence, or of the sail's inclination to the axis, is to radius, as $\sqrt{(2+\frac{9cc}{4aa})}+\frac{3c}{2a}$ to 1.

In order to appply this theorem, let us suppose that the radius or whip ms (Fig. 1.) of the sail $\alpha\beta\delta i$, is divided into 6 equal parts, that the point n is equidistant from m and s, and is the point of the sail which has the same velocity as the wind; then, in the preceding theorem, we will have $c=a$, when the sail is loaded to a maximum; and, therefore, the tangent of the angle, which the surface of the sail at n makes with the axis, when $a=1$, will be $\sqrt{(2+\frac{9}{4})}-\frac{3}{2}=3.561=$ tangent of 74° 19′, which gives 15° 41′ for the angle of weather at the point n. Since, at $\frac{1}{2}$ of the radius $c=a$, and since c is proportional to the distance of the corresponding part of the sail from the centre, we will have, at $\frac{1}{6}$ of the radius sm, $c=\frac{a}{3}$; at $\frac{2}{6}$ of the radius, $c=\frac{2a}{3}$; at $\frac{4}{6}$, $c=\frac{4a}{3}$; at $\frac{5}{6}$, $c=\frac{5a}{3}$; and, at the extremity of the radius, $c=2a$. By substituting these different values of c, instead of c in the theorem, and by making $a=1$, the following table will be obtained, which exhibits the angles of inclination and weather which must be given to different parts of the sails.

Parts of the radius from the centre of motion at s.	Velocity of the sail at these distances—or values of c.	Angle made with the axis.		Angle of weather.	
		Deg.	Min.	Deg.	Min.
$\frac{1}{6}$	$\frac{a}{3}$	63	26	26	34
$\frac{2}{6}$	$\frac{2a}{3}$	69	54	20	6
$\frac{3}{6}$ or $\frac{1}{2}$	a	74	19	15	41
$\frac{4}{6}$ or $\frac{2}{3}$	$\frac{4a}{3}$	77	20	12	40
$\frac{5}{6}$	$\frac{5a}{3}$	79	27	10	33
1	$2a$	81	0	9	0

Having thus pointed out an important error in Parent's theory, and shown how to find the law of variation in the angle of weather, we have further to observe, that, in order to simplify the calculus, Parent supposed the velocity of the wind to be infinite when compared with the velocity of the sail, and that its impulsion upon the sail was in the compound ratio of the square of its velocity and the square of the sine of incidence. The first of these suppositions is evidently inaccurate, as shown by Daniel Bernouilli, in his Hydrodynamique. With regard to the force of impulsion on the sails, the proposition is perfectly true in theory, and has been demonstrated by Pitot* and other philosophers; but it unquestionably appears, from the experiments presented to the French Academy in 1763, by M. le Chevalier de Borda, and from those made in 1776 by M. d'Alembert, the Marquis Con-

* Memoires de l'Acadamie, 1729, p. 540.

dorcet, and the Abbe Bossut,* that this proposition does not hold in practice. The first part of the proposition, indeed, that the force of impulsion is proportional to the square of the velocity of the surface impelled, is true in practice; but when the angles of incidence are small, the latter part of the proposition must be abandoned, as it would afford very false results.—In cases, however, where the angles of incidence are between 50 and 90 degrees, we may regard the impulsion as proportional to the square of the velocity multiplied by the square of the sine of incidence; but we must remember, that the force thus determined by the theory will be a little less than that which would be found by experiment, and that this difference increases, as the angle of incidence recedes from 90 degrees.

Such being the circumstances which Parent has overlooked in his investigations, we need not be surprised to find, from the experiments of Smeaton, that, when the angle which he recommends was adopted, the sails produced a smaller effect than when they were weathered in the common manner, or, according to the Dutch construction.†

The theory of wind-mills has been treated at great length by M. Euler, the most profound and celebrated mathematician of his time. He has shown, that the angle assigned by Parent

* Nouvelles Experiences sur la resistance des fluides, par M. M. d'Alembert, le Marquis de Condorcet, et l'Abbe Bossut. Chap. v. § 35.

† Mons. Belidor has fallen into the same error as Parent, and observes, that the workmen at Paris make the angle of weather 18 degrees, and thereby lose 2-7ths of the effect; whereas, this is nearly the most efficacious angle that can be adopted. See Architecture Hydraulique par Belidor Tom. ii. B. 3, pp. 33—41.

₎o small for a sail in motion, and that the
le of weather should vary with the velocity
he different parts of the sails: but like Pa-
, he has supposed that the force of impul-
upon surfaces, with different obliquities,
roportional to the square of the sines of
r inclination. As the angles of incidence,
ever, are sufficiently great, this circum-
ce will have but a trifling effect upon his
lusions. After Euler has shown in gene-
how to determine the force of impulsion
the sails, whatever be their figure and
osition, and whatever be the celerity of
motion; he then investigates by the me-
de maximis et minimis, what should be
nclination of the sails to the axis, and the
city of their extremities, in order to produce
ximum effect; and he finds, that this in-
tion and velocity are variable, and are in-
ely proportional to the momentum of fric-
in the machine. That the reader may ful-
nderstand this important result, we may
rk, that, in theory, the greatest effect will
roduced when the velocity of the sails is
itely great, and when their surfaces are
endicular to the wind's direction; that is,
the angle of weather is nothing. But
these suppositions are excluded in prac-
for, though the sails receive the greatest
ible impetus from the wind, when they are
ned 90 degrees to the axis, yet this force
not the smallest tendency to put them in
on; and it is not difficult to perceive, that
riction of the machine, and the resistance of
ir to the thickness of the sails, must always
the velocity of their motion. In this case,
ry does not accord with practice; but they

may be easily reconciled by making the angle of inclination 89 degrees instead of 90, and supposing the sails to perform a finite, but a very great number of revolutions in a second, a hundred for example. Then the sails, having still a very disdvantageous position, will receive but a small impetus from the wind, which may be called *one* pound. But this defect in the impelling power is made up by the great velocity of the sails; and since the effect is always equal to the product of the weight and the velocity, we will have $1\times100=100$ for the effect of the machine. Now, let us take friction into the account, and suppose it to be so great as to diminish the rapidity of the sails, from 100 to 50 turns in a second; then, in order that the machine may produce an effect equal to 100, as formerly, we must change the angle of the sail's inclination, till it receives from the wind an impetus equal to 2 pounds; for $2\times50=100$. If the friction be still further increased, the celerity of the machine will experience a proportional diminution, and the angle of inclination must undergo such a change, that the force of impulsion received from the wind may make up for the velocity that is lost by an increase of friction. From these observations it plainly appears, that the celerity of the sails, and their inclination to the axis, depend upon the momentum of friction, and as this is generally a constant quantity in machines, and can easily be determined experimentally, the position of the sails, the velocity of their motion, and the effect of the machine, may be found from the following table, which is calculated from the formulæ of Euler, and adapted to different degrees of friction.

In this table F denotes the force of the wind upon all the sails; d is the radius of the sail, or the distance of its extremity from the centre of the axis or windshaft; v is the velocity of the wind; and s the velocity of the sail's extremity, which is equal to the numbers contained in the fourth column.

TABLE,

Containing the Angle of Inclination and Weather of Wind-mill Sails, the Velocity of their Extremities, and the Effect of the Machine, for any Degree of Friction.

Momentum of friction.	Angle of the sail's inclination to the axis	Angle of weather.	Velocity of the sails at their extremities.	Effect of the machine.	Effect of the machine differently expressed.
2.235702 *Fd*	45°	45°	0.000000 *v*	0.000000 *Fv*	0.000000 *Fs*
0.175837 *Fd*	50	40	0.127686 *v*	0.004718 *Fv*	0.036950 *Fs*
0.122871 *Fd*	55	35	0.281334 *v*	0.017968 *Fv*	0.063869 *Fs*
0.079653 *Fd*	60	30	0.469882 *v*	0.037427 *Fv*	0.079653 *Fs*
0.047001 *Fd*	65	25	0.711154 *v*	0.060147 *Fv*	0.084576 *Fs*
0.024370 *Fd*	70	20	1.042160 *v*	0.083159 *Fv*	0.079795 *Fs*
0.010362 *Fd*	75	15	1.550395 *v*	0.103842 *Fv*	0.066978 *Fs*
0.003084 *Fd*	80	10	2.499421 *v*	0.120105 *Fv*	0.048053 *Fs*
0.000386 *Fd*	85	5	5.208606 *v*	0.130454 *Fv*	0.025046 *Fs*
0.000000 *Fd*	90	0	Infinite.	0.134001 *Fv*	0.000000 *Fs*
1	2	3	4	5	6

The preceding table has been applied by Euler solely to that species of wind-mills in which the sails are sectors of an ellipse, and which intercept the whole cylinder of wind. This construction was recommended also by Parent; but later and more accurate experiments have evinced, that, when the whole area is filled up with sail, the wind, from the want of proper interstices to escape, does not produce its greatest effect. On this account a small number of sails are generally used, and these are either rectangular, or a little enlarged at their extremities. It will be proper, therefore, to show how the table can be applied to this description of sails, for the application is much more difficult than in the other case.

It is evident, from the first column of the table, that before we can use it, we must find the value of F, or the force of the wind upon all the sails. But as this force depends not merely upon the quantity of surface, and the velocity of the wind, which are always given, but also upon the angle of their inclination, which is unknown, some method of determining it, independently of this angle, must be adopted. Euler has shown how to do this, in the case where the whole area is filled with elliptical sectors; but there is no direct method of determining the value of F in the case of rectangular sails, when the angle of inclination is unknown. We must find it, therefore, by approximation, that is, we must take any probable angle of inclination, 70 degrees, for example, and find the value of F suited to this angle, and thence the coefficient of Fd in the first column. With this coefficient enter the table, and take out the corresponding angle of inclination, which will be either less or

greater than 70. With this new angle of inclination, find a more accurate value of F, and consequently a new coefficient of Fd. If this coefficient does not differ very much from that formerly found, it may be regarded as true, and employed for taking out of the table a more accurate angle of inclination, along with the velocity of the sails, and the effect of the machine. We shall now illustrate both these methods by an example, after having shown how to determine by experiment the momentum of friction, and the velocity of the wind.

To find the momentum of friction.

In a calm day, when the wind-mill is unloaded, or performing no work, bring two opposite sails into a horizontal position; and, having attached different weights to the extremities of their radii, find how many pounds are sufficient, not only for impressing the smallest motion on the sails, but for continuing them in that state; and the number of pounds multiplied into the length of the radius, will be the momentum of friction. When this experiment is made, it will always be found that a greater weight is necessary for moving the sails than for continuing them in motion; and, in order that the quantity of friction may be accurately estimated, the wind-mill should be put in motion immediately before the experiment is made, for the friction always increases with the time in which the communicating parts have remained in contact.

To find the velocity of the wind.

Various instruments, denominated anemometers, or anemoscopes, have been invented for measuring the force and velocity of the wind, the best of which are those which were constructed by Mr. Pickering* and Dr. Lind.†—The velocity of the wind has been deduced also from the motion of the clouds, and the change effected by the wind upon the motion of sound.‡ The second of these methods is manifestly inaccurate, and the first takes for granted what is palpably erroneous, that the velocity of the wind is the same in the higher regions of the atmosphere, as at the surface of the earth. The ingenious Professor Leslie having found, in the course of his experiments on heat, that the refrigerant, or cooling power of a current of air, is exactly proportional to its velocity, derives from this principle the construction of a new and simple anemometer. 'It is, in reality, nothing more,' says he, 'than 'a thermometer, only with its bulb larger than 'usual. Holding it in the open still air, the 'temperature is marked: it is then warmed by 'the application of the hand, and the time is 'noted which it takes to sink back to the mid'dle point. This I shall term the fundament'al measure of cooling. The same observa'tion is made on exposing the bulb to the im-

* Philosophical Transactions, No. 473.

† Id. vol. lxv. p. 353.

‡ Brisson, *Traite de Physique*, vol. ii. p. 150, § 1015. For the description of another Anemometer, see Wolfii Opera. Math. tom. i. p. 773.

'pression of the wind, and I shall call the time 'required for the bisection of the interval of 'temperatures, the occasional measure of cool- 'ing. After these preliminaries, we have the 'following easy rule:—*Divide the fundament- 'al by the occasional measure of cooling, and 'the excess of the quotient above unity, being 'multiplied by 4½, will express the velocity of 'the wind in miles per hour.* The bulb of the 'thermometer ought to be more than half an 'inch in diameter, and may, for the sake of 'portability, be filled with alkohol, tinged, as 'usual, with archil. To simplify the observa- 'tion, a sliding scale of equal parts may be 'applied to the tube. When the bulb has ac- 'quired the due temperature, the zero of the 'slide is set opposite to the limit of the colour- 'ed liquor in the stem; and, after having been 'heated, it again stands at 20° in its descent, 'the time which it thence takes until it sinks 'to 10° is measured by a stop-watch. Extem- 'poraneous calculation may be avoided, by 'having a table engraved upon the scale for 'the series of occasional intervals of cooling.'*

The most simple method of determining the velocity of the wind, is that which Coulomb employed in his experiments on wind-mills, and which requires neither the aid of instruments nor the trouble of calculation.† Two persons were placed on a small elevation, at the distance of 150 feet from one another, in the direction of the wind, and, while the one observed, the other measured the time with a small and light feather employed in moving through this space. The distance between the two per-

* Enquiry into the Nature and Propagation of Heat, p. 284.
† See Memoires de l'Academie Royale, 1781, p. 79.

sons, divided by the number of seconds, gave the velocity of the wind per second. Having thus shown how to find the momentum of friction, and the velocity of the wind, we shall now explain the use of the table.

Supposing the radius of the sails to be 20 feet, the velocity of the wind 10 feet per second, and that it requires a force of 10 pounds acting at the extremity of the radius to overcome the friction of the machine—it is required to find the angle of weather, the velocity of the sails, and the effect of the machine.

Let d, the radius of the sails, be $=20$ feet, then the momentum of friction will be $10\times 20=200$ pounds. Let n, the number of sails, be $=12$, while a represents the breadth of the sails at their extremities, and b the breadth into which they are projected, or the breadth which they would occupy if reduced into a plane perpendicular to the wind. Then, since the whole cylinder of wind is supposed to be intercepted, the effect produced upon all the oblique sails, will be equal to the effect that would be produced upon a perpendicular surface, equal to the whole area of the polygon into which the oblique triangular sails are projected. The value of b, therefore, may be found by plane trigonometry, the length of the sail, and the angle of the polygon being given, or by the following theorem, derived from trigonometry, viz. $b=2d\times \text{tang.} \frac{180}{n}$, d being radius, and n the number of sails. In the present case, then, we will have $b=2\times 20\times \text{tang.} \frac{180}{n}$, or $b=40\times \text{tang.}$

$15°$, $= 10.717968$ feet. Now, since the area of any triangle is equal to its altitude multiplied by half its base, the area of a polygon will be equal to the altitude of one of the triangles which compose it, or to the radius of the inscribed circle, multiplied by half the number of its sides. The area of the polygon, therefore, into which the sails are projected, or the quantity of perpendicular surface impelled by the wind, will be $\frac{1}{2}\,ndb$, and, consequently, the force of impulsion, F, upon this surface, will be $\frac{1}{2}\,ndbvv$, where vv is the square of the wind's velocity, to which the force of impulsion is always proportional. In the present case, then, the force F, which impels the sails, will be $6 \times 20 \times 10.717968 \times vv$; and if vv be the altitude which is due to the velocity of the wind, or the height through which a heavy body must fall in order to acquire that velocity, the force of impulsion, F, will be equal to the weight of a mass of air, whose volume is $1286.15616 \times vv$, cubic feet, or to $1\frac{3}{5}\,vv$ cubic feet of water, for water is about 800 times more dense than air, that is, to 100 vv pounds avoirdupois, $62\frac{1}{2}$ of which are equal to a cubic foot of water. But, in order that the machine may move, the momentum of friction, 200 must be less than $0.235702 \times Fd$, or $0.235702 \times 100\,vv \times 20$; for, when it is exactly this, the wind cannot move the machine, as appears from the first line of the table; or, what is the same thing, the height due to the velocity of the wind, viz. vv, must be greater than 0.424 or $\frac{3}{7}$ of a foot, which corresponds to a velocity of 5.222, or $5\frac{2}{9}$. Unless, therefore, the celerity of the wind exceeds $5\frac{2}{9}$ feet per second, it will not be able to move the machine. These things being premised, let us now proceed to deter-

mine the construction and effect of the machine, upon the supposition that the momentum of friction is 200 pounds, and the velocity of the wind 10 feet per second. Now, vv, the height due to this velocity, is $1\frac{3}{5}$ feet;* therefore, the force of impulsion, F, is$=100\ vv$ pounds, or $100\times\frac{3}{5}$, or$=160$ pounds avoirdupois; and $Fd=160\times 20=3200$. But the momentum of friction, viz. Fd, multiplied into its coefficient, should be equal 200 pounds; therefore, the coefficient will be equal to $\frac{200}{Fd}=\frac{200}{3200}=0.062500$, and the momentum of friction will be $0.062500\ Fd$.— With this number enter the first column of the table, and you will find the angle of inclination corresponding to it to be about 63°; the velocity of the sail's extremity$=\frac{3}{5}\ v$, or 6 feet per second; and the effect of the machine$=0.05\ Fv$ $=0.05\times 160\text{lb.}\times 10=8\text{lb.}\times 10$ feet, or 8 pounds raised through 10 feet in a second, which is equal to 1000 pounds raised through 288 feet in an hour. But the force of a man, according to Euler, is equal to 1000 pounds raised through 180 feet in an hour; therefore, the power of the machine, with a wind moving at the rate of 10 feet per second, is not equal to the power of two men.†

* The height answering to any velocity, and the velocity due to any height, may be found by the following theorems, in which v is the velocity, and h the height due to it. $v=2\sqrt{16.087\times h}$, hence $h=\frac{2vv}{129}$. See p. 172, 173.

† Bernouilla makes the force of a man equal to 1000 pounds raised through 216 feet in an hour—*Receuil des Prix, Tom.* VIII. Coulomb makes it equal to 1000 pounds raised through 192 feet in an hour—*Mem. Acad. Roy* 1781, *p.* 74; and M. Schulze makes it 1000 pounds raised through 260 feet in an hour—*Mem. de l'Acad. Berlin* 1783, *p.* 333.

Let us now suppose that the wind-mill is driven by means of 4 rectangular sails, 18 feet in length, and 4 in breadth, and that the momentum of friction, and the radius of the sails, are the same as before. Then the area of each sail will be 18×4, and the whole surface that is acted upon by the wind will be $18\times4\times4=288$ square feet. But before we can determine the force which the wind exerts upon this surface, we must know its inclination to the wind; let us suppose this to be 70 degrees, and the impetus of the wind upon the sails, or F, will be $=288\times(\text{Sin. }70)^2\times vv$, in which v is the wind's velocity, or $F=254\,vv$ cubic feet of air. If vv be the height due to the wind's velocity, dividing this quantity by 800, we will have

$$F=\frac{127}{400}vv$$

cubic feet of water, and multiplying this by $62\frac{1}{2}$, we will have $F=19.8\,vv$ pounds avoirdupois. Now, let the velocity of the wind be 30 feet per second, the height vv due to this velocity will be 14 feet nearly, and, consequently, $F=19.8\times14,=276$ pounds avoirdupois. Fd will therefore be $=5540$; and, since the whole momentum of friction is 200, the coefficient of

$$Fd \text{ will be }=\frac{200}{5540}=0.036101,$$

and the momentum of friction, expressed as the table requires, will be $=0.036101\,Fd$. Having entered the table with this number, the proper angle of inclination will be found to be $67\frac{1}{2}$ degrees. With this angle, instead of 70 degrees, repeat the foregoing calculation, and after finding a new coefficient to Fd, enter the table with it a second time, and you will have the proper angle

of inclination, differing but little from the former, and likewise the velocity of the sails, and the effect of the machine.*

These theoretical deductions, however interesting they may be, must yield in point of practical utility to the observations of our countryman, Mr. Smeaton. From a variety of well-conducted experiments, he found, that the common practice of inclining plane sails, from 72° to 75°. to the axis, was much more efficacious than the angle assigned by Parent, the effect being as 45 to 31. When the sails were weathered in the Dutch manner, that is, when their surfaces were concave to the wind, and when the angle of inclination increased towards their extremities, they produced a greater effect than when they were weathered either in the common way, or according to Maclaurin's theorem.† But when the sails were enlarged at their extremities, as represented at $\alpha\beta$, in Plate XLII, Fig. 1. so that $\alpha\beta$ was one third of the radius ms, and αm to $m\beta$, as 5 to 3, their power was greatest of all, though the surface acted upon by the wind remained the same. If the sails be farther enlarged, the effect is not increased in proportion to the surface; and, besides, when the quantity of cloth is great, the machine is much exposed to injury by sudden squalls of wind. In these experiments of Smeaton, the angle of weather varied with the

* Those who wish to inquire farther into the theory of wind-mills, will find some excellent observations in D'Alembert's Traite de l'equilibre et du movement des fluides, 1770, p. 396, § 368; or in his *Opuscules*, Tom. 5, p. 148, &c. and also by Lambert, in the Mem de l'Acad. Berlin, 1775, p 92

† See page 258

distance from the axis; and he found, from several trials, that the most efficacious angles were those contained in the following table.

Parts of the radius ms, which is divided into 6 parts.	Angle with the axis.	Angle of weather.
1	72	18
2	71	19
3	72	18 middle
4	74	16
5	$77\frac{1}{2}$	$12\frac{1}{2}$
6	83	7

Supposing the radius *ms* of the sail to be 30 feet, then the sail will commence at $\frac{1}{6}$ *ms*, or 5 feet from the axis, where the angle of inclination will be 72°. At $\frac{2}{6}$ *ms*, or 10 feet from the axis, the angle will be 71°, and so on.

On the effect of wind-mill sails.

The following Maxims, deduced by Mr. Smeaton from his experiments, contain the best information which we have upon this subject, if we except a few experiments made by Coulomb.

Maxim 1.—The velocity of wind-mill sails, whether unloaded or loaded, so as to produce a maximum effect, is nearly as the velocity of the wind, their shape and position being the same.

Maxim 2.—The load at the maximum is nearly (but somewhat less than) as the square of the velocity of the wind, the shape and position of the sails being the same.

Maxim 3.—The effects of the same sails at a maximum, are nearly (but somewhat less than) as the cubes of the velocity of the wind.*

Maxim 4.—The load of the same sails at the maximum is nearly as the squares, and their effect as the cubes of their number of turns in a given time.

Maxim 5.—When sails are loaded, so as to produce a maximum at a given velocity, and the velocity of the wind increases, the load continuing the same: 1st. The increase of effect, when the increase of the velocity of the wind is small, will be nearly as the squares of those velocities: 2dly. When the velocity of the wind is double, the effects will be nearly as 10 : $27\frac{1}{2}$: But, 3dly. When the velocities compared are more than double of that where the given load produces a maximum, the effects increase nearly in the simple ratio of the velocity of the wind.

Maxim 6.—In sails where the figure and positions are similar, and the velocity of the wind the same, the number of turns in a given time will be reciprocally as the radius or length of the sail.

Maxim 7.—The load, at a maximum, that sails of a similar figure and position will overcome, at a given distance from the centre of motion, will be as the cube of the radius.

* This maxim was confirmed by Euler, Lambert, and Coulomb, who makes the effect, *exactly* proportional to the cube of the velocity.

Maxim 8. The effects of sails of similar figure and position are as the squares of the radius.

Maxim 9. The velocity of the extremities of Dutch sails, as well as of the enlarged sails, in all their usual positions when unloaded, or even loaded to a maximum, are considerably quicker than the velocity of the wind.*

M. Coulomb made a number of experiments on wind-mills that were employed to raise stampers for the purpose of bruising seed. He found that wind-mills having the dimensions formerly stated,† produced an effect equivalent to 1000 pounds raised through the space of 218 feet in a minute. The quantity of force, which was lost by the action of the wipers upon the stampers, was equal to 1000 pounds, raised through $16\frac{1}{2}$ feet in a minute; and the friction was equivalent to 1000 pounds, raised through $18\frac{1}{2}$ feet in a minute. The total quantity of action, therefore, exerted by the wind in moving the machine, was equal to 1000 pounds elevated to the height of 253 feet in a minute, the velocity of the wind being 20 feet per second.

It appears, too, from Coulomb's experiments, that when the wind moved at the rate of 13 feet per second, the sails made 8 turns in a minute, when the velocity of the wind was 20 feet per second, the sails performed 13 turns in a minute, and when its velocity was 28 feet in a second, the sails made 17 turns in a minute.‡ By

* Mr. Smeaton found, when the radius was 30 feet, that for every 3 turns of the Dutch sails in their common position (when the angle of weather at the extremity is nothing), the windmill moves at the rate of 2 miles an hour; for every 5 turns in a minute of the Dutch sails in their best position, the wind moves 4 miles an hour; and for every 6 turns in a minute, of the enlarged sails in their best position, the wind will move 5 miles an hour.

† See page 251.

‡ Memoires de L'Acadamie Royale, &c 1781, p. 81.

taking the medium of these results, it will be found, that the number of turns made by the sails in a minute, is to the number of feet which the wind moves in a second, as 1 to 1.6. Hence, when the velocity of the sails is given, that of the wind may be easily determined. PLATE XLII.

On horizontal wind-mills.

A variety of opinions have been entertained respecting the relative advantages of horizontal and vertical wind-mills. Mr. Smeaton, with great justice, gives a decided preference to the latter; but when he asserts that horizontal wind-mills haves only $\frac{1}{8}$ or $\frac{1}{10}$ of the power of vertical ones, he certainly forms too low an estimate of their power. Mr. Beatson, on the contrary, who has received a patent for the construction of a new horizontal wind-mill, seems to be prejudiced in their favour, and greatly exaggerates their comparative value. From an impartial investigation, it will probably appear, that the truth lies between these two opposite opinions; but before entering on this discussion, we must first consider the nature and form of horizontal wind-mills.

In Fig. 3. *CK* is the perpendicular axis or windshaft, which moves upon pivots. Four cross bars, *CA*, *CD*, *IB*, *FG*, are fixed to this arbor, which carries the frames *APIB*, *DEFG*. The sails *AI*, *EG*, are stretched upon these frames, and are carried round the axis *CK* by the perpendicular impulse of the wind. Upon the axis *CK*, a toothed wheel is fized, which gives motion to the particular machinery that is employed. In the figure, only two sails are represented; but there are always other two Fig. 3.

placed at right angles to these. Now, let the sails be exposed to the wind, and it will be evident that no motion will ensue; for the force of the wind upon the sail *AI*, is counteracted by an equal and opposite force upon the sail *EG*. In order, then, that the wind may communicate motion to the machine, the force upon the returning sail *EG*, must either be removed by screening it from the wind, or diminished by making it present a less surface when returning against the wind. The first of these methods is adopted in Tartary, and in some provinces of Spain, but is objected to by Mr. Beatson, from the inconvenience and expense of the machinery and attendance requisite for turning the screens into their proper positions. Notwithstanding this objection, however, I am disposed to think that this is the best method of diminishing the action of the wind upon the returning sails, for the moveable screen may easily be made to follow the direction of the wind, and assume its proper position, by means of a large wooden weathercock, without the aid either of men or machinery. It is true, indeed, that the resistance of the air in the returning sails is not completely removed; but it is at least as much diminished as it can be by any method hitherto proposed. Besides, when this plan is resorted to, there is no occasion for any moveable flaps and hinges, which must add greatly to the expense of every other method.

The mode of bringing the sails back against the wind, which Mr. Beatson invented, is, perhaps, the simplest and best of the kind. He makes each sail *AI* to consist of 6 or 8 flaps or vanes, *APb* 1, *b* 1, *c* 2, &c. moving upon hinges represented by the dark lines *AP*, *b* 1,

c 2, &c. so that the lower side *b* 1, of the first flap overlaps the hinge or higher side of the second flap, and so on. When the wind, therefore, acts upon the sail *AI*, each flap will press upon the hinge of the one immediately below it, and the whole surface of the sail will be exposed to its action. But when the sail *AI* returns against the wind, the flaps will revolve round upon their hinges, and present only their edges to the wind, as is represented at *EG*, so that the resistance occasioned by the return of the sail must be greatly diminished, and the motion will be continued by the great superiority of force exerted upon the sails in the position *AI*. In computing the force of the wind upon the sail *AI*, and the resistance opposed to it by the edges of the flaps in *EG*, Mr. Beatson finds, that when the pressure upon the former is 1872 pounds, the resistance opposed by the latter is only about 36 pounds, or $\frac{1}{52}$ part of the whole force; but he neglects the action of the wind upon the arms *CA*, &c. and the frames which carry the sails, because they expose the same surface in the position *AI*, as in the position *EG*. This omission, however, has a tendency to mislead us in the present case, as we shall now see, for we ought to compare the whole force exerted upon the arms, as well as the sail, with the whole resistance which these arms and the edges of the flaps oppose to the motion of the wind-mill. By inspecting Fig. 3. it will appear, that if the force upon the edges of the flaps, which Mr. Beatson supposed to be 12 in number, amounts to 36 pounds, the force spent upon the bars *CD*, *DG*, *GF*, *EF*, &c. cannot be less than 60 pounds. Now, since these bars are acted up-

PLATE XLII.

on with an equal force, when the sails have the position *AI*, $1872+60=1932$ will be the force exerted upon the sail *AI*, and its appendages, while the opposite force upon the bars and edges of the flaps when returning against the wind will be $36+60=96$ pounds, which is nearly $\frac{1}{20}$ of 1932, instead of $\frac{1}{12}$, as computed by Mr. Beatson. Hence, we may see the advantages which will probably arise from using a screen for the returning sail instead of moveable flaps, as it will preserve not only the sails, but the arms and the frame which support it, from the action of the wind.*

We shall now conclude this article with a few remarks on the comparative power of horizontal and vertical wind-mills. It was already stated, that Mr. Smeaton rather under-rated the former, while he maintained that they have only $\frac{1}{8}$ or $\frac{1}{10}$ the power of the latter. He observes, that when the vanes of a horizontal and a vertical mill are of the same dimensions, the power of the latter is four times that of the former, because, in the first case, only one sail is acted upon at once, while, in the second case, all the 4 receive the impulse of the wind. This, however, is not strictly true, since the vertical sails are all oblique to the direction of the wind. Let us suppose that the area of each sail is 100 square feet; then the power of the horizontal sail may be called $100 \times (\text{sine } 70°)^2$ (which is the common angle of inclination) $= 88$ nearly;

* The sails of horizontal wind-mills are sometimes fixed like floatboards on the circumference of a large drum or cylinder. These sails move upon hinges so as to stand at right angles to the drum, when they are to receive the impulse of the wind: and when they return against it, they fold down upon its circumference. See Repertory of Arts, vol. 6

but since there are four vertical sails, the power of them all will be $4 \times 88 = 352$; so that the power of the horizontal sail is to that of the four vertical ones as 1. to 3.52, and not as 1 to 4, according to Mr. Smeaton. But Mr. Smeaton also observes, that if we consider the further disadvantage which arises from the difficulty of getting the sails back against the wind, we need not wonder if horizontal wind-mills have only about $\frac{1}{8}$ or $\frac{1}{10}$ of the common sort. We have already seen, that the resistance occasioned by the return of the sails, amounts to $\frac{1}{20}$ of the whole force which they receive; by subtracting $\frac{1}{20}$, therefore, from $\frac{1}{3.52}$, we will find, that the power of horizontal wind-mills is only $\frac{103}{4.40}$. or little more than $\frac{1}{4}$ less than that of vertical ones. This calculation proceeds upon a supposition, that the whole force exerted upon vertical sails is employed in turning them round the axis of motion; whereas, a considerable part of this force is lost in pressing the pivot of the axis or windshaft against its gudgeon.— Mr. Smeaton has overlooked this circumstance, otherwise he could never have maintained that the power of four vertical sails was quadruple the power of one horizontal sail, the dimensions of each being the same. Taking this circumstance into the account, we cannot be far wrong in saying that, in theory at least, if not in practice, the power of a horizontal windmill is about $\frac{1}{3}$ or $\frac{1}{4}$ of the power of a vertical one, when the quantity of surface and the form of the sails is the same, and when every part of the horizontal sails have the same distance from the axis of motion as the corresponding parts of the vertical sails. But if the horizon-

PLATE XLII.

tal sails have the position *AI, EG,* in Fig. 8. instead of the position *CAdm, CDon,* their power will be greatly increased, though the quantity of surface is the same, because the part *CP3m* being transferred to *B13d,* has much more power to turn the sails. Having this method, therefore, of increasing the power of horizontal sails, which cannot be applied to vertical ones, we would encourage every attempt to improve their construction, as not only laudable in itself, but calculated to be of essential utility in a commercial country.

MECHANICS.

On the Nature of Friction, and the Method of Diminishing its Effects in Machinery.

THE resistance which friction generates in the communicating parts of machinery is so powerful, and the consequent defalcation from the impelling power is so great, that a knowledge of its nature and effects must be of the highest importance to the philosopher and the practical mechanic. The theory of machines must continue imperfect till the nature and effects of friction are thoroughly developed, and their performance must be comparatively small, and the expense of their erection and preservation comparatively great, till some effectual method be discovered for diminishing that retardation of the machine's velocity, and that decay of its materials which arise from the attrition of the connecting parts. The knowledge, however, which has been acquired concerning this abstruse subject, has not been commensurate with the labours of philosophers, and the progress of other branches of mechanical science; and those contrivances which ingenious men

have discovered for diminishing the resistance of friction, have either been overlooked by practical inquirers, or rejected by those vulgar prejudices which prompt the mechanics of the present day to persist in the most palpable errors, and neglect such maxims of construction as are authorized both by theory and experience.—It may be proper, therefore, in a work like this, to give a summary view of the opinions of different philosophers upon the nature of friction, and the means which may be adopted for diminishing its effects.

M. Amontons was the first philosopher who favoured us with any thing like correct information upon this subject. He found that the resistance opposed to the motion of a body upon a horizontal surface, was exactly proportional to its weight, and was equal to *one third* of it, or more generally to one-third of the force with which it was pressed against the surface over which it moved. He discovered also that this resistance did not increase with an increase of the rubbing superficies, nor with the velocity of its motion.*

The experiments of M. Bulfinger authorized conclusions similar to those of Amontons, with this difference only, that the resistance of friction was equal only to *one fourth* of the force with which the rubbing surfaces were pressed together.†

This subject was also considered by Parent, who supposed that friction is occasioned by

* Memoires de l'Acadamie Royale des Sciences 1699, p. 206. Amonton's experiments were confirmed by Bossut and Belidor See Architect. Hydraulique, v. i, chap. 2, p. 70.

† Commentarii Petropolitanæ, Tom. ii, p. 40.

small spherical eminences in one surface being dragged out of corresponding spherical cavities in the other, and proposed to determine its quantity by finding the force which would move a sphere standing upon three equal spheres.—This force was found to be to the weight of the sphere as 7 to 20, or nearly one third of the sphere's weight.* In investigating the phenomena of friction, M. Parent placed the body upon an inclined plane, and augmented or diminished the angle of inclination till the body had a tendency to move, and the angle at which the motion commenced, he called the angle of equilibrium. The weight of the body, therefore, will be to its friction upon the inclined plane as radius to the sine of the angle of equilibrium, and its weight will be to the friction on a horizontal plane, as radius to the tangent of the angle of equilibrium.†

The celebrated Euler seems to have adopted the hypothesis of Bulfinger respecting the ratio of friction to the force of pressure; and in two curious dissertations which he has published upon this subject,‡ has suggested many important observations, to which Mr. Vince seems afterwards to have attended. He observes, that when a body is in motion, the effect of friction will be only one half of what it is when the body has begun to move; and he shows that if the

* Recherches de Mathematique et Physique 1713, tom. ii, p. 462.

† Memoires de l'Academie 1704, p. 174.

‡ The first is entitled, *Sur le frottement des Corps solides*, and the other, *Sur la diminution de la resistance du frottement*, published in the Memoires de l'Academie Royale de Sciences à Berlin, ann. 1748, p. 122, 133.

angle of an inclined plane be gradually increased till the body which is placed upon it begins to descend, the friction of the body at the very commencement of its motion will be to its weight or pressure upon the plane, as the sine of the plane's elevation is to its cosine, or as the tangent of the same angle is to radius, or as the height of the plane is to its length.—But when the body is in motion the friction is diminished, and may be found by the following equation $\mu = \text{Tan.}\ a - \frac{m}{15625\ nn\ \text{cos.}\ a}$, in which μ is the quantity of friction, the weight or pressure of the body being $=1$, a is the angle of the plane's inclination, m is the length of the plane in 1000th parts of a rhinland foot, and n the time of the body's descent. Respecting the cause of friction, Euler is nearly of the same opinion with Parent; the only difference is, that instead of regarding the eminences and corresponding depressions as spherical, he supposes them to be angular, and imagines the friction to arise from the body's ascending a perpetual succession of inclined planes.

Mr. Ferguson found that the quantity of friction was always proportional to the weight of the rubbing body, and not to the quantity of surface, and that it increased with an increase of velocity, but was not proportional to the augmentation of celerity. He found also, that the friction of smooth soft wood, moving upon smooth soft wood, was equal to $\frac{2}{3}$ of the weight; of rough wood upon rough wood $\frac{1}{2}$ of the weight; of soft wood upon hard, or hard upon soft, $\frac{1}{5}$ of the weight; of polished steel upon

polished steel or pewter $\frac{1}{4}$ of the weight; of polished steel upon copper $\frac{1}{5}$, and of polished steel upon brass $\frac{1}{6}$ of the weight.*

The Abbe Nollet† and Bossut‡ have distinguished friction into two kinds, that which arises from one surface being dragged over another, and that which is occasioned by one body rolling upon another. The resistance which is generated by the first of these kinds of friction is always greater than that which is produced by the second; and it appears evidently from the experiments of Muschenbroek, Schoeber, and Meister, that when a body is carried along with a uniformly accelerated motion, and retarded by the first kind of friction, the spaces are still proportional to the squares of the times, but when the motion is affected by the second kind of friction, this proportionality between the spaces and the times of their description does not obtain.

The subject of friction has more lately occupied the attention of the ingenious Mr. Vince, of Cambridge. He found that the friction of hard bodies in motion is a uniformly retarding force, and that the quantity of friction considered as equivalent to a weight drawing the body backwards is equal to $M - \frac{(M+W)\times S}{rt^2}$, where M is the moving force expressed by its weight, W the weight of the body upon the horizontal plane, S the space through which the moving force or weight descended in the

* Tables and Tracts, edit. 2d, p. 258 9.

† Nollet, Leçons de Physique, tom. iii, p 231, ed. 1770.

‡ Bossut Traité Elementaire de Mecanique, § 306-7.

time t and $r=16.087$ feet, the force of gravity. Mr. Vince also found that the quantity of friction increases in a less ratio than the quantity of matter or weight of the body, and that the friction of a body does not continue the same when it has different surfaces applied to the plane on which it moves, but that the smallest surfaces will have the least friction.*

Notwithstanding these various attempts to unfold the nature and effects of friction, it was reserved for the celebrated Coulomb to surmount the difficulties which are inseparable from such an investigation, and to give an accurate and satisfactory view of this complicated part of mechanical philosophy. By employing large bodies and ponderous weights, and conducting his experiments on a large scale, he has corrected several errors which necessarily arose from the limited experiments of preceding writers; he has brought to light many new and striking phenomena, and confirmed others which were hitherto but partially established. As it would be foreign to the nature of this work to follow Monsieur Coulomb through his numerous and various experiments, we shall only present the reader with the new and interesting results which they authorize.†

1. The friction of homogeneous bodies, or bodies of the same kind moving upon one another, is generally supposed to be greater than

* Philosophical Transactions, v. lxxv, p. 167.

† A full account of Coulomb's experiment may be seen in the *Journal de Physique* for September and October 1785, vol. xxvii, pp. 206 & 282, &c. An excellent summary of them may also be found in Van Swinden's Positiones Physico. They were originally published in the Memoires de Mathematique et Physique.

that of heterogeneous bodies;* but Coulomb has shewn that there are exceptions to this rule. He found, for example, that the friction of oak upon oak was equal to $\frac{1}{2.34}$ of the force of pression; the friction of pine against pine was $\frac{1}{1.78}$, and of oak against pine $\frac{1}{1.5}$. The friction of oak against copper was $\frac{1}{5.5}$, and that of oak against iron nearly the same.

2. It was generally supposed, that in the case of wood, the friction is greatest when the bodies are dragged contrary to the course of their fibres;† but Coulomb has shown that the friction is in this case sometimes the smallest. When the bodies moved in the direction of their fibres the friction was $\frac{1}{2.34}$ of the force with which they were pressed together; but when the motion was contrary to the course of the fibres, the friction was only $\frac{1}{3.76}$.

3. The longer the rubbing surfaces remain in contact, the greater is their friction.‡—When wood was moved upon wood, according to the direction of the fibres, the friction was increased by keeping the surfaces in contact for a few seconds; and when the time was prolonged to a minute, the friction seemed to have reached its farthest limit. But when the motion was performed contrary to the course of the fibres, a greater time was necessary before the friction arrived at its maximum. When wood was moved upon metal, the friction did not attain its

* This was the opinion of Muschenbroek, Krafft, Camus, and Bossut.

† Muschenbroek, Introductio ad Philosoph. Nat. § 513, Bc.

‡ This is mentioned by Bossut, Traite de Mecanique, § 310, but Coulomb has the merit of having established the fact.

maximum till the surfaces continued in contact for 5 or 6 days; and it is very remarkable, that, when wooden surfaces were anointed with tallow, the time requisite for producing the greatest quantity of friction is increased. The increase of friction which is generated by prolonging the time of contact is so great, that a body weighing 1650 pound was moved with a force of 64 pounds when first laid upon its corresponding surface. After having remained in contact for the space of 3 seconds, it required 160 pounds to put it in motion, and when the time was prolonged to 6 days, it could scarcely be moved with a force of 622 pounds. When the surfaces of metallic bodies were moved upon one another, the time of producing a maximum of friction was not changed by the interposition of olive oil; it was increased, however, by employing swine's grease as an unguent, and was prolonged to 5 or 6 days by smearing the surfaces with tallow.

4. Friction is in general proportional to the force with which the rubbing surfaces are pressed together; and is, for the most part, equal to between $\frac{1}{2}$ and $\frac{1}{4}$ of that force.—In order to prove the first part of this proposition, Coulomb employed a large piece of wood, whose surface contained 3 square feet, and loaded it successively with 74 pounds, 874 pounds, and 2474 pounds. In these cases the friction was successively $\frac{1}{2.46}$, $\frac{1}{2.16}$, $\frac{1}{2.21}$, of the force of pression; and when a less surface and other weights were used, the friction was $\frac{1}{2.36}$, $\frac{1}{2.42}$, $\frac{1}{2.40}$. Similar results were obtained in all Coulomb's experiments, even when metallic surfaces were employed. The second part of the proposition

has also been established by Coulomb. He found that the greatest friction takes place when oak moves upon pine, and that it amounts to $\frac{1}{1.78}$ of the force of pression; on the contrary, when iron moves upon brass, the least friction is produced, and it amounts to $\frac{1}{4}$ of the force of pression.

5. Friction is in general not increased by augmenting the rubbing surfaces.*—When a superficies of 3 feet square was employed, the friction, with different weights, was $\frac{1}{2.28}$ at a medium; but when a smaller surface was used, the friction, instead of being greater, as might have been expected, was only $\frac{1}{2.33}$.

6. Friction for the most part is not augmented by an increase of velocity. In some cases, indeed, it is diminished.†—M. Coulomb found, that when wood moved upon wood in the direction of the fibres, the friction was a constant quantity, however much the velocity was varied; but that when the surfaces were very

* Muschenbroek and Nollet entertained the opposite opinion. The experiments of Krafft coincide with those of Coulomb. See Commentarii Petropolitanæ, tom, xii, p. 266, § 19, 20, &c.

† The latter part of this proposition is confirmed by a circumstance which occurred in the course of M. Lambert's experiments on undershot mills, but which he imputes to a very different cause. He found that the resistance which is generated by the friction of the communicating parts of a corn-mill, combined with that which arises from the grain between the millstones, always diminished when the velocity was increased. M. Lambert did not hesitate to assert, that the part of this compound resistance which was produced by the friction of the machinery continued invariably the same, and ascribed, without any reason, the diminution which accompanied an increase of velocity to a diminution of the grain's resistance between the millstones; whereas it was probably a diminution of the friction of the connecting parts occasioned by the augmentation of their velocity.

small in respect to the force with which they were pressed, *the friction was diminished by augmenting the rapidity:* the friction, on the contrary, was increased when the surfaces were very large when compared with the force of pression. When the wood was moved contrary to the direction of its fibres, the friction in every case remained the same. If wood is moved upon metals, the friction is greatly increased by an increase of velocity; and when metals move upon wood smeared with tallow, the friction is still augmented by adding to the velocity. When metals move upon metals, the friction is always a constant quantity; but when heterogeneous substances are employed which are not daubed with tallow, the friction is so increased with the velocity, as to form an arithmetical progression when the velocities form a geometrical one.

7. The friction of loaded cylinders rolling upon a horizontal plane, is in the direct ratio of their weights, and the inverse ratio of their diameters. In Coulomb's experiments, the friction of cylinders of guaiacum-wood, which were two inches in diameter, and were loaded with 1000 pounds, was 18 pounds, or $\frac{1}{56}$ of the force of pression. In cylinders of elm, the friction was greater by $\frac{2}{5}$, and was scarcely diminished by the interposition of tallow.

From a variety of experiments on the friction of the axis of pullies, Coulomb obtained the following results.—When an iron axle moved in a brass bush or bed, the friction was $\frac{1}{6}$ of the pression; but when the bush was smeared with very clean tallow, the friction was only $\frac{1}{11}$, when swine's grease was interposed, the fric-

tion amounted to $\frac{1}{8.3}$, and when olive oil was employed as an unguent, the friction was never less than $\frac{1}{8}$ or $\frac{1}{7.3}$. When the axis was of green oak, and the bush of guaiacum-wood, the friction was $\frac{1}{26}$ when tallow was interposed; but when the tallow was removed, so that a small quantity of grease only covered the surface, the friction was increased to $\frac{1}{17}$. When the bush was made of elm, the friction was in similar circumstances $\frac{1}{33}$ and $\frac{1}{20}$, which is the least of all. If the axis be made of box, and the bush of guaiacum-wood, the friction was $\frac{1}{23}$ and $\frac{1}{14}$, circumstances being the same as before. If the axle be of boxwood, and the bush of elm, the friction will be $\frac{1}{29}$ and $\frac{1}{20}$; and if the axle be of iron, and the bush of elm, the friction will be $\frac{1}{20}$ of the force of pression.

Having thus given a brief, though, we trust, a comprehensive view of the interesting results of Coulomb's experiments, we shall conclude this part of the subject, by presenting the reader with some excellent and original observations on the cause of friction, by Mr. John Leslie, Professor of Mathematics in the University of Edinburgh.*

'If the two surfaces which rub against each 'other be rough and uneven, there is a necessary waste of force, occasioned by the grinding and abrasion of their prominences. But 'friction subsists after the contiguous surfaces 'are worked down as regular and smooth as 'possible. In fact, the most elaborate polish 'can operate no other change than to diminish 'the size of the natural asperities. The sur-

* See his ingenious and profound work on the Nature and Propagation of Heat, chap. xv, p. 299, &c.

'face of a body, being moulded by its internal 'structure, must evidently be furrowed, or 'toothed, or serrated. Friction is, therefore, 'commonly explained on the principle of the 'inclined plane, from the effort required to 'make the incumbent weight mount over a suc-'cession of eminences. But this explication, 'however currently repeated, is quite insuffici-'ent. The mass which is drawn along is not 'continually ascending; it must alternately rise 'and fall: for each superficial prominence will 'have a corresponding cavity; and since the 'boundary of contact is supposed to be hori-'zontal, the total elevations will be equalled 'by their collateral depressions. Consequent-'ly, if the lateral force might suffer a diminu-'tion in lifting up the weight, it would, the 'next moment, receive an equal increase by 'letting it down again; and those opposite ef-'fects, destroying each other, could have no 'influence whatever on the general motion.

'Adhesion seems still less capable of ac-'counting for the origin of friction. A perpen-'dicular force acting on a solid can evidently 'have no effect to impede its progress; and 'though this lateral force, owing to the una-'voidable inequalities of contact, may be sub-'ject to a certain irregular obliquity, the ba-'lance of chances must, on the whole, have the 'same tendency to accelerate, as to retard, the 'motion. If the conterminous surfaces were, 'therefore, to remain absolutely passive, no 'friction could ever arise. Its existence de-'monstrates an unceasing mutual change of fi-'gure, the opposite planes, during the passage, 'continually seeking to accommodate them-

'selves to all the minute and accidental varie-
'ties of contact. The one surface, being pres-
'sed against the other, becomes, as it were,
'compactly indented, by protruding some
'points and retracting others. This adaptation
'is not accomplished instantaneously, but re-
'quires very different periods to attain its
'*maximum*, according to the nature and rela-
'tion of the substances concerned. In some
'cases, a few seconds are sufficient; in others,
'the full effect is not produced till after the
'lapse of several days. While the incumbent
'mass is drawn along, at every stage of its ad-
'vance, it changes its external configuration,
'and approaches more or less towards a strict
'contiguity with the under surface. Hence the
'effort required to put it first in motion, and
'hence too, the decreased measure of friction,
'which, if not deranged by adventitious causes,
'attends generally an augmented rapidity. This
'appears clearly established by the curious ex-
'periments of Coulomb, the most original and
'valuable which have been made on that inter-
'esting subject. Friction consists in the force
'expended to raise continually the surface of
'pressure by an oblique action. The upper sur-
'face travels over a perpetual system of inclin-
'ed planes; but that system is ever changing,
'with alternate inversion. In this act, the in-
'cumbent weight makes incessant, yet unavail-
'ing efforts to ascend: for the moment it has
'gained the summits of the superficial promi-
'nences, these sink down beneath it, and the
'adjoining cavities start up into elevations,
'presenting a new series of obstacles, which
'are again to be surmounted; and thus the

'labours of Sisiphus are realized in the phe-
'nomena of friction.

'The degree of friction must evidently de-
'pend on the angles of the natural protuber-
'ances, and which are determined by the ele-
'mentary structure or the mutual relation of the
'two approximate substances. The effect of
'polishing is only to abridge those asperities,
'and increase their number, without altering in
'any respect their curvature or inflexions. The
'constant or successive acclivity produced by
'the ever-varying adaptation of the contiguous
'surfaces, remains, therefore, the same, and
'consequently the expense of force will still
'amount to the same proportion of the pressure.
'The intervention of a coat of oil, soap, or tal-
'low, by readily accommodating itself to the
'variations of contact, must tend to equalize
'it, and, therefore, must lessen the angles, or
'soften the contour, of the successively-emerg-
'ing prominences, and thus diminish likewise
'the friction which thence results.'

Having thus considered the origin, the nature, and the effects, of friction, we shall now attend to the method of lessening the resistance which it opposes to machinery. The most efficacious mode of accomplishing this, is to convert that species of friction which arises from one body being dragged over another, into that which is occasioned by one body rolling upon another. As this will always diminish the resistance, it may be easily effected by applying wheels or rollers to the sockets or bushes which sustain the gudgeons of large wheels, and the axles of wheel-carriages. Casatus* seems to have been

* Mechan. lib. ii, cap. i, p. 130.

the first who recommended this apparatus. It was afterwards mentioned by Sturmius,* and Wolfius;† but was not used in practice till Sully‡ applied it to clocks in the year 1716, and Mondran|| to cranes in 1725. Notwithstanding these solitary attempts to introduce friction-wheels, they seem to have attracted but little attention till the celebrated Euler examined and explained, with his usual accuracy, their nature and advantages.** The diameter of the gudgeons and pivots should be made as small as the weight of the wheel and the impelling force will permit. The gudgeons should rest upon two wheels as large as circumstances will allow, having their axes as near each other as possible, but no thicker than what is absolutely necessary to sustain the superincumbent weight. When these precautions are properly attended to, the resistance which arises from the friction of the gudgeons, &c. will be extremely trifling.††

* Miscellan. Berolinens. Tom. i, p. 306.

† Opera Mathematica, tom. ii, p. 684.

‡ Machines approuvées, tom. No. 177.

|| Id. No. 254.

** Memoires de l'Academie de Berlin 1748, p. 133.

†† Mr. Walker, a lecturer on Experimental Philosophy, has boldly pronounced friction-wheels to be 'expensive nonsense,' (System of Famil. Philos. v. i.) This gentlemen should have recollected that they were recommended by Euler, and many other distinguished philosophers, and, though this is by no means a sufficient reason for their adoption, yet we humbly conceive that the errors of the learned should always be opposed with respectful diffidence. We are of opinion, however, and we presume that every person who understands the subject will agree with us, that the friction-wheels, if properly executed, are of immense service, and that nothing but the ignorance or narrowness of the proprietors of machinery could have prevented them from being more generally adopted.

The effects of friction may likewise in some measure be removed by a judicious application of the impelling power, and by proportioning the size of the friction-wheels to the pressure which they severally sustain. If we suppose, for example, that the weight of a wheel, whose iron gudgeons move in bushes of brass, is 100 pounds; then the friction arising from both its gudgeons will be equivalent to 25 pounds. If we suppose also that a force equal to 40 pounds is employed to impel the wheel, and acts in the direction of gravity, as in the case of overshot wheels, the pressure of the gudgeons upon their supports will thus be 140 pounds, and the friction 35 pounds. But if the force of 40 pounds could be applied in such a manner as to act in direct opposition to the wheel's weight, the pressure of the gudgeons upon their supports would be 100—40, or 60 pounds, and the friction only 15 pounds. It is impossible, indeed, to make the moving force act in direct opposition to the gravity of the wheel, in the case of water-mills; and it is often impracticable for the engineer to apply the impelling power but in a given way: but there are many cases in which the moving force may be so exerted, as at least not to increase the friction which arises from the wheel's weight.

When the moving force is not exerted in a perpendicular direction, but obliquely, as in undershot wheels, the gudgeon will press with greater force on one part of the socket than on any other part. This point will evidently be on the side of the bush opposite to that where the power is applied, and its distance from the lowest point of the socket, which is supposed circular and concentric with the gudgeon, being

called x, we will have Tang. $x=\frac{H}{V}$, that is, the tangent of the arch contained between the point of greatest pressure and the lowest point of the bush, is equal to the sum of all the horizontal forces, divided by the sum of all the vertical forces and the weight of the wheel; H representing the former, and V the latter quantities. The point of greatest pressure being thus determined, the gudgeon must be supported at that part by the largest friction-wheel, in order to equalize the friction upon their axles.

The application of these general principles to particular cases is so simple as not to require any illustration. To aid the conceptions, however, of the practical mechanic, we may mention two cases in which friction-wheels have been successfully employed.

Mr. Gottlieb, the constructor of a new crane, has received a patent for what he calls an anti-attrition axle tree, the beneficial effects of which he has ascertained by a variety of trials. It consists of a steel roller about 4 or 6 inches long, which turns within a groove cut in the inferior part of the axle. When wheel-carriages are at rest, Mr. Gottlieb has given the friction-wheel its proper position; but it is evident that the point of greatest pressure will change when they are put in motion, and will be nearer the front of the carriage. This point, however, will vary with the weight of the load; but it is sufficiently obvious, that the friction-roller should be at a little distance from the lowest point of the axle-tree.

Mr. Garnett, of Bristol,* has applied friction-rollers in a different manner, which does not,

* Now of New-Brunswick, State of New-Jersey.

like the preceding method, weaken the axle-tree. Instead of fixing the rollers in the iron part of the axle, he leaves a space between the nave and the axis, to be filled with equal rollers almost touching each other. The axes of these rollers are inserted in a circular ring at each end of the nave, and these rings, and consequently the rollers, are kept separate and parallel, by means of small bolts passing between the rollers from one side of the nave to the other.

In wheel-carriages constructed in the common manner with conical rims, there is a great degree of resistance occasioned by the friction of the linch-pins on the external part of the nave, which the ingenious mechanic may easily remove by a judicious application of the preceding principles.

As it appears from the experiments of Ferguson and Coulomb, that the least friction is generated when polished iron moves upon brass, the gudgeons and pivots of wheels, and the axles of friction-rollers, should all be made of polished iron, and the bushes in which these gudgeons move, and the friction-wheels should be formed of polished brass.*

When every mechanical contrivance has been adopted for diminishing the obstruction which arises from the attrition of the communicating parts, it may be still farther removed by the judicious application of unguents. The most proper for this purpose are swine's grease and tallow, when the surfaces are made of wood; and oil, when they are of metal. When the force with which the surfaces are pressed toge-

* M. de la Hire recommends the sockets or bushes to be made square and not concave.

ther is very great, tallow will diminish the friction more than swine's grease. When the wooden surfaces are very small, unguents will lessen their friction a little, but it will be greatly diminished if wood moves upon metal greased with tallow. If the velocities, however, be increased, or the unguent not often enough renewed, in both these cases, but particularly in the last, the unguent will be more injurious than useful. The best mode of applying it, is to cover the rubbing surfaces with as thin a stratum as possible, for the friction will then be a constant quantity, and will not be increased by an augmentation of velocity.

In small works of wood, the interposition of the powder of black lead has been found very useful in relieving the motion. The ropes of pulleys should be rubbed with tallow, and whenever the screw is used, the square threads should be preferred.

MECHANICS.

On the Nature and Operation of Fly-wheels.

A FLY in mechanics is a heavy wheel or cylinder which moves rapidly upon its axis, and is applied to machines for the purpose of rendering uniform a desultory or reciprocating motion, arising either from the nature of the machinery employed, from an equality in the resistance to be overcome, or from an irregular application of the impelling power. When the first mover is inanimate, as wind, water, and steam, an inequality of force obviously arises, from a variation in the velocity of the wind, from an increase or decrease of water occasioned by sudden rains, or from an augmentation or diminution of the steam in the boiler, produced by a variation in the heat of the furnace; and, accordingly, various methods have been adopted for regulating the action of these variable powers. The same inequality of force obtains when machines are moved by horses or men. Every animal exerts its greatest strength when first set to work. After pulling for some time its strength will be impaired, and when the resistance is great, it will take frequent, though short relaxations, and then commence its labour with renovated vigour. These intervals of rest and vigorous exertion must always produce a variation in the velocity of the machine, which

ought particularly to be avoided, as being detrimental to the communicating parts as well as the performance of the machine, and injurious to the animal which is employed to drive it. But if a fly, consisting either of cross bars, or a massy circular rim, be connected with the machinery, all these inconveniences will be removed. As every fly-wheel must revolve with great rapidity, the momentum of its circumference must be very considerable, and will consequently resist every attempt either to accelerate or retard its motion. When the machine, therefore, has been put in motion, the fly-wheel will be whirling with a uniform celerity, and with a force capable of continuing that celerity when there is any relaxation in the impelling power. After a short rest, the animal renews his efforts, the machine is now moving with its former velocity, and these fresh efforts will have a tendency to increase that velocity: the fly, however, now acts as a resisting power, receives the greatest part of the superfluous motion, and causes the machinery to preserve its original celerity. In this way the fly secures to the engine a uniform motion, whether the animal takes occasional relaxation or exerts his force with redoubled ardour.

We have already observed, that a desultory or variable motion frequently arises from the inequality of resistance, or work to be performed. This is particularly manifest in thrashing mills, on a small scale, which are driven by water. When the corn is laid unequally on the feeding-board, so that too much is taken in by the fluted rollers, this increase of resistance instantly affects the machinery, and communi

cates a desultory or irregular motion even to the water-wheel or first mover. This variation in the velocity of the impelling power may be distinctly perceived by the ear in a calm evening, when the machine is at work. The best method of correcting these irregularities, is to employ a fly-wheel, which will regulate the motion of the machine, when the resistance is either augmented or diminished. In machines built upon a large scale, there is no necessity for the interposition of a fly, as the *inertia* of the machinery supplies its place, and resists every change of motion that may be generated by an unequal admission of the corn.

A variation in the velocity of engines arises also from the nature of the machinery. Let us suppose that a weight of 1000 pounds is to be raised from the bottom of a well 50 feet deep, by means of a bucket attached to an iron chain which winds round a barrel or cylinder: and that every foot length of this chain weighs two pounds. It is evident that the resistance to be overcome in the first moment is 1000 pounds, added to 50 pounds, the weight of the chain; and that this resistance diminishes gradually, as the chain coils round the cylinder, till it becomes only 1000 pounds, when the chain is completely wound up. The resistance therefore decreases from 1050 to 1000 pounds; and, if the impelling power be inanimate, the velocity of the bucket will gradually increase; but if an animal be employed, it will generally proportion its action to the resisting load, and must therefore pull with a greater or less force according as the bucket is near the top or bottom of the well. In this case, however, the assistance of a fly may be

dispensed with, because the resistance diminishes uniformly, and may be rendered constant, by making the barrel conical, so that the chain may wind upon the part nearest the vertex at the commencement of the motion, the diameter of the barrel gradually increasing as the weight diminishes. In this way, the variable resistance will be equalized much better than by the application of a fly-wheel; for the fly, having no power of its own, must necessarily waste the impelling power.

Having thus pointed out the chief causes of a variation in the velocity of machines, and the method of rendering it uniform by the invention of fly-wheels, the utility, and, in some instances, the necessity of this piece of mechanism, may be more obviously illustrated by showing the propriety of its application in particular cases.

In the description which has been given of Vauloue's pile engine in Vol. I. the reader must have remarked a striking instance of the utility of fly-wheels. The ram *Q* is raised between the guides *bb*, by means of horses acting against the levers *SS*; but as soon as the ram is elevated to the top of the guides, and discharged from the follower *G*, the resistance against which the horses have been exerting their force, is suddenly removed, and they would instantaneously tumble down, were it not for the fly *O*. This fly is connected with the drum *B*, by means of the trundle *X*; and as it is moving with a very great force, it opposes a sufficient resistance to the action of the horses, 'till the ram is again taken up by the follower.

PLATE IX. Fig. 1.

When machinery is driven by a single-stroke steam-engine, there is such an inequality in the impelling power, that, for 2 or 3 seconds, it does not act at all. During this interval of inactivity, the machinery would necessarily stop, were it not impelled by a massy fly-wheel of a great diameter, revolving with rapidity, till the moving power again resumes its energy.

If the moving power be a man acting with a handle or winch, it is subject to great inequalities. The greatest force is exerted when the man pulls the handle upwards from the height of his knee, and he acts with the least force when the handle, being in a vertical position, is thrust from him in a horizontal direction.—The force is again increased when the handle is pushed downwards by the man's weight, and it is diminished, when the handle, being at its lowest point, is pulled towards him horizontally. But when a fly is properly connected with the machinery, these irregular exertions are equalized, the velocity becomes uniform, and the load is raised with an equable and steady motion.

In many cases, where the impelling force is alternately augmented and diminished, the performance of the machine may be increased by rendering the resistance unequal, and accommodating it to the inequalities of the moving power. Dr. Robison observes, that 'there are 'some beautiful specimens of this kind of ad'justment in the mechanism of animal bodies.'

Besides the utility of fly-wheels as regulators of machinery, they have been employed for accumulating or collecting power. If motion be communicated to a fly-wheel by means of a small force, and if this force be continued till

the wheel has acquired a great velocity, such a quantity of motion will be accumulated in its circumference as to overcome resistances, and produce effects, which could never have been accomplished by the original force. So great is this accumulation of power, that a force equivalent to 20 pounds, applied for the space of 37 seconds to the circumference of a cylinder, 20 feet diameter, which weighs 4713 pounds, would, at the distance of one foot from the centre, give an impulse to a musket ball equal to what it receives from a full charge of gunpowder. In the space of 6 minutes and 10 seconds, the same effect would be produced, if the cylinder were driven by a man who constantly exerted a force of 20 pounds at a winch 1 foot long.*

This accumulation of power is finely exemplified in the sling. When the thong which contains the stone is swung round the head of the slinger, the force of the hand is continually accumulating in the revolving stone, till it is discharged with a degree of rapidity which it could never have received from the force of the hand alone. When a stone is projected from the hand itself, there is even then a certain degree of force accumulated, though it only moves through the arch of a small circle. If we fix the stone in an opening at the extremity of a piece of wood 2 feet long, and discharge it in the usual way, there will be more force accumulated than with the hand alone, for the stone describes a larger arch in the same time, and must therefore be projected with greater force.

* This has been demonstrated by Mr. Atwood. See his Treatise on Rectilineal and Rotatory Motion.

When coins or medals are struck, a very considerable accumulation of power is necessary, and this is effected by means of a fly.—The force is first accumulated in weights fixed in the end of the fly, this force is communicated to two levers, by which it is further condensed; and from these levers it is transmitted to a screw, by which it suffers a second condensation. The stamp is then impressed on the coin or medal by means of this force, which was first accumulated by the fly, and afterwards augmented by the intervention of two mechanical powers.*

Notwithstanding the great advantages of fly-wheels, both as regulators of machines, and collectors of power, their utility wholly depends upon the position which is assigned them, relative to the impelled and working points of the engine. For this purpose, no particular rules can be laid down, as their position depends altogether on the nature of the machinery. We may observe, however, in general, that when fly-wheels are employed to regulate machinery, they should be near the impelling power; and, when used to accumulate force in the working point, they should not be far distant from it.—In hand-mills for grinding corn, the fly is for the most part very injudiciously fixed on the axis to which the winch is attached; whereas, it should always be fastened to the upper millstone, so as to revolve with the same rapidity. In the first position, indeed, it must equalize the varying efforts of the power which moves the winch;

* In the article on the Steam-Engine, the reader will see an account of a new kind of fly, called the conical pendulum, which Messrs. Watt and Boulton have very ingeniously employed for regulating the admission of steam into the cylinder.

but when it is attached to the turning mill-stone, it not only does this, but contributes very effectually to the grinding of the corn.

Dr. Desaguliers mentions an instance of a blundering engineer, who applied a fly-wheel to the slowest mover of the machine, instead of the swiftest. The machine was driven by four men, and when the fly was taken away, one man was sufficiently able to work it. The error of the workman arose from his conceiving, like many others, that the fly added power to the machine; but we presume, that Dr. Desaguliers himself has been accessary to this general misconception of its nature, by denominating it a *mechanical power.** By the interposition of a fly, however, as the Doctor well knew, we gain no mechanical force; the impelling power, on the contrary, is wasted, and the fly itself even loses some of the force which it receives, by the resistance of the air.

* Dr. Desaguliers calls it a *mechanical organ;* but he gives the same appellation to the lever, and all the other mechanical powers. See his Experimental Philosophy, Vol. i, p. 344.

MECHANICS.

On Wheel-Carriages.

Mr. Ferguson, in his IVth Lecture, has treated the subject of wheel-carriages with great perspicuity, and has communicated much practical information of considerable importance.—Many of the prejudices, however, which he has there encountered, and several others which have escaped his notice, still continue to prevail in this country; and as some of these have been countenanced even by ingenious men, we are laid under a more urgent necessity of attempting to develope the source of their errors, and of regulating the practice of the mechanic by the deductions of theory. The very assistance which theory has in this case furnished to the artist, has been rendered both useless and injurious by an erroneous application; and we may safely affirm, that there is no species of machinery where less science is displayed, than in the construction and position of carriage-wheels. The few imperfect hints which we are able to convey upon this subject, regard the formation and position of the wheels, the line of traction, and the method of disposing the load which is to be drawn. To some

of these we solicit the reader's attention, as being entirely new, and apparently leading to consequences of high importance. PLATE XXXVIII.

On the formation of carriage-wheels.

When the wheels of carriages either move upon a level surface, or overcome obstacles which impede their progress, they act as mechanical powers, and may be reduced to levers of the first kind. In order to elucidate this remark, which is of great importance in the present discussion, let *A* be the centre, and *BCN* the circumference of a wheel 6 feet in diameter, and let the impelling power *P*, which is attached to the extremity of a rope *ADP*, passing over the pulley *D*, act in the horizontal direction *AD*. Then, if the wheel be not affected by friction, it will be put in motion upon the level surface *MB*, when the power *P* is infinitely small. For, since the whole weight of the wheel rests on the ground at the point *B*, which is the fulcrum of the lever *AB*, the distance of the weight from the centre of motion will be nothing, and therefore the mechanical energy of the smallest power *P*, acting at the point *A*, with a length of lever *AB*, will be infinitely great when compared with the resistance of the weight to be raised; and this will be the case, however small the lever *AB*, and however great the weight of the wheel.—But as the wheels of carriages are constantly meeting with impediments, let *C* be an obstacle 6 inches high, which the wheel is to surmount. Then the spoke *AC* will represent the lever, *C* its fulcrum, *AD* the direction of the power; and if the wheel weigh 100 pounds, Fig. 5.

we may represent it by a weight *W*, fixed to the wheel's centre *A*, or to the extremity of the lever *CA*, and acting in the perpendicular direction *AB*, in opposition to the power *P*.— Now, the mechanical energy of the weight *W* to pull the lever round its fulcrum in the direction *AE*, is represented by *CE*, while the mechanical energy of an equal weight *P* to pull it in the opposite direction *AF*, is represented by *CF*; an equilibrium, therefore, will be produced, when the power *P* is to the weight *W* as *CE* to *CF*, or as the sine is to the cosine of an angle, whose versed sine is equal to the height of the obstacle to be surmounted; for *EB*, the height of the mound *C*, is the versed sine of the angle *BAC*, *CE* is the sine, and *CF* the cosine of the same angle. In the present case, where *EB* is 6 inches, and *AB* 3 feet, *EB*, the versed sine, will be 1666, &c. when *AB* is 1000; and, consequently, the angle *BAC* will be 33° 33′, and *CE* will be to *CF* as 52 to 83, or as 66 to 100. A weight *P*, therefore, of 66 pounds, acting in a horizontal direction, will balance a wheel 6 feet diameter, and 100 pounds in weight, upon an obstacle 6 inches high; and a small additional power will enable it to surmount that obstacle. But if the direction *AD* of the power be inclined to the horizon, so that the point *D* may rise towards *H*, the line *FC*, which represents the mechanical energy of *P*, will gradually increase, till *DA* has reached the position *HA*, perpendicular to *AC*, where its mechanical energy, which is now a maximum, is represented by *AC*, the radius of the wheel; and since *EC* is to *CA* as 53 to 1000, a little more than 53 pounds will be sufficient

for enabling the wheel to overcome the obstacle.

Proceeding in this way, it will be found, that the power of wheels to surmount eminences increases with their diameter, and is directly proportional to it, when their weight remains the same, and when the direction of the power is perpendicular to the lever which acts against the obstacle. Hence we see the great advantages which are to be derived from large wheels, and the disadvantages which attend small ones. There are some circumstances, however, which confine us within certain limits in the use of large wheels. When the radius *AB* of the wheel, is greater than *DM* the height of the pulley, or of that part of the horse to which the rope or pole *DA* is attached, the direction of the power, or the line of traction *AD*, will be oblique to the horizon as *Ad*, and the mechanical energy of the power will be only *Ae*, whereas, it was represented by *AE*, when the line of traction was in the horizontal line *DA*. Whenever the radius of the wheel, therefore, exceeds *four feet and a half*, the height of that part of the horse, to which the traces should be attached,* the line of traction *AD* will incline to the horizon, and by declining from the perpendicular *AH*, its mechanical effort will be diminished; and since the load rests upon an inclined plane, the trams or poles of the cart will rub against the flanks of the horse even in level roads, and

* According to M. Couplet the distance of this part of the horse from the ground is generally *three feet and a half.* (Mem. Acad. Royale 1733. 8vo. p. 75) In horses of a common size, however, it is seldom below *four feet and a half.*

still more severely in descending ground. Notwithstanding this diminution of force, however, arising from the unavoidable obliquity of the impelling power, wheels exceeding four and a half feet radius have still the advantage of smaller ones; but their power to overcome resistances does not increase so fast as before.—Hitherto we have supposed the weight of the large and small wheels to be the same, but, it is evident, that when we augment their diameter, we add greatly to their weight; and, by thus increasing the load, we sensibly diminish their power.

From these remarks we see the superiority of great wheels to small ones, and the particular circumstances which suggest the propriety of making the wheels of carriages less than $4\frac{1}{2}$ feet radius. Even this size is too great, as we shall afterwards show, when speaking of the line of traction; and we may safely assert, that they should never exceed 6 feet in diameter, and should never be less than $3\frac{1}{2}$ feet. When the nature of the machine will permit, large wheels should always be preferred, and small ones should never be adopted, unless we are compelled to employ them by some unavoidable circumstances in the construction.* This maxim, which has been inculcated by every person who has written on the subject, seems

* For the advantage of those who wish to study this subject with greater attention, and with the view also of recommending the use of large wheels, we shall subjoin the following references to the works of eminent men, who have held the same opinion upon this point: Mersennus's Geom. p. 459. Herigon, Mecan. Prob. 16. Schol. Wallis's Mecan. 3. 7, Prob. 3. Schol. § 15. Phil. Trans. v. 15, p. 856.—Camus, Traite des Forces Mouvantes, Prop. 28, 30: and Deparcieux sur le Tirage des Chevaux, Mem. Acad. Royale 1760, p. 263, 4to.

to have been strangely neglected by the practical mechanics of this country. The fore-wheels of our carriages are still unaccountably small, and we have seen carts moving upon wheels scarcely *fourteen* inches in diameter.—The workman, indeed, will tell us, that in the one case the wheels are made small for the conveniency of turning, and in the other for facilitating the loading of the cart; but how trifling are these advantages, when compared with that diminution of the horses' power which necessarily results from the use of small wheels. A convenient place for turning with large fore-wheels, which is not frequently required, may be procured by going to the end of a street; and a few additional turns of a windlass will be sufficient to raise the heaviest load into carts which are mounted upon high wheels. It has been objected against large fore-wheels, that the horses, when going down a declivity, cannot so easily prevent the carriages from running downwards; but this very objection, trifling as it is, is a plain confession, that large fore-wheels are advantageous both in horizontal and inclined planes, otherwise their tendency downwards would not be greater than that of small ones.*

Having thus ascertained the superiority of large wheels, we are now to determine on the shape which ought to be assigned them. Every person who is not influenced by preconceived notions, would affirm, without hesitation, that if

* From some experiments on wheel carriages, Mr. Walker conceives that the greatest advantage was obtained when the hind wheels were 5 feet 6 inches in diameter, and the fore ones 4 feet 8 inches, whereas the large wheels are in general only 4 feet 8 inches, and the small ones 3 feet 8 inches.—System of Familiar Philosophy, v. i, p. 130.

PLATE XXXVIII. the wheels are to consist of solid wood, they should be portions of a cylinder; and if they are to be composed of naves, spokes, and fellies, that the rim of the wheel ought to be cylindrical, and the spokes perpendicular to the naves. But some men, desirous of being inventors, have renounced this simple shape, and
Fig. 6. adopted the more complicated form of Fig. 6. where the rim *ArsB* is conical, and the spokes inclined to the naves.* Philosophers, too, have found a reason for this change, and it has been adopted in every country, more from the authority of names than the force of argument. It is with the greatest diffidence, however, that we presume to contradict a practice which has been defended by the most celebrated mechanics, but we trust that the reader's indulgence will be proportioned to the solidity of the reasons upon which this difference of sentiment is founded.

The form represented in Fig. 6. then, is liable to two objections, namely, the inclination of the spokes, and the conical figure of the rim. When the spokes are inclined to the nave, the wheels are said to be concave, or dishing, and they are recommended by Mr. Ferguson, and every other writer on mechanics, from the numerous advantages which are said to attend them. By extending the base of the carriage, they prevent it from being easily overturned, they hinder the fellies from rubbing against the load, or the sides of the cart, and when one wheel falls into a rut; and, therefore, supports, more than one half of the load, the spokes

* This inclination is about 1 inch out of 11, or *mA* is generally 3 inches when the diameter of the wheel is 5½ feet

are brought into a perpendicular position, which renders them more capable of supporting this additional weight. Now, it is evident, that the second of these advantages is very trifling, and may be obtained when it is wanted, by interposing a piece of board between the wheel and the load. The other two advantages exist only in very bad roads; and if they be necessary, which we very much question, in a country like this, where the roads are so excellently made, and so regularly repaired, the same advantage can easily be procured by making the axle-tree a few inches longer, and increasing the strength of the spokes. But it is allowed on all hands, that perpendicular spokes are preferable on level ground. The inclination of the spokes, therefore, which renders concave wheels advantageous in rugged and uneven roads, renders them disadvantageous when the roads are in good order; and where the good roads are more numerous than the bad ones, as they certainly are in this country, the disadvantages of concave wheels must overbalance their advantages. It is true, indeed, that in concave wheels, the spokes are in their strongest position when they are exposed to the severest strains, that is, when one wheel is in a deep rut, and sustains more than one half of the load; but it is equally true, that in level ground, where the spokes are in their weakest position, a less severe strain, by continuing for a much longer time, may be equally, if not more detrimental to the wheel.*

* Mr. Anstice, in his excellent Treatise on wheel-carriages, recommends concave wheels; but candidly allows that 'some 'disadvantages attend this contrivance; for the carriage thus

PLATE XXXVII. Upon these observations we might rest the opinion which we have been maintaining, and appeal for its truth to the judgment of every intelligent and unbiassed mind; but we shall go a step farther, and endeavour to show that concave dishing wheels are more expensive, more injurious to the roads, more liable to be broken by accidents, and less durable in general than those wheels in which the spokes are perpendicular to the naves. By inspecting Fig. 6. Fig. 6. it will appear that the whole of the pressure which the wheel *AB* sustains, is exerted along the inclined spoke *ps*, and, therefore acts obliquely upon the level ground *nD*, whether the rims be conical or cylindrical.—This oblique action must necessarily injure the roads, by loosening the stones more between *B* and *D* than between *B* and *n*; and if the load were sufficiently great, the stones would start up between *s* and *D*. The texture of the roads, indeed, is sufficiently firm to prevent this from taking place; but, in consequence of the oblique pressure, the stones between *s* and *D* will at least be loosened, and, by admitting the rain, the whole of the road will be materially damaged. But when the spokes are perpendicular to the nave as *pn*, and when the rims *mA*, *nB*, are cylindrical, or parallel to the ground, the weight sustained by the wheel will act perpendicularly upon the road, and however much that weight is increased, its ac-

'takes up more room upon the road, which makes it more un-'manageable; and when it moves upon plane ground, the spokes 'not only do not bear perpendicularly, by which means their 'strength is lessened, but the friction upon the nave and axle 'is made unequal, and the more so the more they are dished.'

tion can have no tendency to derange the materials of which it is composed, but is rather calculated to consolidate them, and render the road more firm and durable. Plate XXXVI.

It was observed that concave wheels are more expensive than plane ones. This additional expense arises from the greater quantity of wood and workmanship which the former require; for, in order that dishing wheels may be of the same perpendicular height as plane ones, the spokes of the former must exceed in length those of the latter, as much as the hypothenuse *oA* of the triangle *oAm* exceeds the side *om*; and therefore the weight and the resistance of such wheels must be proportionably great. The inclined spokes, too, cannot be formed nor inserted with such facility as perpendicular ones. The extremity of the spoke which is fixed into the nave is inserted at right angles to it, in the direction *op*, and if the rims be cylindrical, the other spoke should be inserted in a similar manner, while the intermediate portion has an inclined position. There are, therefore, two flexures or bendings in the spokes of concave wheels, which require them to be formed out of a larger piece of wood than if they had no such flexures, and render them liable to be broken by any sudden strain at the points of flexure. Fig. 6.

In the comparison which we have now been stating, between the merits of concave and plane wheels, we have taken for granted what has been uniformly stated by the advocates of the former, that when one of the wheels falls into a rut, or surmounts an eminence, the lowest sustains much more than one half of the load. Now though it is true that the lower wheel supports

PLATE XXXVIII. more than one half of the load, yet we deny that it bears so much as has generally been supposed,* and we shall prove the assertion, by pointing out a method of ascertaining the additional weight which is transferred to one wheel by any given elevation of the other. Fig. 7. Let *AMOC* represent a cart loaded with coals or lime, or any other material which fills it to the top, and let *AB* be a horizontal line on the surface of a level road. Then, if the wheel *A* remain fixed, and the wheel *C* be raised to any height, its lower extremity *C* will describe the arch *BC* round the centre *A*, while the centre of gravity *D*, of the whole machine and load, will move in the arch *NM* round the same centre. Now let us suppose that *BC* is an eminence which the wheel *C* has to surmount, and that it has arrived at the top of it, it is required to find what proportion of the load is sustained by each wheel. Bisect the horizontal line *AB* in *e*, and from *e* draw *ed* at right angles to *AB*, and meeting the arch *NM* in the point *d*, join *AC*, *Ad*, *AD*, and from the point *D* let fall the perpendicular *DE*. The point *d* will be the centre of gravity of the load when the points *C* and *B* coincide; that is, when the wheels are resting on the horizontal plane *AB*. For, since in this case each wheel bears an equal part of the weight, the line of direction, in a vertical line passing through the centre of gravity, will cut the base *AB*, so that *Ae* will be to *eB* as the weight upon the wheel *A* to the weight upon

* Mr. Ferguson observes (vol. i.) that the wheel which falls into the rut bears *much more* of the weight than the other; and, a little afterwards, that it bears *most of* the weight of the load.

weight upon the wheel *A* to the weight upon *C*; and, therefore, *ed* will be the line of direction. and the point *d* where it cuts the circle *NM* in which the centre of gravity moves, will be the centre of gravity of the load in a horizontal position. Now as *D* is the centre of gravity when the cart is in its inclined position, the perpendicular *DE* will be the line of direction. and the weight sustained by the wheel *A* will be to that sustained by *C* as *EB* to *EA*, or *Ee* will represent the additional weight transferred upon *A*, when *AB* represents the whole of the load. But *Ee* can be easily determined for any value of *BC*, the height of the obstacle. For, while the point *C* moves from *B* to *C*, the centre of gravity rises from *d* to *D*, so that *Dd* and *BC* are similar arches, and *AB*, *Ad*, *BC*, are known, *AB* being the distance between the wheels, and *Ad* being equal to the square-root of the sum of the squares of *Ae*, the half of that distance, and *de* the height of the centre of gravity (Eucl. 1, 47), and *BC* being the height of the eminence. But since *de*, the sine of the arch *dN*, is known, *dN* is known, and also *DN*, the sum of the two arches, *Dd*, *dN*. The cosines *AE*, *Ae*, of the arches *DN*, *dN*, are therefore known, and consequently *Ee*, their difference may be determined; or, otherwise, *Ee* is the difference of the versed sines *EN*, *eN*, of the same arches. Let us now take a particular value of *BC*, or rather of *Co*, the perpendicular height of the eminence, and call it 12 inches, for even in the worst roads there are few eminences which are greater than this. Let *AB*, the distance between the wheels, be 6 feet, and *de*, the height of the centre of gravity, 4 feet, then *Co* will be $\frac{1}{6}$ of the radius *AB*;

and making $AB=1000000$, CO will be 166666, which, being the natural sine of the arch BC, gives 9° 35′ for the arch BC, and for the similar arch Dd. Now, since Ae is 3 feet, and de 4 feet, the sum of their squares will be 25, and its square-root 5 will be the length of the hypothenuse Ad, or the radius of the circle NDM. Then, making Ad radius, or 1000000, de the sine of the arch dN will be $\frac{4}{5}$ of it, or 800000; and, therefore, the arch dN will be 53° 8′, and the arch DN, 62° 43′. But AE, the cosine of the arch DN, is=458391=or $\frac{46}{100}$ nearly, of $AD=5$ feet, and is therefore equal to 2 feet 3 inches and 6 tenths; consequently $Ee=Ae-AE$ will be 8 inches and 4 tenths, which is nearly $\frac{1}{9}$ of AB. We may, therefore, conclude that the additional weight sustained by the wheel A, while the other wheel is rising over an obstacle 12 inches in perpendicular height is $\frac{1}{9}$ only of the whole load; or that $\frac{2}{9}$ of the pressure upon the wheel C is transferred to the wheel A, while surmounting an eminence 12 inches high. If one of the wheels fall into a rut 12 inches deep, the same conclusion will result; and we may affirm, that as the ruts and eminences which are generally to be met with even in bad roads, are for the most part much less than 12 inches in depth or height, such a small proportion of the load will be transferred to the lower wheel, that there is no necessity for inclining the spokes in order to sustain the additional weight. When the cart is loaded with stones, or any heavy substance, the centre of gravity will be lower than d, so that a less proportion of the weight will be transferred to one wheel by the elevation of the other, and when it is loaded with hay, or any light mate-

rial, the lower wheel will sustain a greater proportion of the load. PLATE XXXVIII.

We shall now dismiss the subject of concave wheels with one observation more, and we beg the reader's attention to it, because it appears to be decisive of the question. The obstacles which carriages have to encounter, are almost never spherical protuberances, that permit the elevated wheel to resume by degrees its horizontal position. They are generally of such a nature, that the wheel is instantaneously precipitated from their top to the level ground. Now the momentum with which the wheel strikes the ground is very great, arising from a successive accumulation of force. The velocity of the wheel *C* is considerable when it reaches the top of the eminence, and while it is tumbling into the horizontal line *AB*, the centre of gravity is falling through the arch *Dd*, and the wheel *C* is receiving gradually that proportion of the load which was transferred to *A*, till, having recovered the whole, it impinges against the ground with great velocity and force. But in concave wheels, the spoke which then strikes the ground is in its weakest position; and, therefore, much more liable to be broken by the impetus of the fall, than the spokes of the lower wheel by the mere transference of additional weight. Whereas, if the spokes be perpendicular to the nave, they receive this sudden shock in their strongest position, and are in no danger of giving way to the strain. Fig. 7.

In the preceding observations, we have supposed the rims of the wheels to be cylindrical, as *AC*, *BD*. In concave wheels, however, the rims are uniformly made of a conical form, as *Ar*, *Bs*, which not only increases the disadvan- Fig. 6.

tages that we have ascribed to them, but adds many more to the number. Mr. Cumming, in a late treatise on wheel-carriages, solely devoted to the consideration of this single point, has shown with great ability the disadvantages of conical rims, and the propriety of making them cylindrical; but we are of opinion that he has ascribed to conical rims several disadvantages which arise chiefly from an inclination of the spokes. He insists much upon the injury done to the roads by the use of conical rims, yet though we are convinced that they are more injurious to pavements and highways than cylindrical rims, we are equally convinced, that this injury is occasioned chiefly by the oblique pressure of the inclined spokes. The defects of conical rims are so numerous and palpable that it is wonderful how they should have been so long overlooked. Every cone that is put in motion upon a plane surface, will revolve round its vertex, and if force be employed to confine it to a straight line, the smaller parts of the cone will be dragged along the ground, and the friction greatly increased. Now when a cart moves upon conical wheels, one part of the cone rolls while the other is dragged along, and though confined to a rectilineal direction by external force, their natural tendency to revolve round their vertex occasions a great and continued friction upon the linch-pin, the shoulder of the axle-tree, and the sides of deep ruts.

The shape of the wheels being thus determined, we must now attend to some particular parts of their construction. The iron plates of which the rims are composed, should never be less than 3 inches in breadth, as narrower rims sink deep into the ground, and therefore injure

the roads and fatigue the horses. Mr. Walker indeed attempts to throw ridicule upon the act of parliament which enjoined the use of broad wheels, but he does not assign any sufficient reason for his opinion, and ought to have known that several excellent and well-devised experiments were lately instituted by Boulard and Marguerou,* which evinced in the most satisfactory manner, the great utility of broad wheels. Upon this subject an observation occurs to us which has not been generally attended to, and which appears to remove all the objections which can be urged against broad rims. When any load is supported upon two points in a horizontal plane, each point supports one half of the weight; if the points be increased to four, each will sustain one fourth of the load, and so on, the pressure upon each point of support diminishing as the number of points increases. If a weight, therefore, be supported by a broad surface, the points of support are infinite in number, and each of them will bear an infinitely small portion of the load; and, in the same way, every finite portion of this surface will sustain a part of the weight inversely proportional to the number of similar portions which the surface contains. Let us now suppose that a cart, carrying a load of 16 hundred weight, is supported upon wheels whose rims are four inches in breadth, and that one of the wheels passes over four stones, each of them an inch broad and equally high, and capable of being pulverized only by a pressure of 400 weight. Then, as each wheel sustains one half

* The memoir which contains an account of these experiments, was presented to the Academy of Lyons, and is published in the Journal de Physique, tom xix, p. 424.

of the load, and as the wheel which passes over the stones has 4 points of support, each stone will bear a weight of 200 pounds, and therefore will not be broken. But if the same cart, with rims only 2 inches in breadth, should pass the same way, it will cover only 2 of the stones; and the wheel having now only 2 points of support, each stone will be pressed with a weight of 400 pounds, and will therefore be reduced to powder. Hence we may infer, that narrow wheels are, in another point of view, injurious to the roads, by pulverizing the materials of which they are composed.

As the rims of wheels wear soonest at their edges, they should be made thinner in the middle, and ought to be fastened to the fellies with nails of such a kind, that their heads may not rise above the surface of the rim. In some military waggons, we have seen the heads of these nails rising an inch above the rims, which not only destroy the pavements of streets, but oppose a continual resistance to the motion of the wheel. If these nails were 8 in number, the wheel would experience the same resistance as if it had to surmount 8 obstacles, 1 inch high, during every revolution. The fellies on which the rims are fixed, should, in carriages, be $3\frac{1}{2}$ inches deep; and in waggons, 4 inches. The naves should be thickest at the place where the spokes are inserted, and the holes in which the spokes are placed should not be bored quite through, as the grease upon the axletree would insinuate itself between the spoke and the nave, and prevent that close adhesion which is necessary to the strength of the wheel.

On the position of the wheels.

It must naturally occur to every person reflecting upon this subject, that the axle-trees should be straight, and the wheels perfectly parallel, so that they may not be wider at their highest than at their lowest point, whether they be of a conical or of a cylindrical form. In this country, however, the wheels are always made concave, and the ends of the axletrees are *universally* bent downwards in order to make them spread at the top and approach nearer below.—In some carriages which we have examined, the wheels were only 4 feet 6 inches in diameter, the distance of the wheels at top was fully 6 feet, and their distance below only 4 feet 8 inches. By this foolish practice the very advantages which may be derived from the concavity of the wheels are completely taken away, while many of the disadvantages remain; more room is taken up in the coach-house, and the carriage is more liable to be overturned by the contraction of its base.

With some mechanics it is a practice to bend the ends of the axle-trees forwards, and thus make the wheels wider behind than before.—This blunder has been strenuously defended by Mr. Henry Beighton, who maintains that wheels in this position are more favourable for turning, since, when the wheels are parallel, the outermost would press against the linchpin, and the innermost would rub against the shoulder of the axle-tree. In rectilineal motions, however, these converging wheels engender a great deal of friction both on the axle and the

PLATE XXXVIII. ground, and must therefore be more disadvantageous than parallel ones. This indeed is allowed by Mr. Beighton; but he seems to found his opinion upon this principle, that as the roads are seldom straight lines, the wheels should be more adapted for curvilineal than for rectilineal motion. In what part of the world Mr. Beighton has examined the roads we cannot say; but of this we are sure, that there are no such flexures in the roads of Scotland.

On the line of traction, and the method by which horses exert their strength.

M. Camus, a gentleman of Lorrain, was the first person who treated on the line of traction.* He attempted to show that it should be a horizontal line, or rather that it should always be parallel to the ground on which the carriage is moving, both because the horse can exert his greatest strength in this direction, and because the line of draught being perpendicular to the vertical spoke of the wheel, acts with the largest possible lever. M. Couplet,† however, considering that the roads are never perfectly level, and that the wheels are constantly surmounting small eminences even in the best roads, recommends the line of traction to be oblique to the horizon. By this means the line of draught

Fig. 7. *HJ*, (which is by far too much inclined in the

* Traites des Forces Mouvantes, p. 387.

† Reflexions, sur le tirage des charrettes. Mem. Acad. Royale 1733, 8vo. pp. 57, 86

figure), will in general be perpendicular to the lever *AC* which mounts the eminence, and will therefore act with the longest lever when there is the greatest necessity for it. We ought to consider, also, that when a horse pulls hard against any load, he always brings his breast nearer the ground, and therefore it follows, that if a horizontal line of traction be preferable to all others, the direction of the traces should be inclined to the horizon when the horse is at rest, in order that it may be horizontal when he lowers his breast and exerts his utmost force. PLATE XXXVIII.

The particular manner, however, in which living agents exert their strength against great loads, seems to have been unknown both to Camus and Couplet, and to many succeeding writers upon this subject. It is to M. Deparcieux, an excellent philosopher and ingenious mechanic, that we are indebted for the only accurate information with which we are furnished, and we are sorry to see, that philosophers who flourished after him have overlooked his important instructions. In his memoir on the draught of horses,* he has shown, in the most satisfactory manner, that animals draw by their weight, and not by the force of their muscles. In four-footed animals the hinder feet is the fulcrum of the lever by which their weight acts against the load, and when the animal pulls hard, it depresses its chest, and thus increases the lever of its weight, and diminishes the lever by which the load resists its efforts. Thus, in Fig. 5. let *P* be the load, *DA* the line of traction, and let us suppose *FC* to be the hinder Fig. 5.

* *Sur le Tirage des Chevaux*, published in the Mem de l'Acad. Royale, 1760, 4to, p. 263, 8vo. p. 275.

leg of the horse, *AF* part of its body, *A* its chest or centre of gravity, and *CE* the level road. Then *AFC* will represent the crooked lever by which the horse acts, which is equivalent to the straight one *AC*. But when the horse's weight acts downwards at *A*,* round *C* as a centre, so as to drag forward the rope *AD* and raise the load *P*, *CE* will represent the power of the lever in this position, or the lever of the horse's weight, and *CF* the lever by which it is resisted by the load, or the lever of resistance. Now, if the horse lowers its centre of gravity *A*, which it always does when it pulls hard, it is evident that *CE*, the lever of its weight, will be increased, while *CF* the lever of its resistance, will be diminished, for the line of traction *AD* will approach nearer to *CE*. Hence we may see the great benefit which may be derived from large horses, for the lever *AC* necessarily increases with their size, and their power is always proportioned to the length of this lever, their weight remaining the same.—Large horses, therefore, and other animals will draw more than small ones, even though they have less muscular force and are unable to carry such a heavy burden. The force of the muscles tends only to make the horse carry continually forward his centre of gravity, or, in other words, the weight of the animal produces the draught, and the play and force of its muscles serve to continue it.†

* It may be imagined that the fore-feet of the horse prevent it from acting in this manner; but Deparcieux has shown by experiment that the fore-feet bear a much less part of the horse's weight when he draws than when he is at rest.

† When I first compared Deparcieux's theory with the manner in which horses appear to exert their strength, I was inclined to suspect its accuracy: but a circumstance occurred which remov-

From these remarks, then, we may deduce the proper position of the line of traction.—When the line of traction is horizontal, as *AD*, the lever of resistance is *CF;* but if this line be oblique to the horizon, as *Ad,* the lever of resistance is diminished to *Cf,* while the lever of the horse's weight remains the same. Hence it appears, that inclined traces are much more advantageous than horizontal ones, as they uniformly diminish the resistance to be overcome. Deparcieux, however, has investigated experimentally the most favourable angle of inclination, and found, that when the angle *DAF*, made by the trace *Ad,* and a horizontal line is 14 or 15 degrees, the horses pulled with the greatest facility and force. This value of the angle of draught will require the height of the spring-tree bar, to which the traces are attached in four-wheeled carriages, to be *one half* of the height of that part of the horse's breast to that with which the fore end of the traces is connected.*

Notwithstanding the great utility of inclined traces, it will not be easy to derive complete advantage from them in two-wheeled carriages without diminishing the size of the wheels. In all four-wheeled carriages, however, they may be easily employed; and in many other cases where wheels are not concerned, great advantage may be derived from the discovery of Deparcieux.

ed every doubt from my mind. I observed a horse making continual efforts to raise a heavy load over an eminence. After many fruitless attempts, it raised its fore-feet completely from the ground, pressed down its head and chest, and instantly surmounted the obstacle.

* This height is about 4 feet 6 inches, and therefore the height of the spring-tree bar should be only 2 feet 3 inches, whereas it is generally 3 feet

On the position of the centre of gravity, and the manner of disposing the load.

From Mr. Ferguson's observations on the centre of gravity,* it must be evident, that if the axletree of a two-wheeled carriage passes through the centre of gravity of the load, the carriage will be in equilibrio in every position in which it can be placed with respect to the axletree, and in going up and down hill, the whole load will be sustained by the wheels, and will have no tendency either to press the horse to the ground or to raise him from it. But if the centre of gravity be far above the axletree, as it must necessarily be according to the present construction of wheel-carriages, a great part of the load will be thrown on the back of the horses from the wheels, when going down a steep road, and thus tend to accelerate the motion of the carriage, which the animal is striving to prevent; while in ascending steep roads a part of the load will be thrown behind the wheels, and tend to raise the horse from the ground, when there is the greatest necessity for some weight on his back, to enable him to fix his feet on the earth, and overcome the great resistance which is occasioned by the steepness of the road. On the contrary, if the centre of gravity be below the axle, the horse will be pressed to the ground in going up hill, and lifted from it when going down. In all these cases, therefore, when the centre of gra-

* Vol. i.

vity is either in the axletree, or directly above or below it, the horse will bear no part of the load on level ground. In some situations the animal will be lifted from the ground when there is the greatest necessity for his being pressed to it, and he will sometimes bear a great proportion of the load when he should rather be relieved from it. Plate XXXVIII.

The only way of remedying these evils is to assign such a position to the centre of gravity, that the horse may bear some portion of the load when he must exert great force against it, that is, on level ground, and when he is ascending steep roads; for no animal can pull with its greatest effort, unless it is pressed to the ground. Now, this may in some measure be effected in the following manner. Let *BCN*, Fig. 5. be the wheel of a cart, *AD* one of the shafts, *D* that part of it where the cart is suspended on the back of the horse, and *A* the axletree; then if the centre of gravity of the load be placed at *m*, a point equidistant from the two wheels, but below the line *DA*, and before the axletree, the horse will bear a certain weight on level ground, a greater weight when he is going up hill, and has more occasion for it, and a less weight when he is going down hill, and does not require to be pressed to the ground. All this will be evident from the figure, when we recollect that if the shaft *DA* be horizontal, the centre of gravity will press more upon the point of suspension *D* the nearer it comes to it; or the pressure upon *D*, or the horse's back will be proportional to the distance of the centre of gravity from *A*. If *m*, therefore, be the centre of gravity, *bA* will represent its pressure upon *D*, when the shaft *DA* is horizontal. When

the cart is ascending a steep road, *AH* will be the position of the shaft, the centre of gravity will be raised to *a*, and *aA* will be the pressure upon *D*. But if the cart be going down hill, *AC* will be the position of the shaft, the centre of gravity will be depressed to *n*, and *cA* will represent the pressure upon the horse's back. The weight sustained by the horse, therefore, is properly regulated by placing the centre of gravity at *m*. We have still, however, to determine the proper length of *ba* and *bm*, the distance of the centre of gravity from the axle, and from the horizontal line *DA*; but as these depend upon the nature and inclination of the roads, upon the length of the shaft *DA*, which varies with the size of the horse, on the magnitude of the load, and on other variable circumstances, it would be impossible to fix their value. If the load, along with the cart, weigh 400 pounds: if the distance *DA* be 8 feet, and if the horse should bear 50 pounds of the weight, then *bA* ought to be one foot, which being $\frac{1}{8}$ of *DA*, will make the pressure upon *D* exactly 50 pounds. If the road slope 4 inches in 1 foot, *bm* must be 4 inches, or the angle *bAm* should be equal to the inclination of the road, for then the point *m* will rise to *a* when ascending such a road, and will press with its greatest force on the back of the horse.

When carts are not constructed in this manner, we may, in some degree, obtain the same end by judiciously disposing the load. Let us suppose that the centre of gravity is at *O* when the cart is loaded with homogeneous materials, such as sand, lime, &c. then if the load consist of heterogeneous substances, or bodies of different weight, we should place the heaviest at

the bottom and nearest the front, which will not only lower the point *O*, but will bring it forward, and nearer the proper position *m*.—Part of the load, too, might be suspended below the fore part of the carriage in dry weather, as the centre of gravity would then approach still nearer the point *m*. When the point *m* is thus depressed, the weight on the horse is not only judiciously regulated, but the cart will be prevented from overturning, and in rugged roads the weight sustained by each wheel will be in a great degree equalized.

In loading four-wheeled carriages, great care should be taken not to throw much of the load upon the fore-wheels, as they would otherwise be forced deep into the ground, and require great force to pull them forward; in some modern carriages this is very little attended to. The coachman's seat is sometimes enlarged so as to hold two persons, and all the baggage is generally placed in the front directly above the wheels. By this means the greatest part of the load is upon the small wheels, and the draught becomes doubly severe for the poor animals who must thus unnecessarily suffer for the ignorance and folly of man.

MECHANICS.

On the Thrashing-Machine.

IN a country like this, where agriculture has arrived at such a high state of perfection, the utility of thrashing-machines cannot easily be called in question. The universal prevalence of these engines is a strong proof that they are advantageous to the farmer; and, however much some men may inveigh against the adoption of every kind of machinery that has for its object the abridgement of manual labour, yet we are convinced, that no evil consequences can possibly accrue from their introduction; and that such insinuations have a tendency to inflame the minds of the vulgar, and retard the progress of science. As a proof of this, we might mention the fate of the celebrated Arkwright, the inventor of the fly-shuttle, whom the fury of an English rabble banished from his native country.

The thrashing-machine was invented in Scotland, in 1758, after five year's labour, by Mr. Michael Stirling, a farmer in Perthshire. The honour of this invention has been claimed by Mr. Andrew Meikle, an ingenious mill-wright

in East Lothian, who obtained a patent for one of these machines about the year 1785; and in this country his claims have been generally admitted. Mr. Meikle, however, was merely an improver of the thrashing-machine, and I am assured by a gentleman of the most unquestionable authority, who, from his local situation, had access to the best information, that Mr. Meikle had seen Mr. Stirling's thrashing-machine before he erected any of his own, and that he merely altered and improved it.—About 26 years prior to the date of Mr. Stirling's invention, a thrashing-machine was constructed in Edinburgh by Mr. Michael Menzies, which operated by the elevation and depression of a number of flails, by means of the motion of a crank; and in 1767, the model of a thrashing-mill, invented by Mr. Evers of Yorkshire, was laid before the Society of Arts in London, who rewarded the inventor with a premium of 60 pounds. This machine, which was driven by wind, consisted of a number of stampers, that beat out the grain when laid upon a moveable thrashing-floor, and was actually used on a large scale in Yorkshire, where it received the approbation of several intelligent gentlemen of the county.* All these machines, however, and others of a similar kind, with which the public are perpetually harassed, are completely defective in principle, and are greatly inferior to the worst of those now in use, which operate by the revolution of a thrashing-scutch furnished with beaters—the exclusive invention of our countryman Mr. Stirling.

* Bailey's Drawings of machines laid before the Society of Arts. Vol. i, p. 54—59.

On thrashing-mills driven by water.

PLATE XLIII. Fig. 1.

In Fig. 1. *AB* is an undershot water-wheel, which drives the machinery. On its axis is fixed the spur-wheel *CD*, furnished with 150 teeth, which impel the pinion *b*, containing 25 teeth. On the axis *H* of the pinion *b* is placed another wheel *E*, carrying 72 teeth, which take into the 15 leaves of the pinion *c*. The axle *xx*, of the *thrashing-scutch* represented more distinctly in Fig. 2. by *yxy*, is fastened upon the same axis with the pinion *c*, and is, therefore, carried round with the same velocity.—The thrashing-scutch, a section of which may be seen in Fig. 3. is generally furnished with four, and sometimes with a greater number of *beaters* *y*, *y*, whose surfaces *o*, *o*, are covered with iron rounded off at the edges, in order to prevent them from cutting the straw. When these beaters strike upwards, the scutch must be contained in a hollow cylinder of wood *m*, *n*, so that the tops *y*, *y*, *o*, *o*, of the beaters may be above it; in which case the scutch is called the *thrashing-drum*. But when the beaters strike downwards, there is no occasion for covering it with boards.

The gudgeon of the axis *H* carries a wheel *i* of 22 teeth, which acts upon the wheel *h* with 18 teeth; on the axis *he* is fixed another wheel *e*, with 17 teeth, that drives the crown-wheel *d*, furnished with 3 rows of teeth, 13, 17, and 21, which, by means of the spindle *R*, gives motion to one of the feeding-rollers, not visible in Fig. 1. but represented distinctly by *RR* in Fig. 2. On the axis of the upper feeding-roller *RR*, is placed a small pinion, which drives

the under feeding-roller by acting upon another pinion with the same number of teeth fixed upon its spindle. The two feeding-rollers, which are generally 3½ inches in diameter, are fluted, or cut into small leaves like pinions, so that the leaves of the one may take into the leaves of the other; and their gudgeons move in mortices of such a nature, that the upper roller may rise in its frame, and the under one remove from the beaters, when too much corn is admitted between them. In order that the velocity of the rollers may be increased and diminished at pleasure, according to the nature of the corn to be thrashed, the wheel *c* is made to shift on its axis, so as to act upon any of the 3 rows of teeth in the crown-wheel *d*, which enable us to communicate three different degrees of velocity to the rollers. PLATE XLIII.

As the machinery which drives the straw-shaker interferes in Fig. 1. with that which gives motion to the fluted rollers, it will be seen in Fig. 2. which is a plan of the machine where the corresponding parts are marked by similar letters. The wheels *b*, *E*, *c*, in Fig. 1. are not represented in this figure, but *H* is the extremity of the axle on which *E* and *b* are fixed. The small wheel *i* of 22 teeth, fixed upon the extremity of the gudgeon *i*, *H* gives motion to *m*, a wheel of 17 teeth, which, by the intervention of the spindle *mn* and wheel *n*, of 24 teeth, drives *o*, a wheel carrying 34 teeth.* On the Fig. 2.

* The dimensions of the thrashing-machines here described are chiefly taken from Gray's experienced Mill-wright, a book of great utility in a manufacturing country. It consists chiefly of plans, sections, and elevations, of different machines, which the author himself has either erected, or whose construction he has immediately superintended. We are afraid, however, that Mr. Gray has not rendered these machines sufficiently intelligible to

PLATE XLIII.

same axis with *o* is fixed the straw-shaker *KK'*, on whose cross arms are fastened the rakes *zr*, furnished with a number of iron or wooden teeth, which carry off the straw, while the grain falls down into the fanners. The axis of these fanners *pq*, Fig. 1. is put in motion by the belt *pp* passing over the two rollers *p*, *p*. A section of the straw-shaker is shown in Fig. 4. where *K'* is its axle. *z*, *z*, its arms, and *r*, *r*, &c. the teeth fastened at the extremity of these arms.

That the reader may have a distinct idea of the thrashing-machine, we have calculated the following table, which exhibits the number of teeth in the wheels, and the velocity of its different parts. It is scarcely necessary to premise, that when one wheel drives another, the number of turns, or parts of a turn, performed by the wheel which is driven, is represented by a fraction whose numerator is the number of teeth in the wheel that gives the motion. and whose denominator is the number of teeth in the wheel which receives it. Thus a wheel with 25 teeth, driven by another with 150, will perform $\frac{150}{25}$ or 6 revolutions for one revolution of the impelling wheel; and a wheel with 16 teeth, driven by a pinion with 8 teeth, will make $\frac{8}{16}$ or $\frac{1}{2}$ of a turn, for one revolution of the pinion. When two or more wheels are upon the same axis. they all perform the same number of revolutions, however different be their magnitude, and the number of teeth; though the velocities of their circumferences may be widely different.

the uninstructed mechanic, from the great brevity of his descriptions; and, we hope, if his work reach a second edition, as we trust it will, that he will take advantage of this friendly hint.

In the following table we have calculated merely the number of turns made by each wheel for one turn of the water-wheel; but when the number of revolutions performed by the water-wheel in a second is known, we have only to multiply the quantities in the third column by this number, in order to find the number of turns which each wheel makes in the same time.

Names of the wheels.	Number of teeth in each wheel.	Number of turns, for one of the water-wheel.
Plate XLIII. Fig. 1 and 2.	Teeth.	Turns. Dec.
CD	150	1.000
b	25	6.000
E	72	6.000
c	15	28.800
Thrashing-scutch	0	28.000
i	22	6.000
h	18	7.333
e	17	7.333
Fluted Rollers *d*	13	9.534
Fluted Rollers *d*	17	7.333
Fluted Rollers *d*	21	5.940
m	17	7.764
n	24	7.764
o	34	5.479
Straw-shaker	0	5.479

The working parts of the thrashing-machine being thus described, the manner of its operation will be easily understood. The sheaves of corn are spread upon an inclined board *O*, Fig. 2. called the feeding-board, and introduced between the fluted rollers, a section of which is distinctly visible at *ii*, in Fig. 3. of Plate XLIV. Plate XLIV. Fig. 3. The corn is held fast by these rollers, which

are only about three-quarters of an inch from the beaters, while the thrashing-drum or scutch revolving with immense rapidity and force, separates the grain from the straw by the repeated strokes of the beaters. Part of the grain falls through the heck or searce *ir*, into a large hopper, which conducts it to the fanners, and some of it is carried along with the straw into the other heck *rp*, where it falls into the hopper, while the straw is cleared away by the rakes *z*, of the straw-shaker, and thrown out at the opening *np*, into the lower part of the barn.

In some thrashing-machines driven by water, the motion is conveyed to the thrashing-scutch by means of a long perpendicular axis. The lower extremity of the axis is furnished with a pinion, which is driven by a spur-wheel, with teeth perpendicular to its plane, placed upon the axis of the water-wheel. A large horizontal wheel is fixed on the top of this long axle, which acts upon a pinion fastened upon the axis of the thrashing-drum.

On thrashing-machines driven by horses.

Wherever a sufficient quantity of water can be procured, it should always be employed as the impelling power of thrashing-machines. There are many situations, however, in which it cannot be obtained; and, as the erection of steam-engines and wind-mills would be too expensive for the generality of farmers, they are under the necessity of having recourse to animal power. In Plate XLIV, Fig. 1. is represented a thrashing-machine, which may be driven by 4 or 6 horses. To the vertical axis *M*, six strong bars are fixed, called the horse-poles, 4 of which, *P, R, S, L,*

Plate XLIV. Fig. 1.

are visible in the figure, and to the extremity of each of these poles, 2 pieces of wood, like *op*, are attached, to which the horses are yoked when the machine is to be used. Upon the top of the 6 poles is placed the large bevelled wheel *AB*, containing 270 teeth, which drives the pinion *BC*, of 40 teeth; on the axle *N* is also fixed the wheel *DD*, which carries 84 teeth, and drives the pinion *b*, of 21 teeth, placed upon the axle *bk*. Upon the same axis, the wheel *EE* revolves, carrying 66 teeth, which drive the pinion *c*, of 15 teeth, and consequently the thrashing-drum *xx*, which is fixed upon the same axle. The feeding-rollers are driven by the intervention of the four bevelled wheels, *i*, *h*, *e*, *d*, the latter of which is fastened on the axis of the upper feeding-roller. The wheel *i*, upon the gudgeon *ib*, contains 25 teeth, the wheel *h* 24 teeth, *e* 22 teeth, and *d* 21 teeth: but when the fluted rollers require a greater velocity, *e* is taken from its iron axle, and a greater or less wheel substituted in its room. The short axle *bk* is furnished with a pulley *p*, which, by means of the leathern belt *pp*, gives motion to the fanners placed below the thrashing-scutch and straw-shaker. Plate XLIV.

Fig. 2. represents a plan of the wheels, thrashing-drum, and straw-shaker, where the corresponding parts in Fig. 1. are marked with similar letters. The small wheels *g* and *k*, however, which convey motion to the straw-shaker, are not seen in the first figure. The largest one *g*, is fixed on the axis *N*, and carries 38 teeth.—It drives *k*, which contains 14 teeth, and is placed upon the axis of the straw-shaker *KK*.

An elevation of the working parts of the machine is delineated in Fig. 3. where the cor-

responding parts in the plan and section have the same letters affixed to them. The sheaves of corn are spread on the feeding-board *o*, drawn in by the rollers *i*, *i*, and thrashed by the beaters *o*, *c*, which strike downward. Part of the corn falls through the heck *ir*, and some of it is carried along with the straw into the larger heck *rp*, where it falls into the hopper below, while the straw is thrown out at the opening *np*. The drum and straw-shaker are surrounded with a covering of wood *imn*.—The following table exhibits, at one view, the number of teeth in the wheels, and the different velocities with which they move.

Names of the wheels.	Number of turns for one of the wheel.	Number of teeth in each wheel.
Plate XLIV, Fig. 1, 3, 4	Teeth.	Turns. Dec.
AB	270	1.000
BC	40	6.750
DD	84	6.750
b	24	23.625
EE	66	23.625
c	15	103.950
Thrashing-Scutch,	0	103.950
g	38	6.750
k	14	18.293
Straw-Shaker,	0	18.293
i	25	23.625
h	24	24.617
e	22	26.857
d	21	28.199
Feeding-Rollers,	0	28.199

In situations where there is an occasional supply of water, thrashing-machines are sometimes constructed so as to be driven either by

horses or water. In this case, the water-wheel has the position *LH*, Fig. 1. and is furnished with a large wheel *GH*, consisting of segments of cast iron firmly fixed to the arms of the water-wheel. The wheel *GH* drives *FG*, and thus communicates motion to the horizontal shaft *N*, and the rest of the machinery. When there is no water for impelling the mill, the water-wheel *LH* is either lowered in its frame, or one of the segments is taken from the wheel *GH*, in order to keep it clear of the wheel *FG*; and when there is a sufficient discharge of water, *CB* is either raised above *AB*, or *AB* is deprived of a few teeth, which can be screwed and unscrewed at pleasure. Sometimes when there is a small supply of water, its energy may be combined with the exertion of one or two horses. Plate XLIV.

If the thrashing-machine is to be driven by wind, the motion is conveyed to the axle *N*, by the small wheel *mC*, fixed at the bottom of the vertical axis *n*, which is moved by the wheel upon the windshaft. If the mill is to be moved by steam, the large fly must be fixed on the axis *N*, parallel to the horizon.

Fig. 4. represents a thrashing-machine of a very simple construction, which may be driven by two or three horses. The large wheel and pinion corresponding with *AB*, and *BC*, in Fig. 1. are not delineated in the figure, but the former contains 166, and the latter 19 teeth. On the shaft *N*, is fastened the wheel *DD*, which carries 80 teeth, and drives the pinion *c* of 9 teeth, and consequently the thrashing-drum which is fixed on its axis. The straw-shaker is turned by means of the leathern belt *hi* passing over the pulleys *h* and *i*, the fluted rollers Fig. 4.

PLATE XLIV by the belt *km*, and the fanners by the rope *de*.

I have seen a thrashing-mill of this simple construction belonging to Hercules Ross, Esq. of Rossie, which was driven by 6 horses. The large wheel corresponding with *AB*, (Fig. 1.) had 144 teeth: and the pinion corresponding with *BC* had 14 teeth. The wheel *DD* had 80 teeth, and the pinion *c*, 8. The straw-shaker and fluted rollers were driven by belts, and the fanners by a rope passing over a groove in the large wheel *DD*. The thrashing-drum revolved 103 times for every turn of the horses, whereas the drum in the machine, represented in Fig. 5. performed only 79 revolutions in the same time. In the first case, however, the horse-walk was of such a size, that the horses performed only 3 turns in a minute; while in the latter, the horses are supposed to make 4 in a minute. The velocity, therefore, of the former will be $4 \times 79 = 316$, and the velocity of the latter $3 \times 103 = 309$.

When thrashing mills began to be generally adopted in this country, they were constructed

Fig. 5. according to the plan represented in Fig. 5.—The wheel *AB* has 276 cogs; *b*, 14, the crown-wheel *c*, 84, *d*, 16. The thrashing-drum is fixed on the axis *md*, and the fanners, straw-shaker, and fluted rollers, are moved by leathern belts.

A thrashing-machine for small farms, which can be wrought by a single horse, has long been a desideratum in mechanics, and every attempt to construct one on a small scale, seems to have completely failed. While examining the causes of this failure, I have thought of some methods by which they may be partially removed, and

of a machine which might be impelled by one horse, or by two or three men working at a winch. The description of this simple engine I expected to have communicated in this article; but a desire to improve as much as possible, has induced me to defer its publication to some future opportunity.

On the power of thrashing-machines.

The quantity of corn which a machine will thrash in a given time, depends so much upon the judicious formation and position of its parts, that one machine will often perform double the work of another, though constructed upon the same principles, and driven by the same impelling power. Misled by this circumstance, those who have given an account of the power of their thrashing-mills, have published merely the number of bolls, which they can thrash in a given time, without mentioning the quantity of impelling power, or the number of horses employed to drive them.

Mr. Fenwick, whose labours in practical mechanics we have already mentioned with commendation, has furnished us with some important information upon this point. He found, from a variety of experiments, that a power capable of raising a weight of 1000 pounds with a velocity of 15 feet per minute, will thrash 2 bolls of wheat in an hour; and that a power sufficient to raise the same weight with a velocity of 22 feet per minute, will thrash 3 bolls of the same grain in an hour. From these facts, Mr. Fenwick has computed the following table, which is applicable to machines that are driven either by water or by horses.

Table of the power of thrashing-machines.

Gallons of water per minute, ale-measure, discharged on an over-shot wheel 10 feet in diameter.	Gallons of water per minute, ale-measure, discharged on an over-shot wheel 15 feet in diameter.	Gallons of water per minute, ale-measure, discharged on an over-shot wheel 20 feet in diameter.	Number of horses working 9½ hours.	Bolls of wheat thrashed in an hour.	Bolls thrashed in 9½ hours actual working, or in a day.
230	160	130	1	2	19
390	296	205	2	3	28½
528	380	272	3	5	47½
660	470	340	4	7	66½
790	565	400	5	9	85½
970	680	500	6	10	95
1	2	3	4	5	6

The four first columns of the preceding table contain different quantities of impelling power, and the two last exhibit the number of bolls of wheat in Winchester-measure, which such powers are capable of thrashing in an hour, or in a day. Six horses, for example, are capable of thrashing 10 bolls of wheat in an hour, or 95 in the space of 9½ hours, or a working day; and 680 gallons of water discharged on a 15 feet overshot water-wheel during a minute will thrash the same quantity of grain.

MECHANICS.

ON THE CONSTRUCTION AND EFFECT OF MACHINES.*

By Mr. John Leslie, Professor of Mathematics in the University of Edinburgh.

1. IT is a principle in statics, that, if a body act upon another by the intervention of machinery, an equilibrium will obtain when their *potential* velocities are reciprocally as their masses. If the power exerted be augmented beyond what is barely sufficient to maintain the balance, a motion will immediately commence, and if it be still increased the velocity will continually increase. But this velocity will increase in a smaller ratio than the power; and there will, therefore, be a certain point of augmentation, at which the force employed will produce the greatest proportional effect. Such

* This excellent paper, which Professor Leslie was so kind as to communicate to the editor, was written at London so early as February 1790. The same subject was afterwards (in 1801) treated at great length by the late Dr. Robison, in the art. Machinery, Sup. Encycl. Britan.; and we do not conceive that we are derogating in the least from the talents of that learned and good man, when we say, that the present paper is written with greater perspicuity, and gives a more elementary and connected view of this interesting subject. At some future period Mr. Leslie intends to resume the investigation of this subject. F. Ed.

is the grand object that we ought to have always in view in the construction of machines.

2. Forces have been divided into two kinds; those whose action is supposed to be *instantaneous*, and those whose action is *continued* and *incessant*. The former have been termed *impulsive*, the latter *accelerating* or *retarding*. Though accelerated or retarded motions perpetually occur to our observation, the ancients seem to have admitted no other force but that of impulsion. It is difficult, indeed, to conceive that a body can act at a distance; and the idea that motion is always communicated by contact, is one of our earliest and strongest prejudices. Sir Isaac Newton himself was in this instance carried away by the current of opinion. His theory of æther was an attempt to explain gravitation by impulsive forces.—But there are many facts and experiments which satisfactorily prove, that between the particles of matter there subsists a repulsion, increasing as the distance diminishes, and that no *absolute* contact can ever take place. A body does not acquire its celerity *in an instant*. Nothing material can exist but what is *finite*; and the beautiful law of *continuation*, by which changes are produced by imperceptible shades, can never be violated. But an amazing force may be exerted, and an effect may be produced, in a time so small as to elude the acuteness of our senses. Hence the origin of our idea, that motion is derived from impulse. If, however, we consider the subject with more attention, we shall find that it is really as difficult to conceive action in contiguity as at a distance. In neither case can we deduce the consequences *a priori*. The connexion which subsists between cause and

effect is not necessary and absolute*; it is founded upon the invariable experience of our senses. We may, therefore, conclude, that there is only one kind of force, and that is the *accelerating* or the *retarding*. Hence it will always be possible to determine the proportional intensity of any given force, compared with that of gravity, and to assign a weight, which, by its pressure alone, would in a given time produce the same effect.

3. If the gravity of an elementary point at the earth's surface be denoted by 1, the whole attractive force will be as the number of points, or as M, the mass of the body. Let F express the intensity of another force urging the same body; then $M \times F$ will denote the quantity of force exerted, or Φ; but $\frac{\Phi}{M} = F$; wherefore the intensity of a force is directly as its quantity, and inversely as the mass which is urged.

4. Let $V =$ the velocity of a body, $S =$ the space described, and $T =$ the time of description; the velocity that is acquired may be conceived to be composed of all the successive augmentations which are produced by the continued exertion of the force, and which are proportional to the intensity of its action. But the force may for a moment be conceived to be uniform; whence the increment of velocity is compounded of the force, and of the increment of the time, or $\dot{V} \div F\dot{T}$. Suppose the force to be constant, then $\dot{V} \div FT$; and when the time is

* As very erroneous notions, respecting the connexion between cause and effect, seem to be prevalent in this country, it may be proper to mention, though it is evident from the context, that Mr. Leslie alludes merely to physical causes. E. Ed.

given, the velocity must be as the accelerating force. Let $V=CFT$, and V and T denote feet and seconds of time. When $F=1$, and $T=1$, we shall have $V=C=$ the velocity acquired by descent at the surface of the earth at the end of the first second. Put $d=16.1$ feet, then $C=2d$; whence $\dot{V}=2d\ F\dot{T}$ and $\dot{T}=\frac{\dot{V}}{2dF}$.

5. We may conceive that the velocity is uniform for an indefinitely small portion of time. Whence $\dot{S}=V\dot{T}$ and $\dot{T}=\frac{\dot{S}}{V}$; hence also $V=\frac{\dot{S}}{\dot{T}}$; but, by the last article, $\dot{V}=2dF\dot{T}$, consequently $V\dot{V}=2dF\dot{S}$; from which equation the velocity may be determined, when the relation is given between the accelerating force and the space described. If F be constant, then by integration, $\frac{1}{2}VV=2dFS$ and $V^2=4dFS$; wherefore $V=2\sqrt{dFS}$. At the same time, because $\dot{V}=2dF\dot{T}$, $\dot{S}=2dFT\dot{T}$; whence $S=2d\times FT^2$, and $T=\sqrt{\frac{S}{2dF}}$.

6. We may divide machines into two general kinds; into those where the action is interrupted and renewed at short intervals, and into those where the action is continued for a certain period. In the former, the effect of friction not having time to accumulate, may generally be disregarded, and the motion may be considered as uniformly accelerated. With regard to the latter, if a machine be constructed so that the resistance is great, it increases rapidly with the celerity: it soon counterbalances the accelerating force, and produces a motion which is equal and constant.

7. Let us abstract the momenta and friction of the parts of communication, and consider the effects of a machine which is uniformly impelled. Suppose the motion of a power, equal to

the gravity of the mass p, be connected to that of a weight w, so that the *potential* velocity of the former be constantly to that of the latter as $v : 1$, and let v be termed the *advantage*. It is manifest, from the principles of statics, that a part $\frac{w}{v}$ of the power is able to maintain the equilibrium, and that the remaining part $p - \frac{w}{v}$ only is employed in producing the motion. But the action of this force $p - \frac{w}{v}$ is divided between the power and the weight. Put $y =$ the part which urges the power, and $z =$ the part which is exerted against the weight. But (3) the intensity of the force y, which impels the power, is denoted by $\frac{y}{p}$; and therefore the velocity acquired in a given time is (4) also $\frac{y}{p}$. But the influence of the force z upon the weight, will, in consequence of the mechanism, be equal to the direct action of a force vz; whence the velocity acquired by the weight in the same time will be $\frac{vz}{w}$. Wherefore, by hypothesis, $\frac{y}{p} : \frac{vz}{w} :: v : 1$; consequently $\frac{y}{p} = \frac{v^2 z}{w}$ and $y = \frac{v^2 pz}{w}$. But, from the notation, $p - \frac{w}{v} = y + z$ and $y = p - z - \frac{w}{v}$, or $\frac{pv - zv - w}{v}$; therefore $\frac{v^2 pz}{w} = \frac{pv - zv - w}{v}$, and reducing, $v^3 pz = pvw - zvw - w^2$, and transposing, $v^3 pz - vwz - pvw - w^2$, whence $z = \frac{pvw - w^2}{v^3 p + vw}$, consequently the real action upon the weight, or $vz = \frac{pvw - w^2}{v^2 p + w}$.

8. Hence the intensity of the force which urges the weight, or F is $\frac{1}{w}\left(\frac{pvw-w^2}{v^2p+w}\right)$ or $\frac{pv-w}{v^2p+w}$; consequently this intensity is to that of terrestrial attraction as $pv-w : v^2p+w$. But this action is uniform; whence (4) $V=2dF\dot{T}$, or the actual velocity which the weight acquires in its ascent during the time T is $2dT\left(\frac{pv-w}{pv^2+w}\right)$. Wherefore $\dot{S}=2dT\dot{T}\times\frac{pv-w}{pv^2+w}$, and integrating, the space described is $dT^2\left(\frac{pv-w}{pv^2+w}\right)$.

9. When the velocity of the power is equal to that of the weight, then $v=1$ and the intensity of action $=\frac{p-w}{p+w}$ or $1-\frac{2w}{p+w}$. Whence the effect increases more slowly than the power. Thus, according as the power is equal to the weight, is double, triple, or quadruple, &c. the intensity of action is 0, $\frac{1}{3}$, $\frac{2}{4}$, $\frac{3}{5}$, $\frac{4}{6}$, &c. . . and ultimately $=1$. The comparative intensity is, therefore, 0, $\frac{1}{6}$, $\frac{1}{6}$, $\frac{3}{20}$, $\frac{2}{15}$, $\frac{5}{42}$, $\frac{3}{28}$, &c. and ultimately $=0$. If $p=w$, the intensity of action is $\frac{wv-w}{wv^2+w}$ or $w\left(\frac{v-1}{v^2+1}\right)$. Suppose the advantage to be successively 1, 2, 3, 4, 5, &c. then the intensity of action is 0, $\frac{1}{5}$, $\frac{1}{5}$, $\frac{3}{17}$, $\frac{2}{13}$, $\frac{5}{37}$, $\frac{3}{25}$; &c. and ultimately vanishing. Both these series commence at *zero*, increase, become stationary, and then continually decrease, till they vanish. In the present case, the *maximum* must lie between the 2d and 3d terms; for these are equal in both.

10. Since the intensity of action produced by the power p is $\frac{pv-w}{pv^2+w}$, the comparative intensity, or the effect produced by a given force 1,

will be $=\frac{1}{p}\times\frac{pv-w}{pv^2+w}$ or $\frac{pv-w}{p^2v^2+pw}$. This quantity is, therefore, the proper measure of effect, and to increase it must be our great object in improving the machine.—Let the power and weight be constant; to find the value of v, when the comparative intensity of action is a *maximum*. By taking the fluxion of $\frac{pv-w}{p^2v^2+pw}$, we obtain $\frac{p\dot{v}(p^2v^2+pw)-2p^2v\dot{v}(pv-w)}{(p^2v^2+pw)^2}=0$; whence by transposition and reduction, $p^3v^2+p^2w=2p^3v^2-2p^2wv$, or $p^2v^2+w=2pv^2-2wv$, and transposing, $pv^2-2wv=w$, and dividing, $v^2-\frac{2w}{p}v=\frac{w}{p}$, and resolving the quadratic, $v\mp\frac{w}{p}=\sqrt{\left(\frac{w}{p}+\frac{w^2}{p^2}\right)}$ and $v=\frac{w}{p}+\sqrt{\left(\frac{w}{p}+\frac{w^2}{p^2}\right)}$, or $v=\frac{w+\sqrt{(w^2+wp)}}{p}$. Hence, according as the weight is equal to the power, is double, triple, or quadruple, &c. the *advantage* ought to be $1+\sqrt{2}$, $2+\sqrt{6}$, $3+\sqrt{12}$, $4+\sqrt{20}$, $5+\sqrt{30}$, $6+\sqrt{42}$, &c.

11. When w is very large compared with p, the expression $\frac{w+\sqrt{(w^2+wp)}}{p}$ is nearly $\frac{2w}{p}+\frac{1}{2}$. In most cases, it will be sufficiently accurate to suppose $v=\frac{2w}{p}$; and hence, in order that a machine may produce the greatest possible effect, without increasing the power applied, the *advantage* which would procure an equilibrium ought to be at least doubled. Substituting this value in the formulæ in art. 9, we obtain $V=2dT\left(\frac{pv-w}{pv^2+w}\right)=2dT\left(\frac{p}{4w+p}\right)$, and $S=dT^2\left(\frac{p}{4w+p}\right)$.

12. If the true value of v, or $\frac{w+\sqrt{(w^2+wp)}}{p}$

be substituted in those formulæ, we shall obtain $V=2dpT\left(\frac{\sqrt{(w^2+wp)}}{2w^2+2wp+2w\sqrt{(w^2+wp)}}\right)$, and $S=dpT^2\left(\frac{\sqrt{(w^2+wp)}}{2w^2+2wp+2w\sqrt{(w^2+wp)}}\right)$. Whence, when the power is equal to the weight, the greatest intensity is $\frac{\sqrt{2}}{4+2\sqrt{2}}$ or $\frac{\sqrt{2}-1}{2}$, or about one fifth of the force of gravity. If w be supposed to be successively $=2p$, $3p$, $4p$, &c. the intensity of action will be $\frac{\sqrt{6}}{24+8\sqrt{6}}$, $\frac{\sqrt{12}}{72+18\sqrt{12}}$, $\frac{\sqrt{20}}{160+32\sqrt{20}}$, $\frac{\sqrt{30}}{300+50\sqrt{30}}$, &c. nearly equal to $\frac{1}{9}$, $\frac{1}{13}$, $\frac{1}{17}$, $\frac{1}{21}$, &c. derived from the expressions in the last article. If the weight be great in respect of the power, the intensity of action will be nearly $\frac{w+\frac{1}{2}p}{2w^2+2wp+2w^2+wp}$, or $p\left(\frac{2w+p}{8w^2+6wp}\right)$. Hence, the other formulæ will be found; $V=2dT\left(\frac{2wp+p^2}{8w^2+6wp}\right)$ and $S=dT^2\times\frac{2wp+p^2}{8w^2+6wp}$. Wherefore, in a machine constructed in the best manner, the accelerating force which impels the weight never amounts to one fourth of the gravity of the power.

13. Let the weight and advantage be given, and it be required to find the power, when the measure of effect or comparative intensity of action is a *maximum*. Suppose p to be variable in the expresssion $\frac{pv-w}{p^2v^2+pvw}$ of art. 10; and taking the fluxion, we shall have $\frac{\dot{p}v(p^2v^2+pvw)-(2p\dot{p}v^2+\dot{p}vw)(pv-w)}{(p^2v^2+pvw)^2}=0$; whence $p^2v^3+pvw=(2pv^2+w)(pv-w)$, and reducing, $v^3p^2+vwp=2v^3p^2+pvw-2v^2wp-w^2$, and transposing, $v^3p^2-2v^2wp=w^2$, and dividing,

$p^2 - \frac{2w}{v}p = \frac{w^2}{v^3}$ and resolving the quadratic, $p - \frac{w}{v} = \sqrt{\left(\frac{w^2}{v^3} + \frac{w^2}{v^2}\right)}$ and $p = \frac{w}{v} + \sqrt{\left(\frac{w^2}{v^2} + \frac{w^2}{v^3}\right)}$. Hence if the advantage be 1, 2, 3, 4, 5, &c. the power ought to be $w\ (1+\sqrt{2})$, $w\ (\frac{1}{2}+\sqrt{\frac{3}{8}})$, $w\ (\frac{1}{3}+\sqrt{\frac{4}{27}})$, $w\ (\frac{1}{4}+\sqrt{\frac{5}{64}})$, $w\ (\frac{1}{5}+\sqrt{\frac{6}{125}})$, &c.

14. If v be large, the value of p will be nearly $= \frac{2w}{v} + \frac{w}{2v^2}$, or $\frac{2w}{v}\left(1+\frac{1}{4v}\right)$, or, in general $= \frac{2w}{v}$ nearly; that is, the power sufficient to maintain an equilibrium, must at least be doubled to produce a *maximum* effect. Substituting the proximate value of p in art. 8, we shall have $V = 2dT \left(\frac{2w(1+\frac{1}{4v})-w}{2wv(1+\frac{1}{4v})+w}\right)$, or $2dT\left(\frac{2v+1}{4v^2+3v}\right)$. Hence also $S = dT^2\left(\frac{2v+1}{4v^2+3v}\right)$.

15. Since, by the last article, $p = \frac{2w}{v} + \frac{w}{2v^2}$ nearly, reducing and transposing, $4v^2p = 8wv + 2w$, and dividing, $v^2 - \frac{2w}{p}.v = \frac{w}{2p}$, and resolving the quadratic, we obtain $v = \frac{w}{p} + \sqrt{\left(\frac{w^2}{p^2} + \frac{w}{2p}\right)}$, or $v = \frac{2w}{p} + \frac{1}{4}$ nearly. Substituting this value of v in the above formulæ, we shall obtain in terms of p, after proper reductions, $V = 2dT \left(\frac{8wp+3p^2}{32w^2+20wp+2p^2}\right)$ and $S = dT^2\left(\frac{8wp+3p^2}{32w^2+20wp+2p^2}\right)$. But when v is large, these formulæ will be expressed with sufficient accuracy thus, $V = 2dT \left(\frac{p}{4w}\right)$ and $S = dT^2\left(\frac{p}{4w}\right)$.

16. Let the true value of p or $\frac{w}{v} + \sqrt{\left(\frac{w^2}{v^2} + \frac{w^2}{v^3}\right)}$ be substituted, and we shall obtain

$V=2dT\ \left(\frac{\sqrt{(w^2v^2+wv)}}{wv^2+wv+\sqrt{(w^2v^4+wv^3)}}\right.$ and $S=dT^2$ $\frac{\sqrt{(w^2v+wv)}}{wv^2+wv+\sqrt{(w^2v^4+wv^3)}}$. If $w=1$, and v be denoted by 1, 2, 3, 4, 5, *&c.* the corresponding absolute intensity will be $\frac{\sqrt{2}}{2+\sqrt{2}}$, or $\sqrt{2}-1$, $\frac{\sqrt{6}}{6+\sqrt{24}}$, $\frac{\sqrt{12}}{12+\sqrt{108}}$, $\frac{\sqrt{20}}{20+\sqrt{320}}$, $\frac{\sqrt{30}}{30+\sqrt{750}}$, *&c.*

17. If the accurate value of p be substituted in the expression $\frac{pv-w}{p^2v^2+prw}$ for the comparative intensity we obtain $\frac{1}{w}\ \left(\frac{\sqrt{(v^2+v)}}{2v+2+2\sqrt{(v^2+v)}+\sqrt{(1+\frac{1}{v})}}\right.$. Suppose $w=1$ and v successively $=1, 2, 3, 4, 5$, *&c.* then the measure of the effect will be

$$\frac{\sqrt{2}}{4+\sqrt{8}+\sqrt{2}} \text{ or } 3-\sqrt{8},\quad \frac{\sqrt{24}}{12+\sqrt{96}+\sqrt{6}},$$

$$\frac{\sqrt{108}}{24+\sqrt{432}+\sqrt{12}},\quad \frac{\sqrt{300}}{40+\sqrt{1280}+\sqrt{20}},\quad \frac{\sqrt{750}}{70+\sqrt{3000}+\sqrt{25}}$$

&c. Let v be a large number, the proximate measure of the effect will then be $=$ $\frac{v+\frac{1}{2}}{2v+2+2v+1+1+\frac{1}{2v}}$ 0 $\frac{2v+1}{8v+8+\frac{1}{v}}$; and this expression will be ultimately $=\frac{1}{4}$. Wherefore, comparing this result with that in art. 12, it appears that, in whatever way the *maximum* be procured, the force which impels the weight can never amount to one fourth part of the direct action of the power.

18. Hitherto we have not taken into account the force expended in impressing motion upon the parts of the machine which connect the power and weight. Let a, b, c, d, &c. denote the masses of the communicating parts, and let α, β, γ, δ, &c. be proportional to their corresponding velocities, and Q to that of the weight.

It is obvious, that the momentum of the part a is equal to the momentum of the mass $\frac{a\alpha}{Q}$: In the same manner, the momentum of b, c, d, &c. will be equal to that of the quantities of matter $\frac{b\beta}{Q}$, $\frac{c\gamma}{Q}$, $\frac{d\delta}{Q}$, &c. moving with the celerity of the weight to be raised. But from the permanency of the construction, these quantities are constant. Whence the total quantity of motion is the same with that of a mass $\frac{a\alpha+b\beta+c\gamma+d\delta}{Q}$, which is given. Let it be equal to q, and supposing, as before, the power $=p$ and the weight $=w$, the whole mass, on which the celerity of the weight to be raised must be impressed, will be denoted by $w+q$.

19. It is obvious that $\frac{w}{v}$ is still that part of the power which is sufficient to maintain the equilibrium, and that the motion is produced by the remaining part $p-\frac{w}{v}$ or $\frac{pv-w}{v}$. This accelerating force may be resolved into x, which is exerted against the compound weight $w+q$, and $\frac{pv-w}{v}-x$, or $\frac{pv-vx-w}{v}$, which acts directly upon the power p. But the velocity generated in a given time is (5) as the intensity of the force divided by the mass. The velocity of the power, therefore, will be denoted by $\frac{1}{p}\left(\frac{pv-vx-w}{v}\right)$ or $\frac{pv-vx-w}{v}$. But the exertion of the force x, which urges the compound weight, is, in consequence of the mechanism, equal to the direct action of vx. Whence the celerity acquired in the same time will be ex-

pressed by $\frac{vx}{w+q}$. Therefore, from the conditions of the motion $\frac{vx}{w+q}^2 : \frac{pv-vx-w}{pv} :: 1 : v$; consequently $\frac{v^2 x}{w+q}=\frac{pv-vx-w}{pv}$, and reducing, $pv^3+qvx \times wvx = pvw - w^2 + pqv - qw$, and dividing, $x = \frac{pvw-w^2+pqv-qw}{pv^3+qv+wv}$, or $\frac{(pv-w)(w+q)}{pv^3+qv+wv}$. Whence vx, the real force exerted upon the compound weight, is $=\frac{(pv-w)(w+q)}{pv^2+q+w}$. The intensity of force is, therefore, $=\frac{pv-w}{pv^2+q+w}$. Hence we shall obtain expressions for the space described and the velocity of description. For $V=2dT\left(\frac{pv-w}{pv^2+q+w}\right)$ and $S=dT^2\left(\frac{pv-w}{pv^2+q+w}\right)$.

20. Since the intensity of action is $= \frac{pv-w}{pv^2+q+w}$, the measure of effect will be $= \frac{pv-w}{p^2v^2+pq+pw}$. Supposing p to be variable, and taking the fluxion, we shall obtain, when the effect is a *maximum*,

$$\frac{pv(p^2v^2+pq+pw)-(2p^2v^2+pq+pw)(pv-w)}{(p^2v^2+pq+pw)^2} = 0.$$

Whence $p^2v^3+pqv+pwv = (2pv^2+q+w)(pv-w) = 2p^2v^3+pqv+pwv-2pv^2w = qw - w$, and transposing, $p^2v^3-2pv^2w = qw + w^2$, and dividing, $p^2-\frac{2w}{v}.p=\frac{qw+w^2}{v^3}$, and resolving the quadratic, $p=\frac{w}{v}+\sqrt{\left(\frac{w^2}{v^2}+\frac{w^2}{v^3}+\frac{qw}{v^3}\right)}$. If v be large, the value of p will be nearly $\frac{2w}{v}+\frac{w}{v^2}+\frac{q}{v^2}$. Whence in machines, where the advantage is great, we may disregard the mo-

ments of the connecting parts, and consider the force which ought to be employed as double of what is barely able to maintain the equilibrium. Plate XLV.

21. In our investigations, we have always supposed that the same accelerating force is uniformly exerted. But instances frequently occur, where the power applied increases or diminishes during the action of the machine. This variation may be affected by numberless circumstances, and the general hypothetical solution of the problem would involve tedious and complicated formulæ. We shall content ourselves with a familiar example. Suppose that a weight *P* is attached to one of the extremities of a rope, *WAPB*, of equal and uniform texture, and applied to the circumference of a wheel, and to the other extremity a smaller weight *W* is appended.* Fig. 4.

It is manifest, that *P* will at first descend solely by its excess of weight; but its exertion will be continually increased, from the addition of the portion of the rope *BP*, while the antagonist power *W* suffers an equal diminution.

* A machine, constructed upon this principle, is actually employed in some coal-works. *P* is a light capacious bucket, *W* another that is strong and massy. When both are empty, *W* descends and elevates *P*; *W* is then loaded with coals, and, at the same time, a cock is opened which fills *P* with water. *P* then descends, by its superior weight, and raises the load. But when it reaches the bottom of the pit, it pushes up a valve, the water is discharged, and the action of the machine is renewed.

PLATE XLV. 22. It will be proper to take into account the momentum of the wheel. Let it be supposed to be solid and homogeneous, and let the radius of the whole $AC=r$, and of the variable circle $CD=x$, and let $\pi=6.2832$; then the minute annulus $DEde$ is equal to the rectangle of its length and its breadth, or $\pi x\dot{x}$; but the velocity is directly at the distance from the centre of motion; whence the momentum of the annulus will be equal to that of $\frac{\pi x^2 \dot{x}}{r}$ applied at A. And since $\int \frac{\pi x^2 \dot{x}}{r} = \frac{\pi x^3}{3r}$, the momentum of the whole matter of the wheel is equal to the momentum of a quantity of matter $\frac{\pi r^3}{3r}$ or $\frac{\pi r^2}{3}$, having the velocity of the point A. But $\frac{\pi r^2}{2}$ = area or A and $\frac{\pi r^2}{3} = \frac{2}{3}A$, and hence the momentum of the wheel will be the same, if $\frac{2}{3}$ of its matter were collected and accumulated at its circumference.

Fig. 3.

23. Now let us denote the weight to be raised, together with that of the rope and $\frac{2}{3}$ of the wheel, p = the power applied, h = the length of the rope, or the whole height of ascent, a = the weight of the rope, and $b = \frac{2}{3}$ of the weight of the wheel, and x = the space through which the ascent is already made. The force applied is therefore $= p + \frac{ax}{h}$ and the resistance opposed $= w - b - \frac{ax}{h}$, consequently the accelerating force, which must be the

difference of these, is $= p+b-w+\frac{2ax}{h}$, or $\frac{ph+bh-wh+2ax}{h}$. But this must be diffused through the whole mass $w+p$; wherefore the intensity of action is $= \frac{ph+bh-wh+2ax}{ph+wh}$. It was shown, in art. 5, that $V\dot{V}=2d\times\frac{ph\dot{x}+bh\dot{x}-wh\dot{x}+2ax\dot{x}}{ph+wh}$, and integrating, $\frac{1}{2}V^2 = 2d\times\frac{phx+bhx-whx+ax^2}{ph+wh}$; consequently $V=2\surd d\surd\left(\frac{phx+bhx-whx+ax^2}{ph+wh}\right)$; hence the final velocity is$=2\surd dh\surd\left(\frac{p+b-w+a}{p+w}\right)$.

24. Let the power applied be equal to the whole weight of the rope, and suppose that nothing is appended to the other; then if the momentum of the wheel be disregarded, the final velocity will be $=2\surd dh\times\surd\frac{p}{2p}$, or $\surd 2dh$. But the velocity which a body would acquire by descending through the same space, if entirely disengaged, is $\surd\ 4dh$; its velocity is, therefore, to that of the former, as $1:\surd 2$.

25. It was shown in art. 5, that $\dot{T}=\frac{\dot{s}}{V}$; whence, from the formulæ in art. 23; $\dot{T}= \frac{\dot{x}}{2\surd d\surd\left(\frac{phx+bhx-whx+ax^2}{ph+wh}\right)}$, or $\dot{T} = \frac{1}{2\surd d}\times \frac{\dot{x}}{\surd x^2\times\frac{a}{ph+wh}\times x\times\frac{ph+bh-wh}{ph-wh}\times x)}$, or multiplying the numerator and denominator by $\surd\left(\frac{ph+wh}{a}\right)$, we shall obtain, $\dot{T} = \frac{1}{2\surd d}\times$

$$\sqrt{\left(\frac{ph+wh}{a}\right)} \times \frac{\dot{x}}{\sqrt{\left(x^2 \times \frac{ph+bh-wh}{a} \times x\right)}}.$$ Put $\frac{ph+bh-wh}{a} = n$; then $\dot{T} = \sqrt{\left(\frac{ph+wh}{4ad}\right)} \times \frac{\dot{x}}{\sqrt{(x^2+nx)}}$; wherefore $T = \sqrt{\frac{ph+wh}{4ad}} \times \sqrt{,}\frac{\dot{x}}{\sqrt{(x^2 \times nx)}}$.

26. To find the fluent of $\frac{\dot{x}}{\sqrt{(x^2+nx)}}$, put $\sqrt{(x^2+nx)}=x+z$; then, $x^2+nx=x^2+2zx+z^2$ and $nx=2zx+z^2$; and transposing, $nx-2zx=z^2$, and dividing, $x=\frac{z^2}{n-2z}$, and taking the fluxion, $\dot{x}. = \frac{2z\dot{z}(n-2z)+2z^2\dot{z}}{(n-2z)^2}$, or $\frac{2nz\dot{z}-2z^2\dot{z}}{(n-2z)^2}$. But $\sqrt{(x^2+nx)}=x+z$, or substituting the value of x, $\sqrt{(x^2+nx)} = \frac{z^3}{n-2z} + z$, or $\frac{z^2+nz-z^2}{n-2z}$, or reducing $= \frac{nz-z^2}{n-2z}$; whence the expression $\frac{\dot{x}}{\sqrt{(x^2+nx)}} = \frac{2nz\dot{z}-2z2\dot{z}}{(n-2z)^2}$ divided by $\frac{nz-z^2}{n-2z}$, or $\frac{2\dot{z}}{n-2z}$. The fluent of $\frac{2\dot{z}}{n-2z}$ is C—Hyperbolic Logarithm, $n-2z$. To find the correction, suppose $2z=o$; in this case the fluent vanishes, and C=Hyp. Log. n; whence the true fluent is Hyp. Log .n—Hyp. Log. $n-2z$, or Hyp. Log. $\frac{n}{n-2z}$. But $z=\sqrt{(x^2+nx)}-x$, and substituting the fluent, is Hyp. Log. $\frac{n}{n+2x-2\sqrt{(x^2+nx)}}$.

To procure a more convenient expresssion, multiply the numerator and denominator by $n+2x+2\surd(x^2+nx)$; then we have Hyp. Log. $\frac{n(n+2x+2\surd(x^2+nx))}{n^2}$, or Hyp. Log. $\frac{n+2x+2\surd(x^2+nx)}{n}$, and resuming the value of n,

$$T=\surd\left(\frac{ph+wh}{4ad}\right)\times H,\ L,\ \frac{2x+\frac{ph+bh-wh}{a}+2\surd(x^2\frac{ph+bh-wh}{a}.x)}{\frac{ph+bh-wh}{a}}$$

or by reduction, $T=\surd\frac{ph+wh}{4ad}\times H,\ L,$

$$\frac{2ax+ph+bh-wh+2\surd(a^2x^2+aphx+abhx-awhx)}{ph+bh-wh}.$$

When $x=h$, we obtain for the whole time of the performance $T=\surd\left(\frac{ph+wh}{4ad}\right)\times H,\ L,$ $\frac{2a+p+b-w+2\surd(a^2+ap+ab-aw)}{p+b-w}$. The value of p, when the comparative quantity of action is a *maximum*, may also be determined, but it will be involved in a transcendant equation.

27. When the resistance of the parts of a machine is inconsiderable, we perceive from all these investigations the importance of continuing the action. The successive impulses are retained and accumulated, and the performance constantly increases. The whole quantity of action, produced by the machine, is not in the simple ratio of the time of continuance, but in that of the square.

28, When we attempt to take the resistance of the moving parts of the machine into the account, we have great difficulties to encounter. Friction is affected by numberless circumstan-

ces; by the nature of the substances employed in the construction; by their form; by the degree of polish; by their velocity, &c. Nor is it probable that its quantity can be derived from general principles; it must often be determined from the individual case, and can never be accurately ascertained. Friction may be considered as a continually retarding force. It may therefore be compared with that of gravity, and its effect may be estimated from that of a counteracting weight. The mass of the connecting parts, and their friction, both contribute to diminish the celerity of the motion; but they produce this retardation in different ways. The momentum which must be impressed upon the connecting parts of the machine requires a greater diffusion of power, and thus diminishes in some degree its effect. Friction does not alter the general mass, but reduces the quantity of accelerating force, and consequently the intensity of its action. If the quantity of friction were equal and constant, it is obvious, that if the moving power exceed it, the motion would be perpetually accelerated. But this is very far from the fact; for all the motions with which we are acquainted tend to a uniform celerity, and in a certain time would actually attain it. We may therefore conclude, that in the same machine, the friction increases in a certain ratio with the velocity. The great desideratum in mechanics is to determine the law of progression, and our deductions upon this subject must be considered as merely hypothetical.

29. Suppose, as before, w = the weight to be raised; q = the mass, which, if attached to the weight, would have the momentum of the connecting parts; p = the power employed;

v = the *advantage*; and let the friction be equal to ϕ, some function of the celerity of the weight. Because the moving parts of the machine remain the same as before, it is manifest that the intensity of action will be proportioned to the quantity of accelerating force; whence $p-\frac{w}{v}:p-\frac{w}{v}-\phi$ or $pv-w:pv-w-w\phi$: : $\frac{pv-w}{pv^2+q+w}:\frac{pv-w-v\phi}{pv^2+q+w}$, the true force which constantly acts upon the weight. Whence, if the quantity $v\phi$ be less than $pv-w$, the motion will be accelerated; if $v\phi$ be greater than $pv-w$, the motion will be retarded. In the natural progress of motion, the celerity at first increases, or $v\phi$ constantly approaches to an equality with the constant quantity $pv-w$, and when this equality takes place, the velocity is perfectly uniform. Hence the final celerity may be determined from the equation $\phi=\frac{pv-w}{v}$. Thus, if we suppose that the friction increases in the simple ratio of the velocity, then $mV=\frac{pv-w}{v}$ or $V=\frac{pv-w}{mv}$. If $\phi=mV\frac{1}{n}$, then $V=\left(\frac{pv-w}{mv}\right)^n$.

30. We may neglect the performance which is made during the first acceleration of motion as inconsiderable, when compared with the whole. The quantity of action will therefore be as V; and if the power affects the friction only by altering the velocity, the comparative action will be denoted by $\frac{V}{p}$; whence the performance will be a *maximum*, when $p\dot{V}=V\dot{p}$. If, as before, $\phi=mV\frac{1}{n}$; then $\left(\frac{pv-w}{mv}\right)^{n-1}\times\frac{n\dot{p}}{m}\times p$

$=\dot{p}\ \left(\frac{pv-w}{mv}\right)$, and reducing $\frac{np}{m}=\frac{pv-w}{mv}$, and $nvp=pv-w$, and transposing, $w=pv-pvn$, and dividing, $p=\frac{w}{v-vn}$. Or put $\frac{w}{v}$, the power which would barely maintain an equilibrium $=\pi$; then $p=\pi\times\frac{1}{1-n}$.

31. The law of the increase of velocity at first may also be ascertained. For (art. 4.) the time corresponding to the velocity V is $=\sqrt{\frac{\dot{V}}{2dF}}$, and in the present case $T=\sqrt{\frac{1}{2d}}\times\frac{fv^2+q+w}{pv-w-v\varphi}\times\dot{V}$. As φ is a function of V, the fluent may always be expressed at least by an infinite series. These formulæ might also be applied to rectilineal motions performed in resisting *media;* but this would rather be a digression from the subject.

MECHANICS.

Description of a simple and powerful Capstane.

THIS capstane is represented in Fig. 1. Plate XLV, where *AD* is a compound barrel, consisting of two cylinders *CD* of different radii. The rope *DEC* is fixed at the extremity of the cylinder *D*, and after passing over the pulley *E*, which is attached to the load by means of the hook *F*, it is coiled round the other cylinder *C*, and fastened at its upper end. *AB* is the bar by which the compound barrel *CD* is urged about its axis, so that the rope may coil round the cylinder *D*, while it unwinds itself from the cylinder *C*. Let us now suppose that the diameter of the part *D* of the barrel is 21 inches, while the diameter of the part *C* is only 20, and let the pulley *E* be 20 inches in diameter. It is evident then that when the barrel *AD* is urged round by a pressure exerted at the point *B*, 63 inches of rope will be gathered upon the cylinder *D*, and 60 inches will be unwinded from the cylinder *C* by one revolution of the bar *AB*, these numbers representing the circumference of each cylinder. The quantity of wound rope, therefore, PLATE XLV. Fig. 1.

exceeds the quantity that is unwound by 63—60, or 3 inches, the difference of their respective perimeters; and the half of this quantity, or $1\frac{1}{2}$ inch will be the space through which the load or the pulley *E* moves by one turn of the bar. But if a simple capstane of the same dimensions had been employed, the length of rope coiled round the barrel by one revolution of the bar would have been 60 inches, and the space described by the pulley or load to be overcome would have been 30 inches. Now, it is a maxim in mechanics, that the power of any engine is universally equal to the velocity of the impelled point divided by the velocity of the working point, or to the velocity of the power divided by the velocity of the weight, that is, to the velocity of the point *B* divided by the velocity of the pulley *E*; consequently if the lever in both capstanes be the same, and the diameter of their barrels equal, the power of the common one will be to the power of the improved one as $1\frac{1}{2}$ to 30, that is, inversely as the velocity of their weights, and the power of the latter will be $\frac{30}{1.5}=20$, or in other words, will be equivalent to a 20 fold tackle of pulleys.* If it be wished to double the power of the machine, we have only to cover the cylinder *C* with laths a quarter of an inch thick, so that the difference between the radii of each cylinder may be half as little as before; for the power of the capstane increases as the difference between the radii of the cylinders is diminished. As we increase the power, there-

* In practice it will be found equivalent to a 26 fold tackle of pulleys, as about *one third* of the power of a system of pulleys is destroyed by friction and the bending of the ropes.

fore, we increase the strength of our machine, while all other engines are proportionably enfeebled by an augmentation of power. Were we, for example, to increase the power of the common capstane, we must diminish the barrel in the same proportion, supposing the bar or handspike not to admit of being lengthened, and we not only weaken its strength, but destroy much of its power by a greater flexure or bending of the ropes.

The reader will perceive that this capstane may be converted into a crane or windlass for raising weights, merely by giving the compound barrel *AB* a horizontal position, and substituting a winch instead of the bar *AB*. The superiority of such a crane above the common ones is obvious from what has been said; but it has this additional advantage, that it allows the weight to stop at any part of its progress, without the aid of a ratchet wheel and catch, from the two parts of the rope pulling on contrary sides of the barrel. The rope, indeed, which coils round the larger part of the barrel acts with a larger lever, and consequently with greater force than the other; but as this excess of force is not sufficient to overcome the friction of the gudgeons, the weight remains stationary in any part of its path.

A crane of this kind was erected in 1797 at Bordenton in New-Jersey by Mr. M'Kean for the purpose of raising logs of wood to the frame of a saw mill, which was 10 feet distant from the ground. The diameter of the largest cylinder was 2 feet, and its length 3 feet; the other cylinder was 1 foot in diameter, and of the same length with the largest. The difference of their

circumferences, therefore, was 3 feet, and the log would move through a space of 18 inches with 1 turn of the handspike; and through the required height with only 8 turns. The length of the bar or handspike was 6 feet, which, at the point where the power was applied, described a circle of about 30 feet, so that the power of the crane was as 1 to 20. The length of the rope was only 55 feet, whereas, if the weight had been raised through the same height with a similar power by means of a tackle of pulleys, 270 feet of rope must have been employed. In the latter case, however, the rope sustains only $\frac{1}{20}$ of the weight, but in the former it supports one half of the load.

In describing a capstane of this kind, Dr. Robison asserts,* that when the diameters of the cylinders which compose the double barrel are as 16 to 17, and their circumferences as 48 to 51, the pulley is brought nearer to the capstane by about 3 inches for each revolution of the bar. This, however, is a mistake, as the pulley is brought only $1\frac{1}{2}$ inch nearer the axis. This will be evident, if we conceive a quantity of rope equal to the circumference of the larger cylinder to be winded up all at once, and a quantity equal to the circumference of the lesser one to be unwinded all at once. In the present case, 51 inches of rope will be coiled round the larger part of the barrel by one revolution of the capstane bar, and consequently the load would be raised $25\frac{1}{2}$ feet, the rope being doubled. Let 48 inches of rope be now unwinded from the lesser cylinder, and

* Encyclopædia Britannica Supplement Art. *Machinery*, vol IX: p [illegible].

the load will sink 24 feet; therefore, $25\frac{1}{2}-24=1\frac{1}{2}$ foot is the whole height or distance through which the weight has been moved.

Dr. Robison affirms that the capstane now described was invented by an untaught but ingenious country tradesman. It appears, however, to be the invention of the celebrated George Eckhardt, and likewise that of the late Mr. Robert M'Kean, of Philadelphia, son to the present governor of Pennsylvania.

MECHANICS.

A Mechanical Method of finding the Centre of Gravity.

AS it is frequently necessary in mechanical operations to find the centre of gravity, the following practical method may probably be acceptable to some readers, as it is not to be met with in any of the elementary treatises in our language.—

1. If the body, whose centre of gravity, is to be found, can be easily suspended by a thread or cord, then the centre of gravity will be situate in some point in the direction of the cord prolonged. Suspend the body at another part, so that the new direction of the cord may be nearly at right angles with its former direction, then, as the centre of gravity must lie somewhere in the new direction of the cord prolonged, the point where these two lines (formed by prolonging the cord) intersect each other, will determine the centre of gravity.

2. If the body be of such a size or quality that it cannot be conveniently suspended, place it upon a horizontal edge so that its parts may be in equilibrio. The horizontal edge will make a line or mark on the body in the same direction with itself, and the centre of gravity will be in some point in this line. Balance the bo-

dy a second time, so that the line upon the body may be nearly at right angles to the horizontal edge, which will make a new line or mark upon the body, the centre of gravity therefore will be somewhere in this new line, and consequently in the point where it intersects the former line.

3. If the body be so flexible that it can neither be suspended nor balanced, then let a board be balanced, as in case 2d, and upon it, when balanced, lay the body, whose centre of gravity is to be found, in such a manner that the board may still be in equilibrio; then the centre of gravity will be in a line opposite to that which is made on the board by the horizontal edge, and by shifting the position of the board, and again balancing it, a new line will be found, the intersection of which, with the former line, will determine the centre of gravity.*

* Or rather the centre of gravity of the body will be somewhere in the common section of the two vertical planes passing through the above horizontal edges or lines.—A. Ed.

HYDRAULICS.

On the Steam-Engine.

THE superiority of inanimate power to the exertions of animals in turning machinery has been universally acknowledged. In the former, the power generally continues its action without the smallest intermission, but frequent and long relaxations are necessary for restoring the strength and activity of exhausted animals. There are many places, however, where a sufficient quantity of water cannot be procured, or where it cannot be employed for the want of proper declivities; and there are situations also which are highly unfavourable for the erection of wind-mills. But even when water and wind-mills can be conveniently erected, there is such a variation in the impelling power, arising from accidental and unavoidable causes, that sometimes, in the case of water, and often in the case of wind, there is not a sufficient force for putting the machinery in motion. In such circumstances, the discovery of steam as an impelling power, may be regarded as a new æra in the progress of the arts. Wherever fire and water can be obtained, we can procure a quantity of the steam capable of overcoming the most powerful resistance, and free from those accidental variations

of power which affect every inanimate agent that has hitherto been employed as the first mover of machines.

The invention of the steam-engine has been universally ascribed by the English to the Marquis of Worcester, and to Papin by the French,* but there can be little doubt that about 34 years prior to the date of the Marquis's invention, and about 61 years before the publication of Papin's, steam was applied as the impelling power of a stamping-engine by one *Brancas,* an Italian, who published an account of his invention in the year 1629. It is extremely probable, however, that the Marquis of Worcester was unacquainted with the discovery of Brancas, and that the fire-engine which he mentions so obscurely in his Century of Inventions, was the result of his own ingenuity.

The utility of steam as an impelling power being thus known, the ingenious Captain Savary took advantage of the discovery, and invented an engine which raised water by the expansion and condensation of steam. Several of Savary's engines were actually erected in England and in France, but they were never capable of raising water from a depth which exceeded 35 feet.

The steam-engine received great improvements from the hands of Newcomen, Beighton, Blakey, and other ingenious men; but it was brought to its present high state of perfection by the celebrated Mr. Watt, of Birmingham, one of the most accomplished philosophers and engineers of the present age. Hitherto the

* Bossut, Traite d'Hydrodynamique.

steam-engine has been employed merely as a hydraulic machine for draining mines or for raising water; but in consequence of Mr. Watt's improvements, it has for a series of years been employed as the impelling power or first mover of almost every species of machinery.

PLATE XLVI. Fig. 1. Fig. 1. of Plate XLVI. represents one of Mr. Watt's latest engines. *CD* is the boiler in which the water is converted into steam by the heat of the furnace *D*. It is sometimes made of copper, but more frequently of iron, its bottom is concave, and the flame is made to circulate round its sides, and is sometimes conducted by means of flues through the very middle of the water, so that as great a surface as possible may be exposed to the action of the fire. In some of Mr. Watt's engines, the fire contained in an iron vessel was introduced into the very middle of the water, and the outer boiler was formed of wood, as being a slow conductor of heat. When the furnaces are constructed in the most judicious manner, 8 square feet of the boiler's surface must be acted upon by the fire or the flame in order to convert 1 cubic foot of water into steam in the space of an hour; and this effect will be produced by between $\frac{1}{8}$ and $\frac{1}{12}$ of a bucket of good Newcastle coals. When fire is applied to the boiler, the water does not evaporate into steam till it has reached the temperature of 212° of Fahrenheit, or the boiling point. This arises from the superincumbent weight of the atmosphere, for when the water is heated in a vessel exhausted of air, steam is generated even below the temperature of 96°, or blood-heat. When the water, however, is pressed by air or steam more condensed than

the atmosphere, a temperature greater than 212° is necessary for the production of steam, but the heat necessary for this purpose increases in a less ratio than the pressures to be overcome.—The steam which is produced in the boiler *CD*, is about 1800 times rarer than water, and is conveyed through the steam-pipe *CE* into the cylinder *G*, where it acts upon the piston *q*, and communicates motion to the great beam *AB*. But before we proceed to consider the manner by which this motion is conveyed, we shall point out the very ingenious method which Mr. Watt has employed for supplying the boiler regularly with water, and preserving it at the same height *OP*. This is absolutely necessary in order that the quantity and elasticity of the steam in the boiler may be always the same. The small cistern *u*, placed above the boiler, is supplied with water from the hot well *h* by means of the pump *z* and the pipe *f*. To the bottom of this cistern is fitted the pipe *ur* which is immersed in the water *OP*, and is bent at its lower extremity in order to prevent the entrance of the rising steam. A crooked arm *u'd* attached to the side of the cistern *u*, supports the small cistern *a b'*, which moves upon *'d* as a centre.—The extremity *'b* of this lever carries, by means of the wire *'bP*, a stone or piece of metal *P*, which hangs just below the surface of the water in the boiler, and the other extremity *'a* is connected by the wire *a'u* with a valve at the bottom of the cistern *u*, which covers the top of the pipe *ur*. Now, it is a maxim in hydrostatics, that when a heavy body is suspended in a fluid, the body loses as much of its weight as the quantity of water which it displaces.

When the water *OP*, therefore, is diminished by part of it being converted into steam, the upper surface of the body *P* will be above the water, and its weight will consequently be increased in proportion to the quantity of the body that is out of the water, or to speak more precisely, the additional weight which the body *P* receives, by a diminution of the water in the boiler, is equal to the weight of a quantity of the fluid, whose bulk is the same as the part of the body *P* which is above the water. By this addition to its weight, the stone *P* will cause the extremity '*b* of the lever to descend, and by elevating the arm '*a*'*d*, will open the valve at the top of the pipe *ur*, and thus gradually introduce a quantity of water into the boiler, equal to that which was lost by evaporation. This process is continually going on while the water is converting into steam; and it is evident that too much water can never be introduced, for as soon as the surface of the water coincides with the upper surface of the body *P*, it recovers its former weight, and the valve at *u* shuts the top of the pipe *ur*. In order to know the exact height of the water in the boiler, two cocks *k* and *l* are employed, the first of which, *k*, reaches to within a little of the height at which the water should stand, and the other, *l*, reaches a very little below that height. If the water stand at the desired height, the cock *k* being opened, will give out steam, and the cock *l* will emit water, in consequence of the pressure of the superincumbent steam on the water *OP*; but if water should issue from both cocks, it will be too high in the boiler, and if steam issue from both, it will be too low. As there would

be great danger of the boiler's bursting if the steam should become too strong, it is furnished with the safety-valve *x*, which permits a part of the steam to escape till its strength be sufficiently diminished. Plate XLVI.

From the dome of the boiler proceeds the steam-pipe *CE*. which conveys the steam into the top of the cylinder *G* by means of the steam-valve *a*, and into the bottom of the cylinder by means of the valve *c*. The branch of the pipe which extends from *a* to *c* is cut off in Fig. 1. in order to show the valve *b*, but is distinctly visible in Fig. 2. which is a view of the pipes and valves in the direction *FM*. The cylinder *G* is sometimes inclosed in a wooden case, in order to prevent it from being cooled by the ambient air; and sometimes in a metallic case, that it may be surrounded and kept warm by a quantity of steam which is brought from the steam-pipe *EC*, through the pipe *EG*, by turning a cock. It is generally thought, however, that little benefit is obtained by encircling the cylinder with steam, as the quantity thus lost is almost equal to what is destroyed by the coldness of the cylinder. After the steam, which was admitted above the piston *q* by the valve *a*, and below it by the valve *c*, has performed its respective offices of depressing and elevating the piston, and consequently the great beam *AB*, it escapes by the eduction-valves, *b* and *d* Fig. 1 & 2. into the condenser *i*, where it is converted into water by means of a jet playing in the inside of it. The water thus collected in the condenser is carried off, along with the air which it contains, into the hot well *h*, by the air-pump *e*, which is wrought by the piston-rod *TM*, attached to the

great beam *AB*. From the hot well *h* this water is conveyed by the pump *z* and the pipe *f*, into the cistern *u*, for the purpose of supplying the boiler. The water *w* which renders air-tight the pump *e*, and supplies the jet of water in the condenser, is furnished by the pump *g*, which is worked by the great beam. The steam and eduction-valves *a*, *c*. *b*, *d*, are opened and shut by the spanners *aM*, *dM*, *cN*, *bN*, whose handles *M* and *N* are moved by the plugs 1, 2, fixed to *TN*, the piston-rod of the air-pump. This part of the machinery has been called the *working-geer*; and is so constructed that the steam and eduction-valves can be worked either by the hand or by the pistons of the air-pump. The piston-rod *R*, which moves the piston *q*. passes through a box or collar of leathers fixed in a strong metallic plate on the top of the cylinder. The rod is turned perfectly cylindrical, and is finely polished in order to prevent any air from passing by its sides. The top *V* of the piston-rod *R* is fixed to the machinery *TV*, which is called the parallel joint, and is so contrived as to make the rod *VR* ascend and descend in a vertical or perpendicular direction. When the lever or beam rises into its present position from a horizontal one, the piston-rod *VR* has a tendency to move towards μ, and would move towards it were the bar $\mu\nu$ fixed in its present position, for while the point *V* rises, the bar μV also rises, at the same time the angle $V\mu\nu$ increases, and likewise the angle $\lambda V\mu$, so that the vertex *V* of the angle $\lambda V\mu$ would move towards *T*. The bar $\mu\nu$, however, is not at rest, but moves round the fixed point ν, and rises along with the point *V*: while $\mu\nu$, therefore,

rises upon ν as a centre, the adjoining bar $\mu\nu T$ moves round the point T towards V, the angle $T\mu\nu$ increases, and the point μ approaches to V, and keeps VR in a perpendicular position, so that whatever tendency the point V has towards T by the increase of the angle $\nu V\mu$, it has an equal tendency in the contrary direction, by the increase of the angle $T\mu\nu$: but as the beam AB falls into a horizontal position, all these motions are reversed. When the piston-rod VR rubs most upon the side of the collar of leathers nearest to a, the fixed point ν must be shifted a little in the contrary direction, viz. to the right hand of R. That the nature of this parallel joint may be better understood, it may be proper to observe, that all the bars which have been mentioned are double, as may be seen in the figure,—that they move round points at ν, T, V, μ, and ν, and that the two bars between μ and V move between the bars at $\mu\nu$.*

In the steam-engines of Newcomen and Beighton, where the piston was raised merely by a counterweight at the extremity A of the great beam, the piston-rod was connected with its other extremity by means of a chain bending round the arch of a circle fixed at B; but in Mr. Watt's improved engines with a double stroke, in which the piston receives a strong impulse upwards as well as downwards, the chain would slacken, and could not communicate motion to the beam. An inflexible rod, therefore, must be employed for connecting the piston with the beam, or the piston must be suspended by

* As the authors who have written on the steam-engine have not taken any notice of the operation of the parallel joint, it was thought proper to give the preceding description of it, which we trust will be satisfactory to the reader.

double chains like those of engines for extinguished fire. In some of Mr. Watt's engines the latter of these methods was adopted. He then employed a toothed rack working in a toothed sector fixed at *B*. and afterwards fell upon the very superior method which we have now been describing.

All the engines which were constructed before the time of Mr. Watt were employed merely for raising water, and were never used as the first movers of machinery. Mr. R. Fitzgerald indeed, published in the Transactions of the Royal Society, a method of converting the irregular motion of the beam into a continued rotatory motion, by means of a crank and a train of wheel-work connected with a large and massy fly, which, by accumulating the pressure of the machine during the working stroke, urged round the machinery during the returning stroke, when there is no force pressing it forward. For this new and ingenious contrivance, Mr. Fitzgerald received a patent, and proposed to apply the steam-engine as the moving power of every kind of machinery, but it does not appear that any mills were erected under this patent. In order to convert the reciprocating motion of the beam into a circular motion, Mr. Watt fixed a strong and inflexible rod *AU* to the extremity of the great beam. To the lower end of this rod, a toothed wheel *U* is fastened by bolts and straps, so that it cannot move round its axis. This wheel is connected with another toothed wheel *S* of the same size, by means of iron bars, which permits the former to revolve round the latter, but prevents them from quitting each other. This apparatus is called the Sun-and-Planet wheels,

from the similarity of their motion to that of the two luminaries. On the axis of the wheel *S* is placed the large and heavy fly-wheel *F*, which regulates the desultory motion of the beam.—When the extremity *A* of the great beam rises from its lowest position, it will bring along with it the wheel *U*, and cause it to revolve upon the circumference of the wheel *S*, so that the interior part of the former, or the part next the cylinder, will act upon the exterior part of the latter, or the part farthest from the cylinder, and put it in motion along with the fly *F*. After the wheel *U* has got to the top of the wheel *S*, the end *A* of the beam will have reached its highest position, and the wheel *S*, along with the fly, will have performed one complete revolution. When the wheel *U* passes from the top of *S* into its former position below it, the extremity *A* of the beam will also descend from its highest to its lowest position, so that for every ascent or descent of the piston or the great beam, the planet-wheel *U* will make one turn, while the sun-wheel and fly will perform two complete revolutions.

When the steam-engine is employed to drive machinery in which the resistance is very variable, and where a determinate velocity cannot properly be dispensed with, Mr. Watt has applied a conical pendulum, which is represented at *mn*, for procuring a uniform velocity. This regulator consists of two heavy balls *m*, *n*, suspended by iron rods which move in joints at the top of the vertical axis *op*, and is put in motion by the rope *oo*, which passes over the pulleys *o*, *o*, and round the axis *o* of the fly. Since the velocity of the fly and sun-wheel increases and diminishes with the quantity of steam that is

admitted into the cylinder, let us suppose that too much is admitted—then the velocity of the fly will increase, but the velocity of the vertical axis *op* will also increase, and the balls *mn* will recede from the axis by the augmentation of their centrifugal force. By this recess of the balls, the extremity *p* of the lever *ps*, moving upon *y* as a centre, is depressed, its other extremity *s* rises, and by forcing the cock at *a* to close a little, diminishes the supply of steam. The impelling power being thus diminished, the velocity of the fly and the axis *op* decrease in proportion, and the balls *m*, *n*, resume their former position.

In Mr. Watt's improved engine, the steam and eduction-valves are all puppet-clacks. One of these valves, and the method of opening and shutting it, is represented in Fig. 1. of Plate XLVII. Let it be one of the eduction-valves, and let *AA* be part of the pipe which conducts the steam into the cylinder, and *MM* the superior part of the pipe which leads to the condenser. At *OO*, the seat of the valve, a metallic ring, of which *nn* is a section, is fitted accurately into the top of the pipe *MM*, and is conical on the outer edge, so as to suit the conical part of the pipe. These two pieces are ground together with emery, and adhere very firmly when the contiguous surfaces are oxidated or rusted. The clack is a circular brass plate *m*, with a conical edge ground into the inner edge of the ring *nn*, so as to be air-tight, and is furnished with a cylindrical tail *mP*, which can rise or fall in the cavity of the crossbar *NN*. To the top of the valve *m*, a small metallic rack *mF* is firmly fastened, which can be raised or depressed by the portion *E* of a

Plate XLVII. Fig. 1.

toothed wheel, moveable upon the centre *D*. The small circle *D* represents a section of an iron cylindrical axis, whose pivots move in holes in the opposite sides of the pipe *AA*. Its pivots are fitted into their sockets, so as to be air-tight; and the admission of air is further prevented by screwing on the outside of the holes, necks of leather soaked in rosin or melted tallow. One end of this axis reaches a good way without the pipe *AA*, and carries a handle or spanner *bN*, which may be seen in Fig. 1. Plate XLVI, and which is actuated by the plugs 1, 2, of the rod *TN*. When the plug 2, therefore, elevates the extremity of the spanner *Nb*, during the ascent of the piston-rod *TN*, the axle *D*, Plate XLVII, Fig. 1. is put in motion, the valve *m* is raised by means of the toothed racks *E* and *F*, and the steam rushes through the cavity of the circular ring *nn*, by the sides of the cross piece of metal *OO*, *NN*. When the valve needs repair, the cover *B*, which is fastened to the top of the valve-box by means of screws, can easily be removed.

Plate XLVI. Fig. 1.

Plate XLVII. Fig. 1.

Having thus described the different parts of the most improved steam-engine, it will be proper to attend to the mode of its operation. Let us suppose, that the piston is at the top of the cylinder, as is represented in the figure, and that the upper steam-valve *a*, and the lower eduction or condensing-valve *d*, are opened by means of the spanner *M*, while the lower steam-valve *c*, and the upper eduction-valve *b*, are shut; then the steam in the boiler will issue through the steam-pipe *CE*, and the valve *a*, into the top of the cylinder, and depress the piston, by its elasticity, to the very bottom.—But when the piston *q* is brought to the bottom

of the cylinder, the extremity *B* of the great beam is dragged down by the parallel joint *TV*, its other extremity *A* rises, and the wheel *U* having passed over ½ of the circumference of *S*, will have urged forward the fly-wheel *F*, and consequently, the machinery attached to it, one complete revolution. When the piston *q* has reached the bottom of the cylinder, the piston-rod *TX* of the air-pump, by the pressure of the plug 1 upon the spanner *M*, has shut the steam-valve *a*, and the eduction-valve *d*, while the plug 2 has, by means of the spanner, opened the eduction-valve *b*, and the steam-valve *c*. The steam, therefore, which is above the piston, rushes through the eduction-valve *b* into the condenser *i*, where it is converted into water by the jet in the middle of it, and by the coldness arising from the surrounding fluid *w*, while, at the same time, a new quantity of steam from the boiler issues through the open steam-valve *c*, into the cylinder, forces up the piston, and, by raising one end of the working beam, and depressing the other, makes the wheel *U* describe the other semi-circumference of *S*, and causes the fly and the machinery on its axis to perform another complete revolution. As the plugs 1, 2, ascend with the piston *q*, they open or shut the steam and eduction-valves, and the operation of the engine may be thus continued for any length of time.

From this brief description of the steam-engine, the reader will be enabled to perceive the nature, and appreciate the value of Mr. Watt's improvements. It had hitherto been the practice to condense the steam in the cylinder itself, by the injection of cold water; but the water which is injected acquires a considerable degree of heat.

from the cylinder, and being placed in air, highly rarefied, part of it is converted into steam, which resists the piston, and diminishes the power of the engine. When the steam is next admitted, part of it is converted into water by coming in contact with the cylinder, which is of a lower temperature than the steam, in consequence of the destruction of its heat by the injection-water. By condensing the steam, therefore, in the cylinder itself, the resistance to the piston is increased by a partial reproduction of this elastic vapour, and the impelling power is diminished by a partial destruction of the steam which is next admitted. Both these inconveniencies Mr. Watt has in a great measure avoided, by using a condenser separate from the cylinder, and incircled with cold water;* and by surrounding the cylinder with a wooden case, and interposing light wood-ashes, in order to prevent its heat from being abstracted by the ambient air.

The greatest of Mr. Watt's improvements consists in his employing the steam both to elevate and depress the piston. In the engines of Newcomen and Beighten, the steam was not the impelling power, it was used merely for producing a vacuum below the piston, which was forced down by the pressure of the atmosphere, and elevated by the counterweight at the farther extremity of the great beam. The cylinder, therefore, was exposed to the external air at every descent of the piston, and a considerable portion of its heat being thus abstracted,

* Even in Mr. Watt's best engines, a very small quantity of steam remains in the cylinder, having the temperature of the hot-well *p*, or of the water, into which the ejected steam is converted. Its pressure is indicated by a barometer, which Mr. Watt has ingeniously applied to his engines for exhibiting the state of the vacuum.

a corresponding quantity of steam was of consequence destroyed. In Mr. Watt's engines, however, the external air is excluded by a metal plate at the top of the cylinder, which has a hole in it for admitting the piston-rod; and the piston itself is raised and depressed merely by the force of steam.

When these improvements are adopted, and the engine constructed in the most perfect manner, there is not above $\frac{1}{5}$ part of the steam consumed in heating the apparatus; and, therefore, it is impossible that the engine can be rendered $\frac{1}{4}$ more powerful than it is at present. It would be very desirable, however, that the force of the piston could be properly communicated to the machinery without the intervention of the great beam. This, indeed, has been attempted by Mr. Watt, who has employed the piston-rod itself to drive the machinery; and Mr. Cartwright has, in his engine, converted the perpendicular motion of the piston into a rotatory motion, by means of two cranks fixed to the axis of two equal wheels which work in each other. Notwithstanding the simplicity of these methods, none of them have come into general use, and Mr. Watt still prefers the intervention of the great beam, which is generally made of hard oak, with its heart taken out, in order to prevent it from warping. A considerable quantity of power, however, is wasted by dragging, at every stroke of the piston, such a mass of matter from a state of rest to a state of motion, and then from a state of motion to a state of rest. To prevent this loss of power, a light frame of carpentry* has been employed by se-

* The great beam in Mr. Hornblower's engine, is constructed in this manner, and is formed upon truly scientific principles.

veral engineers, instead of the solid beam; but after being used for some time, the wood was generally cut by the iron bolts, and the frame itself was often instantaneously destroyed. In some of the engines lately constructed by Mr. Watt, he has formed the great beam of cast iron, and while he has thus added to its durability, he has at the same time diminished its weight and increased the power of his engine.

Encouraged by Mr. Watt's success, several improvements upon the steam engine have been attempted by Hornblower, Cartwright, Trevethick, and other engineers of this country. But it does not appear that they have either increased the power of the engine, or diminished its expense. It appears, on the contrary, that most of these pretended improvements, excepting those of Hornblower, consist merely in having adopted Mr. Watt's discoveries, in such a manner as not to infringe upon his patent.

About ten months ago, Mr. Arthur Woolf announced to the public a discovery respecting the expansibility of steam, which promises to be of very essential utility. Mr. Watt had formerly ascertained, that steam which acts with the expansive force of four pounds per square inch, against a safety-valve exposed to the weight of the atmosphere, after expanding itself to four times the volume it thus occupies, is still equal to the pressure of the atmosphere. But Mr. Woolf has gone much farther, and has proved that quantities of steam, having the force of 5, 6, 7, 8, 9, 10, &c. pounds on every square inch

Dr. Robison observes, that it is stronger that a beam of the common form, which contains 20 times its quantity of timber.

If the great beam were so centered as that its *natural vibrations* might correspond with those communicated to it by the engine, the resistance arising from the vis inertiæ of the beam would be, in a great measure, obviated.—A. Ed.

may be allowed to expand 5, 6, 7, 8, 9, 10, &c. times its volume, and will still be equal to the atmosphere's weight, provided that the cylinder in which the expansion takes place, has the same temperature as the steam before it began to expand. It is evident. however, that an increase of temperature is necessary both to produce and to maintain this augmentation of the steam's expansive force above the pressure of the atmosphere. At the temperature of 212° of Fahrenheit, the force of steam is equal only to the pressure of the atmosphere, and, in order to give it an additional elastic force of 5 pounds per square inch, the temperature must be increased to about $227\frac{1}{3}$°, as is evident from the following table.

TABLE,

Of the Pressures, Temperatures, and Expansibility, of Steam, equal to the Force of the Atmosphere.

Elastic force of steam predominating over the pressure of the atmosphere, and acting upon a safety-valve.	Degrees of temperature requisite for bringing the steam to the different expansive forces in the preceding column.	Number of times its volume that steam of the preceding force and temperature will expand, and still continue equal to the pressure of the atmosphere.
lbs. per square inch.	Degrees of heat.	Expansibility.
5	$227\frac{1}{3}$	5
6	$230\frac{1}{4}$	6
7	$232\frac{3}{4}$	7
8	$235\frac{1}{4}$	8
9	$237\frac{1}{2}$	9
10	$239\frac{1}{2}$	10
15	$250\frac{1}{2}$	15
20	$259\frac{1}{2}$	20
25	267	25
30	273	30
35	278	35
40	282	40

In this manner, by small additions of temperature, an expansive power may be given to steam, which will enable it to expand 50, 100, 200, 300, &c. times its volume, and still have the same force as the atmosphere.

Upon this principle Mr. Woolf has taken out a patent for various improvements on the steam-engine, a short account of which we shall subjoin in the words of the specification.

'If the engine be constructed originally with the intention of adopting the preceding improvement, it ought to have two steam-vessels of different dimensions, according to the expansive force to be communicated to the beam, for the smaller steam-cylinder must be a measure for the larger. For example, if steam of 40 pounds the square inch be fixed on, then the smaller steam-vessel should be at least $\frac{1}{40}$ part of the contents of the larger one. Each steam-vessel should be furnished with a piston, and the smaller cylinder should have a communication both at its top and bottom, with the boiler which supplies the steam; which communications, by means of cocks or valves, are to be alternately opened and shut during the working of the engine. The top of the smaller cylinder should have a communication with the bottom of the large cylinder, and the bottom of the smaller one with the top of the larger, with proper means to open and shut these alternately by cocks, valves, or any other contrivance. And both the top and bottom of the larger cylinder should, while the engine is at work, communicate alternately with a condensing-vessel, into which a jet of water is admitted to hasten the condensation. Things being thus arranged when the engine is at work, steam of high tem-

When the piston has descended to the bottom of the cylinder, through the space of 4 feet, the weight will have risen through the same space, and 100 pounds raised through the height of 4 feet, during one descent of the piston, will express the mechanical power of the engine. But if the area of the piston q, and the length of the cylinder be doubled, while the expansive force of the steam, and the time of the piston's descent remain the same, the mechanical energy of the engine will be quadruple, and will be represented by 200 pounds raised through the space of 8 feet during the time of the piston's descent. The power of steam-engines, therefore, is, *cæteris paribus*, in the compound ratio of the area of the piston, and the length of the stroke. These observations being premised, it will be easy to compute the power of steam-engines of any size.

Thus, let it be required to determine the power of steam-engines, whose cylinder is 24 inches diameter, and which make 22 double strokes in a minute, each stroke being 5 feet long, and the force of the steam being equal to a pressure of 12 pounds avoirdupois upon every square inch.* The diameter of the piston being multiplied by its circumference, and divided by 4, will give its area in square inches; thus, $\frac{24\times3.1416\times24}{4}=452.4$, the number of square inches exposed to the pressure of the steam. Now, if we multiply this area by 12 pounds, the pressure upon every square inch, we will

* The working pressure is generally reckoned at 10 pounds on every circular inch, and Smeaton makes it only 7 pounds on every circular inch. The effective pressure which we have adopted is between these extremes, being equivalent to 9.42 pounds on every circular inch.

have $452.4 \times 12 = 5428.8$ pounds, the whole pressure upon the piston, or the weight which the engine is capable of raising. But since the engine performs 22 *double* strokes, 5 feet long, in a minute, the piston must move through $22 \times 5 \times 2 = 220$ feet in the same time; and therefore the power of the engine will be represented by 5428.8 pounds avoirdupois, raised through 220 feet in a minute, or by 10.4 hogsheads of water, ale measure, raised through the same height in the same time. Now this is equivalent to $542.80 \times 220 = 1194336$ pounds, or $10.4 \times 220 = 2288$ hogsheads raised through the height of 1 foot in a minute. This is the most unequivocal expression of the mechanical power of any machine whatever, that can possibly be obtained. But as steam-engines were substituted in the room of horses, it has been customary to calculate their mechanical energy in *horse-powers*, or to find the number of horses which could perform the same work. This indeed is a very vague expression of power, on account of the different degrees of strength which different horses possess. But still, when we are told that a steam-engine is equal to 16 horses, we have a more distinct conception of its power, than when we are informed that it is capable of raising a number of pounds through a certain space in a certain time.

Messrs. Watt and Boulton suppose a horse capable of raising 32,000 pounds avoirdupois, 1 foot high in a minute, while Dr. Desaguliers makes it 27,500 pounds, and Mr. Smeaton only 22,916. If we divide, therefore, the number of pounds which any steam-engine can raise 1 foot high in a minute, by these three numbers, each quotient will represent the number of

horses to which the engine is equivalent. Thus, in the present example, $\frac{1194336}{32000} = 37\frac{1}{3}$ horses according to Watt and Boulton; $\frac{1194336}{27500} = 43\frac{1}{3}$ horses, according to Desaguliers; and $\frac{1194336}{22916}$ $= 52\frac{1}{7}$ horses, according to Smeaton. In this calculation it is supposed that the engine works only 8 hours a-day; so that if it wrought during the whole 24 hours, it would be equivalent to thrice the number of horses found by the preceding rule. We cannot help observing, and it is with sincere pleasure that we pay that tribute of respect to the honour and integrity of Messrs. Watt and Boulton which has every where been paid to their talents and genius.—that in estimating the power of a horse, they have assigned a value the most unfavourable to their own interests. While Mr. Smeaton and Dr. Desaguliers would have made the engine in the preceding example equivalent to 52 or 43 horses, the patentees themselves state that it will perform the work only of 37. How unlike is this conduct to some of our modern inventors, who ascribe powers to their machines which cannot possibly belong to them, and employ the meanest arts for ensnaring the public.

Before concluding this article, we shall state the performance of some of these engines, as determined by experiment. An engine whose cylinder is 31 inches in diameter, and which makes 17 double strokes per minute, is equivalent to 40 horses, working day and night, and burns 11,000 pounds of Staffordshire coal per day. When the cylinder is 19 inches, and the engine makes 25 strokes of 4 feet each per minute, its power is equal to that of 12 horses working constantly, and burns 3,700 pounds of coal per day. And a cylinder of 24 inches,

which makes 22 strokes of 5 feet each, performs the work of 20 horses, working constantly, and burns 5,500 pounds of coal. Mr. Boulton has estimated their performance in a different manner. He states, that one bushel of Newcastle coal, containing 84 pounds, will raise 30 million pounds 1 foot high; that it will grind and dress 11 bushels of wheat; that it will slit and draw into nails 5 cwt. of iron: that it will drive 1,000 cotton spindles, with all the preparation-machinery, with the proper velocity; and that these effects are equivalent to the work of 10 horses.

HYDRAULICS.

Description of a Water-blowing Machine.

THIS machine is so useful for conveying wind to the furnaces of iron forges, and the principle by which it operates is so curious, as to entitle it to the particular attention of the practical mechanic, as well as of the speculative philosopher. Although it has been known and generally adopted on the continent for above a century, yet it has neither been generally introduced into the forges of this country, nor has it found its way into many of our treatises upon machinery.

PLATE XLV. Fig. 2.

Let *AB*, Fig. 2. be a cistern of water, with the bottom of which is connected the bended leaden pipe *BCH*. The lower extremity *H* of the pipe is inserted into the top of a cask or vessel *DE*, called the condensing vessel, having the pedestal *P* fixed to its bottom, which is perforated with two openings *M*, *N*. When the water, which comes from the cistern *A*, is falling through the part *CH* of the pipe, it is supplied by the openings or tubes, *m*, *n*, *o*, *p*, with a quantity of air which it carries along with it. This mixture of air and water issuing from the aperture *H*, and impinging upon the surface of the stone pedestal *P*, is driven back and dis-

persed in various directions. The air being thus separated from the water, ascends into the upper part of the vessel, and rushes through the opening *F*, whence it is conveyed by the pipe *FG* to the fire at *G*, while the water falls to the lower part of the vessel, and runs out by the openings *M*, *N*.

In order that the greatest quantity of air may be driven into the vessel *DE*, the water should begin to fall at *C* with the least possible velocity; and the distance of the lowest tubes *o*, *p*, from the extremity of the pipe *H* should be to the length of the vertical tube *CH* as 3 to 8, in order that the air may move in the pipe *FG* with sufficient velocity. The part of the tube between *op* and *H*, and the vessel *DE*, must be completely closed, to prevent the escape of the internal air.

Fabri and Dietrich imagined that the wind is occasioned by the decomposition of the water, or its transformation into gas, in consequence of the agitation and percussion of its parts. But M. Venturi,* to whom we are indebted for the first philosophical account of this machine, has shown that this opinion is erroneous, and that the wind is supplied from the atmosphere; for when the lateral openings *m*, *n*, *o*, *p*, were shut up, no wind was generated.

Hence the principal object in the construction of these machines is to combine as much air as possible with the descending current. With this view the water is often made to pass through a kind of cullender placed in the open air, and perforated with a great number of

* Experimental Enquiry concerning the lateral communication of motion in Fluids. Prop. 8.

small triangular holes. Through these apertures the water descends in many small streams, and by exposing a greater surface to the atmosphere, it carries along with it an immense quantity of air, and is conveyed to the pedestal *P* by a tube *CH*, open and enlarged at *C*, so as to be considerably wider than the end of the pipe which holds the cullender.

It has been generally supposed that the water-fall should be very high,* but Dr. Lewis has shown, by a variety of experiments, that a fall of 4 or 5 feet is sufficient, and that when the height is greater than this, two or more blowing machines may be erected, by conducting the water from which the air is extricated into another reservoir, from which it again descends and generates air as formerly. That the air, which is necessarily loaded with moisture, may arrive at the furnace in as dry a state as possible, the condensing vessel *DE* should be made as high as circumstances will permit; and in order to determine the strength of the blast, it should be furnished with a gage *ab* filled with water.

Franciscus Tertius de Lanis observes,† that he has seen a greater wind generated by a blowing machine of this kind, than could be produced by bellows 10 or 12 feet long.

* Wolfius makes the length of the tube *CH*, 5 or 6 feet. Opera Mathematica, tom. i, p. 830.

† In Magisterio Naturæ et Artis, lib. v. cap. 3.

OPTICS.

ON ACHROMATIC TELESCOPES.

On Achromatic Object-glasses.

NOTWITHSTANDING the claims of foreigners to the invention of the achromatic telescope, we have the most unquestionable evidence that this instrument was first invented and constructed about the year 1758, by our countryman, Mr. John Dolland. As telescopes of this description are not affected with the prismatic colours, the late Dr. Bevis proposed to distinguish them by the name of *achromatic*, which they have ever since retained, though some have erroneously stated that this appellation was first given them by the astronomer Lalande.

During the 17th century, when every branch of science was cultivated with unwearied assiduity, the attention of philosophers was particularly directed to the improvement of the *refracting* telescope. But as the different refrangibility of the rays of light was then unknown, men of science employed themselves chiefly in trying to remove the spherical aber-

ration, or the error which arises from the spherical figure of the object-glass. For this purpose they ground their object-lenses of a parabolic or hyperbolic figure, or of a spherical form with the radius of the surface next the object six times greater than that of the surface next the eye, in which case Huygens had shown that the aberration is less than when the radii of curvature have any other proportion. After all these trials, however, the refracting telescope still retained its former imperfections, and the opticians of those days, completely despairing of bringing it to perfection, turned the whole of their attention to the construction of the *reflecting* telescope.

It was reserved for Sir Isaac Newton to discover the cause of these imperfections, and for Dolland to point out their cure. From Newton's Theory of Colours, it plainly appeared that the imperfections of the dioptric telescope arose from the different refrangibility of the rays of light, and that, when compared with this, the spherical aberration was extremely trifling. But though Newton, by thus pointing out the cause of the indistinctness of refracting telescopes, contributed indirectly to their improvement, he may certainly be said to have checked the progress of this branch of science, when he stated,* 'that all refracting substan- 'ces diverged the prismatic colours in a con- 'stant proportion to their mean refractions.— 'that refraction could not be produced without 'colour, and, consequently, that no improve- 'ment could be expected in the refracting tel- 'escope.' In this conclusion philosophers had

* Newton's Optics, p. 112.

acquiesced for above half a century, till Mr. Dolland, having examined the premises from which it was drawn, obtained a result very different from that of Sir Isaac Newton. He found that substances which had the same refractive power had different dispersive powers, or, in his own words,* 'that there is a difference in the dispersion of the colours of light, 'when the mean rays are equally refracted by 'different mediums;' and thence concluded that the object-glasses of refracting telescopes were capable of being made, without being affected by the different refrangibility of the rays of light. PLATE XLV.

That our readers may understand this illustrious discovery, and the method of its application to the construction of achromatic object-glasses, let *ABC* be a prism, *O* a beam of white light proceeding in the direction *ON*, but refracted from its rectilineal course by the interposition of the prism, and forming the prismatic spectrum *RMV*. The line *nM* being the direction of the mean refracted light, the angle *NnM* is called the *angle of deviation*, and *RnV* the *angle of dissipation*, or *dispersion*. In the same medium, the angle of dispersion is always proportional to the angle of deviation, or to the mean angle of refraction, and Newton imagined that this was universally the case in different media, *i. e.* that the angle *NnM* is always proportional to the angle *RnV* whether the prism be of crown, or flint, or any other kind of glass. Dolland, however, found that these angles were not proportional to each other in Fig. 5.

* See Transactions of the Royal Society of London, vol. l, p. 743.

PLATE XLV. different media, but that in some the angle of deviation is larger when the angle of dispersion is smaller, and that in others the angle of deviation is smaller when the angle of dispersion is larger. Thus, if the prism *ABC* be made of crown-glass, the angle of deviation will be *NnM*, and that of dispersion *RnV*, but if a flint-glass prism with a less refracting angle be substituted in its room, the angle of deviation may be *NnM*, while that of dispersion becomes *rnv*.

The application of these principles to the improvement of the refracting telescope will be easily comprehended, if we consider that light is refracted and dispersed by lenses in the same manner as by prisms. Fig. 6. Thus, in Fig. 6, let *AB* be a convex lens, *O* a beam of light incident at *m*, emerging at *m*, and proceeding in the direction *mN*; then if we suppose *abc* to be a prism whose sides *ab*, *ac*, are tangents to the surfaces of the lens in the points *n*, *m*, the beam of light *O*, incident at *n*, will emerge at *m*, and proceed in the same direction *mN* as formerly; and if the lens be concave, as *CD*, it will refract and disperse the rays in the same way as a prism *acd*, placed in a contrary direction with its base *ad* uppermost. Now, if we apply to the prism *abc* another prism *acd*, having a similar refracting angle, and the same refractive or dispersive power, or, if to the convex lens *AB* we apply the concave one *CD*, having the same curvature and the same refractive and dispersive power, then the ray of white light *O*, incident at *n*, will emerge colourless at *p*, and proceed in the direction *pN*, parallel to *On*, because the change which is pro-

duced on the incident ray by the first prism or convex lens, is counteracted by an equal and opposite change produced by the second prism or concave lens. But if the second prism or lens has a different refractive and dispersive power from the first, and if the refractive angle of both be the same, the ray *pN* will be coloured after refraction, because the second prism more than counteracts the effects of the first, and it will be bent to or from the axis of the lenses according as the refracting power of the second prism or lens is greater or less than that of the first. PLATE XLV.

From these observations, the attentive reader will easily understand the construction of the double achromatic object-glass, in which *AB* (Fig. 6.) is the convex lens of *crown-glass*, and *CD* the concave one of *flint-glass*. As the refractive and dispersive powers of the lens *CD* is greater than those of the lens *AB*, the curvatures of the lenses or the refracting angles of the corresponding prisms being equal, the ray *pN* will be bent from the axis of the lenses, and it will be coloured by the excess of the dispersive power of the flint above that of the crown-glass. In this case, therefore, the combined lenses will not have a positive focus.—But, since, in the same medium, the angle of dispersion increases or diminishes with the angle of deviation, we can diminish the refraction and dispersion of the concave lens by diminishing its concavity, or the refracting angle of the corresponding prism. Now, let the concavity of the lens *CD* be diminished till its dispersion be equal to the dispersion of the lens *AB*, its refraction or power of bending the incident rays will also be diminished; then, since the

dispersion of the concave lens is equal to the dispersion of the convex one, and its curvature less, the ray *pV* will emerge perfectly colourless, and will be bent *towards* the axis of the lenses, as the convergency of the incident ray occasioned by the convex lens, is not wholly counteracted by the divergency produced by the concave one. In the same manner, every other ray falling upon the surface of *AB* will be refracted colourless into a positive focus, and an image will be formed perfectly achromatic.

From what has been said concerning the double achromatic object-glass, it will be easy to comprehend how a colourless image is formed by a combination of three lenses, which is now universally adopted for the purpose of diminishing the spherical aberration. In Fig. 7. let *AB*, *CD*, *EF*, be the three lenses which compose the triple object-glass,* *AB*, and *EF*, being convex, and of crown-glass, and *CD* concave, and made of flint-glass; and let *ab*, *cd*, *ef*, be corresponding prisms, which, if substituted instead of the lenses, would refract and disperse, in a similar manner, any ray of light which falls upon the points *q*, *m*, *r*, *t*, *g*, *p*, where the sides of the prisms are supposed to touch the surfaces of the lenses. Suppose, also, which is generally the case, that the two convex lenses have equal focal distances, and that the focal distances of either lens is greater, or their curvature less, than that of the concave one, whose dispersion exceeds that of the lens *AB*; a ray then, *O*, of white light, incident at *q*, will,

PLATE XL. Fig. 7.

* The lenses are placed at a distance from each other in the figure, that the progress of the incident ray may be more easily perceived.

after refraction by the lens *AB*, be separated into its component parts, and proceed in the direction *mrR*, *nsV*; *mrR* being the extreme red ray, and *nsV* the extreme violet. But as these rays are intercepted by the lens *CD*, at the points *r*, *s*, they will undergo another refraction in a contrary direction, and will proceed according to the dotted lines *tr*, *ov*. These rays will diverge after refraction, and be bent *from* the axis of the lenses, since the refraction as well as the refracting angle of the prism *cd*, or lens *CD*, exceeds the refraction, and refracting angle of the prism *ab*, or lens *AB*; for though the violet ray *qns* is bent from the red ray *qmr*, by the refraction of the lens *AB*, it is again bent towards it by the superior refraction of the concave lens, and they will therefore converge to one another in the direction *tr*, *ov*.

In this case, the excess of dispersive power in the concave lens tends only to delay the mutual convergency of the red and violet rays, or to remove the point where they would meet farther from the lens *CD*. Now, it is evident, that two rays of different refrangibility falling upon a prism or lens with different angles of incidence, may emerge with the same angle of refraction, or may be united at their emersion from the prism or lens; for, in this case, their difference of refrangibility counteracts the difference between their angles of incidence. The red and violet rays *or*, *tv*, therefore, which fall upon the lens *EF*, with different angles of incidence, will, after refraction by the third lens, proceed perfectly colourless in the direction *pN*. In the same manner, all the rays which proceed from any object, emerging colourless

from the triple object-glass, will unite in one point, and form an image completely achromatic.

Having thus discovered that light could be refracted without colour, the next object of philosophers was to ascertain the curvature which must be given to lenses, in order to produce this effect, and likewise to correct the spherical aberration. This subject has been treated with the greatest ability by several foreign mathematicians, but particularly by Euler,* D'Alembert,† Clairault,‡ Boscovich,§ and Klugel.‖ The writings of these philosophers furnish us with the most complete and accurate information upon this point; and art has in this case received from science all the assistance which she can possibly bestow. It shall be our object at present to reduce the results of their investigations either into tables, or into such a form as may be easily comprehended by the practical optician, and thus to furnish the artist with a popular view of this interesting subject. For this purpose, the celebrated Euler has given, in his dioptrics,¶ two formulæ, from which we have calculated the two following tables, containing

* Commentarii Novi Academ. Petropolitanæ, Tom. 18, p. 407.

† Memoires de L'Acad. Royale, Par. 1764, 8vo, p. 139; 1765, 8vo, p. 81, and 1767, 4to, p. 43.

‡ Memoires de l'Acad. Royale, Par. 1756, 8vo, p. 612; 1757, 8vo, p. 853, and 1762.

§ R. J. Boscovichii Opera pertinentia ad Opticam et Astronomiam, Bassani. 1785, Tom. 1, Opusc. 2, p. 169.

‖ Commentationes Reg. Soc. Gottingensis, 1795 to 1798, Tom. 13, p. 28.

¶ The mean refraction of the crown-glass is supposed to be 1.53, and that of the flint-glass 1.58, and their dispersive powers as 2 to 3.

the radii of curvature for the lenses of a triple object-glass.—The first column contains the focal distance of the lenses when combined; and the six following columns contain the radii of their curvature in inches and decimals, beginning with the surface next the object.

TABLE I.

Table of the radii of curvature of the lenses of a Triple Achromatic Object-Glass, *according to Euler's first formula.*

Focal length.	Convex lens of crown-glass.		Concave lens of flint-glass.		Convex lens of crown-glass.		Semi-aperture
Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.
6	3.00	22.00	3.10	2.91	3.13	2.85	0.71
9	4.50	33.00	4.65	4.36	4.70	4.28	1.07
12	6.00	44.00	6.20	5.82	6.26	5.70	1.43
18	9.00	66.00	9.30	8.72	9.40	8.56	2.14
24	12.00	88.00	12.40	11.64	12.52	11.40	2.86
30	15.01	109.99	15.50	14.53	15.65	14.27	3.56
36	18.01	131.99	18.60	17.44	18.80	17.12	4.28
42	21.01	153.99	21.70	20.37	21.91	19.97	4.99
48	24.02	175.99	24.80	23.28	25.04	22.80	5.72
54	27.02	197.99	27.90	26.16	28.18	25.69	6.42
60	30.02	219.99	31.00	29.06	31.31	28.54	7.13

TABLE II.

Table of the radii of curvature of the lenses of a TRIPLE ACHROMATIC OBJECT-GLASS, *according to Euler's second formula.*

Focal length.	Convex lens of crown-glass.		Concave lens of flint-glass.		Convex lens of crown-glass.		Semi-aperture
Inches.	Inches.	Inches.	Inches.	Inches.	Inches	Inches.	Inches.
6	1.70	12.44	12.88	1.77	3.56	15.00	0.42
9	2.55	18.66	19.31	2.66	5.34	22.51	0.64
12	3.40	24.87	25.75	3.55	7.13	30.01	0.85
18	5.09	37.31	38.63	5.32	10.69	45.01	1.27
24	6.80	49.74	51.50	7.10	14.25	60.02	1.70
30	8.49	62.19	64.38	8.86	17.81	75.02	2.12
36	10.19	74.63	77.26	10.63	21.37	90.02	2.54
42	11.89	87.07	90.14	12.40	24.93	105.03	2.96
48	13.60	99.48	103.02	14.17	28.49	120.03	3.39
54	15.27	111.93	115.87	15.96	32.07	135.04	3.82
60	16.97	124.37	128.75	17.73	35.63	150.04	4.24

The only person in our country who has written upon the theory of achromatic object-glasses, is the late learned Dr. Robison of Edinburgh, who, following the steps of Clairault and Boscovich, has given an able dissertation upon this subject.* From the formulæ contained in that dissertation, the following table is computed.

* Article *Telescope*, Encyclopœdia Britannica, vol. 18, p. 338.

TABLE III.*

Table of the apertures and radii of curvature of the lenses of a Triple Object-Glass.

Focal length.	Convex lens of crown-glass.		Convex lens of flint-glass.		Convex lens of crown-glass.	
Inches.	Inches.	Inches	Inches.	Inches	Inches.	Inches.
6	4.54	3.03	3.03	6.36	6.36	0.64
9	6.83	4.56	4.56	9.54	9.54	0.92
12	9.25	6.17	6.17	12.75	12.75	1.28
18	13.67	9.12	9.12	19.08	19.08	1.92
24	18.33	12.25	12.25	25.50	25.50	2.56
30	22.71	15.16	15.16	31.79	31.79	3.20
36	27.33	18.25	18.25	38.17	38.17	3.84
42	31.87	21.28	21.28	44.53	44.53	4.48
48	36.42	24.33	24.33	50.92	50.92	5.12
54	40.96	27.36	27.36	57.28	57.28	5.76
60	45.42	30.33	30.33	63.58	63.58	6. 4

The reader will observe, that only three pair of grinding-tools are necessary for constructing a telescope according to the preceding table; but the work may be performed by only two grinding-tools, if the radii of curvature be employed, which are contained in the following table, computed from the formulæ of Boscovich.

* A telescope 30 inches in focal length constructed according to this table, bore an aperture of 3 1-5 inches.

TABLE IV.

Focal Length.	Radii of the four surfaces of the two lenses of crown-glass.	Radius of the two surfaces of the concave lens of flint-glass.
Inches.	Inches.	Inches.
6	3.84	3.17
9	5.76	4.75
12	7.68	6.34
18	11.52	9.50
24	15.36	12.68
30	19.20	15.84
36	23.04	19.00
42	26.88	23.17
48	30.72	25.36
54	34.66	28.51
60	38.40	31.68

TABLE V.

The radii of curvature employed by the London opticians are pretty nearly represented in the following table, which is calculated from Dr. Robison's measurements.

Focal Length.	Convex lens of crown-glass.		Radius of both the surfaces of the concave lens of flint-glass.	Convex lens of crown-glass.	
Inches.	Inches.	Inches.	Inches.	Inches.	Inches.
6	3.77	4.49	3.47	3.77	4.49
9	5.65	6.74	5.21	5.65	6.74
12	7.54	8.99	6.95	7.54	8.99
18	11.30	13.48	10.42	11.30	13.48
24	15.08	17.98	13.90	15.08	17.98
30	18.34	22.47	17.37	18.34	22.47
36	22.61	26.96	20.84	22.61	26.96
42	26.38	31.45	24.31	26.38	31.45
48	30.16	35.96	37.80	30.16	35.96
54	33.91	40.45	31.27	33.91	40.45
60	37.68	44.94	34.74	37.68	44.94

Two of Dolland's best achromatic telescopes being examined, were found to have their lenses of the following curvatures, reckoning from the surface next the object. Crown-glass lens 28 inches, and 40. Concave lens 20.9 inches, and 28. Crown-glass lens 28.4, and 28.4. The focal length of this instrument was 46 inches. In the other telescope, whose focal length was 46.3 inches, the curvature of the 1st lens was 28 and 35.5 inches; the second lens 21.1 and 25.75, and the 3d, 28 and 28. Both these telescopes magnified from 100 to 200 times, according to the powers applied.

The Duc de Chaulnes having in his possession one of Dolland's best telescopes, whose focal length was 3 feet 5 inches 4.25 lines, made a variety of accurate experiments in order to determine the curvature, thickness, and distance, of its lenses, and found them to be of the following dimensions.* Radius of the 1st surface, or the surface next the object, 25 inches 11.5 lines. Radius of the 2d surface 32 inches 8 lines. Radius of the 3d surface 17 inches 10 lines. Radius of the 4th surface $24\frac{1}{2}$ inches. Radius of the 5th $24\frac{1}{2}$ inches; and the radius of the 6th 26 inches and 10.6 lines. Thickness of the 1st lens at its axis 2.11 lines; thickness of the 2d 1.59 lines; thickness of the 3d 2.18; and the thickness of the whole lens 5.91 lines.†

* These experiments are detailed at great length in the Memoires de l'Acadamie, Royal. Par. 1767, 4to, p. 423.

† For the dimensions of the eye-piece of this telescope, see the article *on Achromatic eye-pieces*, p. 425.

Tables for double achromatic object-glasses.

The following table, calculated from the formulæ of Boscovich, contains the radii of curvature for the lenses of a DOUBLE ACHROMATIC OBJECT-GLASS.

TABLE VI.

Focal Length	Convex lens of crown-glass.		Concave lens of flint-glass.	
Inches.	Inches.	Inches	Inches.	Inches.
6	1.94	1.91	1.91	9.49
9	2.91	2.86	2.86	14.24
12	3.88	3.82	3.82	18.99
18	5.82	5.73	5.73	28.48
24	7.76	7.63	7.63	36.99
30	9.70	9.54	9.54	47.47
36	11.64	11.45	11.45	56.97
42	13.58	13.36	13.36	66.46
48	15.51	15.27	15.27	73.98
54	17.45	17.17	17.17	85.47
60	19.39	19.08	19.08	94.95
70	22.62	22.26	22.26	110.77
80	25.86	25.44	25.44	126.60
90	29.09	28.62	28.62	142.42
100	32.32	31.80	31.80	158.25

In the following table, calculated from Dr. Robison's measurements, the reader will find the radii of curvature which are employed by the London artists in the construction of the double achromatic object-glass.

TABLE VII.*

Focal Length	Convex lens of crown-glass.		Concave lens of flint-glass.	
Inches.	Inches.	Inches.	Inches.	Inches.
6	1.76	2.12	2.07	6.88
9	2.64	3.17	3.10	10.33
12	3.53	4.23	4.13	13.77
18	5.29	6.35	6.20	20.65
24	7.05	8.46	8.26	27.54
30	8.81	10.58	10.33	34.42
36	10.58	12.69	12.39	41.30
42	12.34	14.81	14.46	48.18
48	14.11	16.92	16.52	55.07
54	15.87	19.04	18.59	61.96
60	17.63	21.16	20.66	68.84
70	20.57	24.68	24.10	80.32
80	23.50	28.21	27.54	91.79
90	26.44	31.73	30.99	103.27
100	29.38	35.26	34.43	114.74

Although it has been the practice in this country to construct only double and triple achromatic object-glasses, yet they may be composed even of four or five lenses, the convex ones of crown-glass, and the concave ones of flint-glass

* In this and the six preceding tables, the sine of incidence to the sine of refraction is supposed to be as 1.526 to 1 in the crown-glass, and as 1.604 to 1 in the flint-glass; and the ratio of the differences of the sines of the extreme rays in the crown and flint-glass 0.6054.

being placed in an alternate order. By augmenting the number of media, indeed, a quantity of light must be lost, and the labour of the artist greatly increased; but M. Jeaurat informs us, that he constructed a compound object-glass 5 inches and 10 lines in focal length, which bore an aperture of $1\frac{1}{2}$ inch, while the best achromatic telescopes of 6 inches focus, constructed in England, had an aperture of only an inch and a quarter. To such, therefore, as wish to construct object-glasses of this description, the following table, containing their radii of curvature, may probably be acceptable.

TABLE VIII.*

Focal length of the compound object-glass.		Radius of the six interior surfaces.		Radius of the two exterior surfaces.		Aperture of the object-glass.
Feet	Inches.	Feet.	Inch. Dec.	Feet.	Inch. Dec	Inch. Dec.
0	4	0	3.08	0	3.58	1.25
0	6	0	4.50	0	5.33	1.50
0	8	0	5.92	0	7.08	1.83
0	10	0	7.33	0	8.83	2.17
1	0	0	8.83	0	10.58	2.25
2	0	1	5.33	1	9.17	2.58
3	0	2	1.83	2	7.67	2.92
4	0	2	4.42	3	6.25	3.25
5	0	3	6.92	4	4.75	3.58
6	0	4	3.50	5	3.33	3.92
7	0	5	0.00	6	1.83	4.17
8	0	5	8.58	7	0.42	4.50
9	0	6	5.08	7	11.00	4.83
10	0	7	1.58	8	9.50	5.17

* This table supposes that the mean refraction of the crown-glass is to that of the flint-glass as 1000 to 1045, and their dispersive powers as 200 to 353.

Though it is demonstrable that a telescope constructed according to the preceding tables, and formed of glass, whose refractive and dispersive power is similar to that which was employed in the formulæ upon which these tables are founded, will form an image perfectly distinct and colourless; yet it is so difficult to procure flint-glass of the same refractive and dispersive power, that it is almost impossible for a private individual to succeed, even after several trials. The London opticians have always at hand a number of lenses of various curvatures, and different powers of refraction and dispersion, and by selecting such as answer best upon trial, they are enabled without much trouble to construct an object-glass in which the spherical and chromatic aberrations are almost wholly corrected. Those, therefore, who are not furnished with a sufficient number of lenses must necessarily meet with frequent disappointments in their attempts to construct achromatic telescopes; and the only way of preventing these disappointments, and rendering success more certain, is to have a variety of tables, which, being founded on different conditions, give different curvatures to the lenses. If the artist should be unsuccessful, either from the nature of the refracting media which he employs, or from giving the lenses a greater or less curvature than the table requires, he may with very little trouble, sometimes with altering the radius of a single surface, adapt the lenses to the conditions of some other table, and in all probability obtain a more favourable result. With the view of facilitating these attempts, we have computed the eight preceding tables, and for the same purpose we shall sub-

join the following different forms of achromatic object-glasses from Boscovich and Klugel.

In these forms a represents the first surface of the compound lens, or that which is next the object, b the second surface, a' the third, b' the fourth, a'' the fifth, and b'' the sixth; a, b, a'', b'', representing the radii of curvature for the convex lenses of crown-glass, and a', b', the curvature of the concave lens of flint-glass. The focal distance of the first lens, or that whose surfaces are marked a, b, is represented by x, that of the second by y, and that of the third by z, while the focal length of the compound lens is distinguished by the letter F.

Forms for triple object-glasses.

I.

$a=b=a''=b''=0.6412$	$x=0.6096$
$a'=0.5227$	$y=0.4384$
$b'=0.5367$	$z=0.6096$
$F=1$	

In this form the two lenses of crown-glass are isosceles,* and have the same curvature and focal distance. The middle lens of flint-glass is nearly isosceles, and may be made so in practice, so that only two grinding-tools are necessary for this form.

* A lens is called *isosceles* when both its surfaces have the same curvature.

II.

THE TWO FIRST LENSES ISOSCELES.

$a=b=a'=b'=0.530$ $\quad x=0.5038$

$a''=1.215$ $\quad y=0.4388$

$b''=0.3046$ $\quad z=0.7727$

$F=1$

III.

THE FIRST AND THIRD LENSES ISOSCELES.

$a=b=a''=b''=0.616$

$a'=0.6356$ $\quad F=1$

$b'=0.3790$*

IV.

THE TWO FIRST LENSES ISOSCELES.

$a=b=a'=b'=0.4748$

$a''=0.3514$ $\quad F=1$

$b''=0.4383$

V.

THE SECOND AND THIRD LENSES ISOSCELES.

$a=0.5721$

$b=1.8744$

$a'=b'=a''=b''=0.4748$

* In these two forms the refractive and dispersive power of the glass is supposed the same as in the note on p. 417.

pressed in feet or inches, and the result will be the proper radii of curvature in feet or inches. Thus let it be required to construct a double achromatic object-glass according to the first of these forms, whose focal length shall be 20 inches, we will have

$a=b=20\times0.3206=$ 6 inches and $\frac{4}{10}$ nearly;
$a'=20\times0.3201=$ 6 inches and $\frac{4}{10}$ nearly;
$b'=20\times1.553\ =30$ inches and $\frac{7}{10}$ nearly;
$x=20\times0.3408=$ 6 inches and $\frac{8}{10}$ nearly;
$y=20\times0.4384=$ 8 inches and $\frac{8}{10}$ nearly;
and $F=20\times1\ =20$ inches.

Achromatic object-glasses may be much improved by interposing pure turpentine varnish between the concave and convex lenses. By this means the reflection from the internal surfaces is removed, and that loss of light prevented which arises from an imperfect polish of the surfaces. M. Putois, an optical instrument maker in Paris, is said to have discovered that the best medium for this purpose is mastich,* a transparent resinous substance, which is exuded from the lentiscus-tree in the island of Chio, and brought to this country in grains or tears.

Achromatic telescopes have also been constructed, by substituting transparent fluids instead of the concave lens of flint-glass. For this discovery we are indebted to the ingenious Dr. Robert Blair, who has given an account of his experiments in the 3d volume of the Transactions of the Royal Society of Edinburgh, to which we must refer the reader, after giving a description of one of these fluid object-glasses.

* Traite Elementaire de Physique par Brisson, tom. ii. § 1657, p. 428.

If pure spirit of turpentine be interposed between two convex lenses of crown-glass, having the radii of their surfaces as 6 to 1 with the most convex sides turned inwards, the image formed by this combination will be perfectly achromatic. The spirit of turpentine has the form of a double concave lens, and as its refractive and dispersive powers differ from those of crown-glass, it will act in every respect like a lens of flint-glass. The writer of this article constructed an object-glass of this kind, having 36 inches of focal length, but found it troublesome to keep it in order.

On achromatic eye-pieces.

Although a brief account of the achromatic telescope has been given by those who have written upon optics, since the invention of that instrument, yet these authors have unaccountably overlooked the construction of achromatic eye-pieces. Dr. Robison, indeed, has treated this subject at considerable length, after Boscovich, but has furnished almost no information to the practical optician. On this account we shall dwell a little longer upon this point, than what might otherwise be thought necessary in a work like ours. In order to correct the error arising from the unequal refrangibility of light in the eye-pieces of telescopes, we are not under the necessity of using compound lenses of crown and flint-glass, as this species of aberration can be completely removed by a particular arrangement of the eye-glasses which are employed for erecting the object. In small pocket-telescopes, however, as opera-glasses, &c. where it would be very inconvenient to

apply a long eye-piece, these compound lenses should be adopted, and may consist either of two or three glasses, with the following curvatures.

Forms for a double eye-glass.

I.

BOTH LENSES ISOSCELES.*

$a=b=0.320$ $\quad$ $x=0.304$

$a'=b'=0.529$ $\quad$ $y=0.438$

II.

FIRST LENS ISOSCELES.

$a=a'=b=0.320$ $\quad$ $x=0.304$

$b'=1.517$ $\quad$ $y=0.438$

Forms for a triple eye-glass.

I.

ALL THE THREE LENSES ISOSCELES.

$a=b=a''=b''=0.640$ $\quad$ $z=x=0.608$

$a'=b'=0.529$ $\quad$ $y=0.438$

II.

FIRST LENS ISOSCELES.

$a=b''=0.810$ $\quad$ $z=x=0.608$

$b=a'=b'=a''=0.529$ $\quad$ $y=0.438$

* The letters, a, b, x, y, *&c.* represent the same quantities as in page 420.

If it is required to erect the object as in the Galilean telescope, the middle lens of flint-glass must be made convex, and the other lenses concave, but with the same radii of curvature, so that the concavity of the compound lens may predominate.

On eye-pieces with three lenses, which remove the chromatic aberration.

The three lenses must be made of the same kind of glass, and may be of any focal length. The distance between the first and second, or the two next the object, must be equal to the sum of their focal distances, and the distance between the second and third must exceed the sum of their focal distances, by a quantity which is a third proportional to the distance between the first and second, and the focal length of the second lens; or, in other words, the distance between the second and third lenses must be equal to the sum of their focal distances, added to the quotient arising from the square of the focal distance of the second lens, divided by the sum of the focal distances of the first and second. These, and other circumstances which should be attended to in the construction of achromatic eye-pieces, will be better understood by expressing them algebraically.

Thus, let F be the focal length of the object-glass, and x, y, z, the focal distances of these eye-glasses, reckoning from that which is nearest the object. Then we will have

1 The distance between the first and second lenses - - $x+y$

2 The distance between the second and third - - $y+z+\frac{y^2}{x+y}$

3 Distance of the first lens from the focus of the object-glass - $\frac{xy}{x+y}$

4 Magnifying power of the eye-piece $\frac{Fy}{xz}$

5 Focal distance of a single lens, with the same magnifying power $\frac{xz}{y}$

6 Distance of the eye from the third lens z

7 Length of the whole eye-piece $x+3y+2z$

8 Length of the whole telescope $F+x+3y+2z$

9 Aperture of the lenses* $a, a', a'' \ldots a'=a'', a=\frac{xz}{y}$

10 The aperture of the diaphragm, or field bar, or m, should be a little less than - - a
and should be placed in the focus of the object-glass.

11 The field of view is nearly - $\frac{3438m}{F}$

Although the aberration of colour will be completely removed by making the lenses of any focal length, and placing them at the distances indicated by the preceding formulæ, yet it is preferable to make the first and second lenses of the same focal length, and to give the third a less focal distance, and make its distance from the second equal to its own focal length, added to $1\frac{1}{2}$ the focal distance of one of the other

* The apertures of the lenses may be made equal to one another, but should never be greater than half the focal distance of the third lens.

lenses; for, in this case, where x and y are equal, the expression $\frac{y^2}{x+y}$, which, when added to $y+z$, expresses the distance between the second and third lenses, becomes $\frac{1}{2}y$.* Beside the simplicity of this combination, it has another advantage, for, the magnifying power of the eye-piece is always equal to the magnifying power of the third lens. This is evident from the fifth formula $\frac{xz}{y}$, which becomes $=z$ when x and y have the same value. So that in this construction, when we wish to give a certain magnifying power to a telescope, we have only to take such a focal length for the third lens as will produce this magnifying power, and make the focal length of the other two a little greater than that of the third. By increasing the focal lengths of the two first lenses, the image is not injured by any particles of dust which may be lying on their surface, and the spherical aberration is also diminished.† By augmenting the curvature of the third lens, however, we contract the field of view, which ought, if possible, to be avoided. This may be avoided, indeed, as Boscovich has shown, by making the third lens consist of two convex ones of the same glass, their surfaces being in contact, and their focal lengths equal. From long experience, he found that eye-pieces of this construction are superior to all others, and that the error arising from the

* Since $x=y$ in this case, $\frac{y^2}{x+y}$ is $=\frac{y^2}{2y}=\frac{y}{2}$ or $\frac{1}{2}y$ for $\frac{1}{2}y\times 2y=y^2$.

† In all the eye-pieces the spherical aberration will be diminished by making the lenses plano-convex, the plane surfaces being turned to the eye.

spherical figure of the glass is greatly diminished, by making all the lenses plano-convex, and turning the plane sides to the eye, excepting the second lens, whose plane surface should be turned to the object. All the lenses may be made of the same focal length, and then the distance between the first and second, and the second and third, will be equal to the sum of their focal distances. In this case, the third and fourth lenses, which are joined together, are considered as a single lens, whose focal length is equal to one half the focal length of either of the two. The apertures, too, may be all equal, and the field-bar must be a little less than any of the apertures.

In all kinds of achromatic eye-pieces which are composed of single lenses, flint-glass should be employed, because it has the greatest refractive power, and therefore requires a less curvature to have the same focal distance. The spherical aberration, consequently, which always increases with the curvature of the lenses, will be less in a flint-glass eye-piece, than in one of crown-glass. Flint-glass, indeed, produces a greater separation of colours, but the error arising from this cause is completely removed by the proper arrangement of the lenses.

On eye-pieces with four lenses, which remove the chromatic aberration.

A good achromatic eye-piece may be made of four lenses, if their focal lengths, reckoning from that next the object, be as the numbers 14, 21, 27, 32, their distances 23, 44, 40, and their apertures 5.6; 3.4; 13.5; 2.6.

In one of Ramsden's small telescopes, whose object-glass was $8\frac{1}{2}$ inches in focal length, and

the magnifying power 15.4, the focal lengths of the eye-glasses were 0.77 of an inch; 1.025; 1.01; 0.79, and their respective distances, reckoning from that next the object, were 1.18; 1.83; 1.10.

In the excellent telescope of Dolland's construction, which belonged to the Duc de Chaulnes,* the focal length of the eye-glasses, beginning with that next the object, were $14\frac{1}{4}$ lines, 19, $22\frac{3}{4}$, 14, their distances 22.48 lines; 46.17; 21.45; and their thickness at the centre 1.23 lines; 1.25; 1.47. The fourth lens was plano-convex, with the plane side to the eye, and the rest were double convex lenses.

On achromatic eye-pieces for astronomical telescopes.

In eye-pieces of this kind which invert the object, the focal length of the first lens should be triple that of the second, and their distance double the focal length of the second, or $\frac{2}{3}$ of the focal length of the first. The lenses should be plano-convex, the plane surfaces turned to the eye, in order that the aberration of sphericity may be diminished as much as possible.

The telescope of Dolland's, belonging to the Duc de Chaulnes, had two astronomical eye-pieces, one of which was furnished with a micrometer. In the eye-piece which carried the micrometer, the first lens was $12\frac{3}{4}$ lines in focal length, and 1.62 lines thick; the second was 5.45 lines in focal length, and 1.25 thick, the distance between their interior surfaces 4.20

* See page 415.

lines, and the distance of the first lens from the focus of the object-glass $13\frac{3}{4}$ lines. In the other eye-piece, the focal length of the first lens was 8.30 lines, and its thickness 1.60; the focal length of the second was 3.53, and its thickness 0.97 lines. In both these eye-pieces the lenses were plano-convex, with the plane surfaces turned to the eye.

OPTICS.

On the construction of Optical Instruments, with Tables of their Apertures and Magnifying Powers, and the method of grinding the Lenses and Mirrors of which they are composed.

On the method of grinding and polishing lenses.

HAVING fixed upon the proper aperture and focal distance of the lens, take a piece of sheet copper, and strike upon it a fine arch, with a radius equal to the focal distance of the lens, if to be equally convex on both sides, or with a radius equal to half that distance, if to be plano-convex, and let the length of this arch be a little greater than the given aperture.—Remove with a file that part of the copper which is without the circular arch, and a *convex gage* will be formed. Strike another arch with the same radius, and having removed that part of the copper which is within it, a *concave gage* will be obtained. Prepare two circular plates of brass, about $\frac{1}{10}$ of an inch thick, and ¹ an inch greater in diameter than the breadth of the lens, and solder them upon a cylinder of lead of the same diameter, and about an inch high. These tools are then to be fixed upon a turning lathe, and one of them turned into a portion of a concave sphere, so as to suit

the convex gage; and the other into a portion of a convex sphere, so as to answer the concave gage. When the surfaces of the brass plates are turned as accurately as possible, they must be ground upon one another alternately with flour emery till the two surfaces exactly coincide, and the grinding tools will then be ready for use.

Procure a piece of glass whose dispersive power is as small as possible, if the lens be not for achromatic instruments, and whose surfaces are parallel; and by means of a pair of large scissars or pincers, cut it into a circular shape, so that its diameter may be a little greater than the aperture of the lens. After the roughness is removed from its edges by a common grindstone,* it is then to be fixed with black pitch to a wooden handle of a smaller diameter than the glass, and about an inch high, so that the centre of the handle may exactly coincide with the centre of the glass.

When the glass is thus prepared for use, it is then to be ground with fine emery upon the concave tool, if to be convex, and upon the convex tool, if to be concave. To avoid circumlocution, we shall suppose that the lens is to be convex. The concave tool, therefore, which is to be used, must be firmly fixed to a table or bench, and the glass wrought upon it with circular strokes, so that its centre may never go beyond the edges of the tool.—

* When the focal distance of the lens is to be short, the surface of the piece of glass should be ground upon a common grindstone, so as to suit the gage as nearly as possible; and the plates of brass, before they are soldered on the lead, should be hammered as truly as they can into their proper form. By this means much labour will be saved, both in turning and grinding.

For every six circular strokes, the glass should receive 2 or 3 cross ones along the diameter of the tool, and in different directions. When the glass has received its proper shape, and touches the tool in every point of its surface, which may be easily known by inspection, the *emery is to be washed away, and finer kinds successively substituted in its room, till, by the same alternation of circular and transverse strokes, all the scratches and asperities are removed from its surface. After the finest emery has been used, the roughness which remains may be taken away, and a slight polish superinduced by grinding the glass with pounded pumice-stone, in the same manner as before. While the operation of grinding is going on, the convex tools should, at the end of every five minutes, be wrought upon the concave one for a few seconds, in order to preserve the same curvature to the tools and the glass. When one side is finished off with the pumice-stone, the lens must be separated from its handle by inserting the point of a knife between it and the pitch, and giving it a gentle stroke. The pitch which remains upon the glass may be removed by rubbing it with a little oil, or spirits of wine; and after the finished side of the glass is fixed upon the handle, the other surface is to be ground and finished in the very same manner.

* Emery of different degrees of fineness may be made in the following manner. Take five or six clean vessels, and having filled one of them with water, put into it a considerable quantity of flour emery. Stir it well with a piece of wood, and after standing for 5 seconds, pour the water into the second vessel. After it has stood about 12 seconds, pour it out of this into the 3d vessel, and so on with the rest; and at the bottom of each vessel will be found emery of different degrees of fineness, the coarsest being in the first vessel, and the finest in the last.

When the glass is thus brought into its proper form, the next and the most difficult part of the operation is to give it a fine polish. The best, though not the simplest, way, of doing this, is to cover the concave tool with a layer of pitch, hardened by the addition of a little rosin, to the thickness of $\frac{1}{15}$ of an inch. Then having taken a piece of thin writing paper, press it upon the surface of the pitch with the convex tool, and pull the paper quickly from the pitch before it has adhered to it; and if the surface of the pitch be marked every where with the lines of the paper, it will be truly spherical, having coincided exactly with the surface of the convex tool. If any paper remains on the surface of the pitch, it may be removed by soap and water; and if the marks of the paper should not appear on every part of it, the operation must be repeated till the polisher, or bed of pitch, is accurately spherical. The glass is then to be wrought on the polisher by circular and cross strokes, with the calx of tin, called the flowers of putty in the shops, or with the red oxide of iron, otherwise called colcothar of vitriol, till it has received on both sides a complete polish.* The polishing will advance slowly at first, but will proceed rapidly when the polisher becomes warm with the friction. When it is nearly finished, no more putty or water should be put upon the polisher, which should be kept warm by breathing upon it; and if the glass moves with difficulty from its adhe-

* As colcothar of vitriol is obtained by the decomposition of martial vitriol, it sometimes retains a portion of this salt. When this portion of martial vitriol is decomposed by dissolution in water, the yellow ochre which results penetrates the glass, forms an incrustation upon its surface, and gives it a dull and yellowish tinge, which is communicated to the image which it forms.

sion to the tool, it should be quickly removed, lest it spoil the surface of the pitch. When any particles of dust or pitch insinuate themselves between the glass and the polisher, which may be easily known from the very unpleasant manner of working, they should be carefully removed, by washing both the polisher and the glass, otherwise the lens will be scratched, and the bed of pitch materially injured.

The operation of polishing may also be performed by covering the layer of pitch with a piece of cloth, and giving it a spherical form by pressing it with the convex tool when the pitch is warm. The glass is wrought as formerly, upon the surface of the cloth with putty or colcothar of vitriol, till a sufficient polish is induced. By this mode the operation is slower, and the polish less perfect; though it is best fitted for those who have but little experience, and would therefore be apt to injure the figure of the lens by polishing it on a bed of pitch.

In this manner the small lenses of simple and compound microscopes, the eye-glasses, and the object-glasses, of telescopes, are to be ground. In grinding concave lenses, Mr. Imison* employs leaden wheels with the same radius as the curvature of the lens, and with their circumferences of the same convexity as the lens is to be concave. These spherical zones are fixed upon a turning lathe, and the lens, which is held steadily in the hand, is ground upon them with emery, while they are revolving on the spindle of the lathe. In the same way convex lenses may be ground and polished, by fixing the concave tool upon the lathe, but these

* School of Arts, part ii, p. 145.

methods, however simple and expeditious they may be, should never be adopted for forming the lenses of optical instruments, where an accurate spherical figure is indispensable. It is by the hand alone that we can perform with accuracy those circular and transverse strokes, the proper union of which is essential to the production of a spherical surface.

On the method of casting, grinding, and polishing, the mirrors of reflecting telescopes.

The metals of reflecting telescopes are generally composed of 32 parts of copper, and 15 of grain-tin, with the addition of two parts of arsenic, to render the composition more white and compact. From a variety of experiments, the Rev. Mr. Edwards found, that if one part of brass, and one of silver, be added to the preceding composition, and only one part of arsenic used, a most excellent metal will be obtained, which is the whitest, hardest, and most reflective, that he ever met with. The superiority of this composition, indeed, has been completely evinced by the excellence of Mr. Edwards's telescopes, which excel other reflectors in brightness and distinctness, and show objects in their natural colours. But as metals of this composition are extremely difficult to cast, as well as to grind and polish, it will be better for those who are inexperienced in the art, to employ the composition first mentioned.

After the flasks of sand* are prepared, and a mould made for the metal by means of a

* The best sand which I have met with in this country, is to be found at Roxburgh castle, in the neighbourhood of Kelso.

wooden or metallic pattern, so that its face may be downwards, and a few small holes left in the sand at its back, for the free egress of the included air;—melt the copper in a crucible by itself, and when it is reduced to a fluid state, fuse the tin in a separate crucible, and mix it with the melted copper, by stirring them together with a wooden spatula. The proper quantity of powdered arsenic, wrapt up in a piece of paper, is then to be added, the operator retaining his breath till its noxious fumes are completely dissipated; and when the scoria is removed from the fluid mass, it is to be poured out as quickly as possible into the flasks. As soon as the metal is become solid, remove it from the sand into some potashes or coals, for the purpose of annealing it, and let it remain among them till they are completely cold. The in-gate is then to be taken from the metal by means of a file, and the surface of the speculum must be ground upon a common grindstone, till all the imperfections and asperities be taken away. When Mr. Edwards's composition is employed, the copper and tin should be melted according to the preceding directions, and, when mixed together, should be poured into cold water, which will separate the mass into a number of small particles. These small pieces of metal are then to be collected and put into the crucible, along with the silver and brass; after they have been melted together in a separate crucible, the proper quantity of arsenic is to be added, and a little powdered rosin thrown into the crucible before the metal is poured into the flasks.

When the metal is cast, and prepared by the common grindstone for receiving its proper

figure, the gages and grinding tools are to be formed in the same manner as for convex lenses, with this difference only, that the radius of the gages must always be double the focal length of the speculum. In addition to the convex and concave brass tools, which should be only a little broader than the metal itself, a convex elliptical tool of lead and tin should also be formed with the same radius, so that its transverse may be to its conjugate diameter as 10 to 9, the latter being exactly equal to the diameter of the metal. On this tool the speculum is to be ground with flour emery, in the same manner as lenses, with circular and cross strokes alternately, till its surface be freed from every imperfection, and ground to a spherical figure. It is then to be wrought with great circumspection, on the convex brass tool, with emery of different degrees of fineness, the concave tool being sometimes ground upon the convex one, to keep them all of the same radius, till every scratch and appearance of roughness be removed from its surface, and it will then be ready for receiving the final polish. Before the speculum is brought to the polisher, it has been the practice to smooth it on a bed of hones, or a convex tool made of common blue hones. This additional tool, indeed, is absolutely necessary, when silver and brass enter into the composition of the metal, in order to remove that roughness which will always remain after the finest emery has been used; but when these metals are not ingredients in the speculum, there is no occasion for the bed of hones.—Without the intervention of this tool I have finished several specula, and given them as ex-

quisite a lustre as they could possibly have received. Mr. Edwards does not use any brass tools in his process, but transfers the metal from the elliptical leaden tool to the bed of hones. By this means the operation is simplified, but we doubt much if it be, in the least degree, improved. As a bed of hones is more apt to change its form than a tool of brass, it is certainly of great consequence that the speculum should have as true a figure as possible before it is brought to the hones; and we are persuaded, from experience, that this figure may be better communicated by a brass tool, which can always be kept at the same curvature by its corresponding tool, than by an elliptical block of lead. We are, moreover, certain, that when the speculum is required to be of a determinate focal length, this length will be obtained more precisely with the brass tools than without them. But Mr. Edwards has observed, that these tools are not only unnecessary, but 'really detrimental.' That Mr. Edwards found them unnecessary, we cannot doubt, from the excellence of the specula which he formed without their assistance, but it seems inconceivable how the brass tools can be in the least degree detrimental. If the mirror be ground upon 20 different tools before it is brought to the bed of hones, it will receive from the last of these tools a certain figure, which it would have received even if it had not been ground on any of the rest; and it cannot be questioned, that a metal wrought upon a pair of brass tools, is equally, if not more, fit for the bed of hones, than if it had been ground merely on a tool of lead.

When the metal is ready for polishing, the elliptical leaden tool is to be covered with black

pitch,* about $\frac{1}{20}$ of an inch thick, and the polisher formed in the same way as in the case of lenses, either with the concave brass tool, or with the metal itself. The colcothar of vitriol should then be triturated between two surfaces of glass, and a considerable quantity of it applied at first to the surface of the polisher. The speculum is then to be wrought in the usual way upon the polishing-tool, till it has received a brilliant lustre, taking care to use no more of the colcothar, if it can be avoided, and only a small quantity of it, if it be found necessary. When the metal moves stiffly on the polisher, and the colcothar assumes a dark muddy hue, the polish advances with great rapidity. The tool will then grow warm, and would probably stick to the speculum, if its motion were discontinued for a moment. At this stage of the process, therefore, we must proceed with great caution, breathing continually on the polisher, till the friction is so great as to prevent the motion of the speculum. When this happens, the metal is to be slipped off the tool at one side, and placed in a tube for the purpose of trying its performance; and if the polishing has been conducted with care, it will be found to have a true *parabolic* figure.

ON MICROSCOPES.

On the single microscope.

In the first volume, we have described the method of forming small glass globules for the

* In summer, or when the pitch is soft, it should be hardened by the addition of a little rosin.

magnifiers of single microscopes; and have also explained the manner in which the enlarged picture is formed upon the retina. When the lenses are made, either by fusion, or, which is by far the more accurate way, by grinding them on spherical tools, they are then to be fitted up for the purpose of examining minute objects. The method which Mr. Wilson has adopted in his pocket-microscope, is very ingenious, though rather devoid of simplicity, as it obliges us to screw and unscrew the magnifiers, when we wish to view the object with a larger or a smaller power. The simplest and the most convenient method of mounting single microscopes, is to fix the lenses *a, b, c, d,* in a flat circular piece of brass, *CD,* which can be moved round *I* as a centre, by the action of the endless screw *AB,* upon the toothed circumference of the circular plate. After the object has been viewed by some of the magnifiers, it may be examined successively with all the rest, by a few turns merely of the endless screw. PLATE XLVII. Fig. 3,

In the first volume, Mr. Ferguson has already shown how to find the magnifying power of single microscopes; but in order to save the trouble of calculation, we have computed the following new table of the magnifying powers of convex lenses, from 1 inch to $\frac{1}{100}$ of an inch in focal length, upon the supposition that the nearest distance at which we see distinctly is *seven* inches. The first column contains the focal length of the convex lens in $\frac{1}{100}$ths of an inch. The second contains the number of times which such a lens will magnify the diameters of objects. The third contains the number of

times that the surface is magnified; and the fourth the number of times that the cube of the object is magnified. A table of a similar kind, though upon a much smaller scale, has already been published by Mr. Barker; but he supposes the nearest distance at which the eye can see distinctly, to be *eight* inches, which, I am confident from experience, is too large an estimate for the generality of eyes.

A NEW TABLE,

Of the Magnifying Power of Small Convex Lenses, or Single Microscopes, not exceeding an inch in focal length.

Focal distance of the lens or microscope.		Number of times that the *diameter* of an object is magnified.	Number of times that the *surface* of an object is magnified.	Number of times that the *cube* of an object is magnified.
100ths of an inch.		Times. Dec. of a time.	Times.	Times.
1	100	7.00	49	343
3/4	75	9.33	87	810
1/2	50	14.00	196	2744
2/5	40	17.50	306	5360
3/10	30	23.33	544	12698
2/10	20	35.00	1225	42875
	19	36.84	1354	49836
	18	38.89	1513	58864
	17	41.18	1697	69935
	16	43.75	1910	83453
	15	46.66	2181	101848
	14	50.00	2500	125000
	13	53.85	2894	155721
	12	58.33	3399	198156
	11	63.67	4045	257259
1/10	10	70.00	4900	343000
	9	77.78	6053	470911
	8	87.50	7656	669922
	7	100.00	10000	1000000
	6	116.66	13689	1601613
1/20	5	140.00	19600	2744000
1/25	4	175.00	30625	5359375
	3	233.33	54289	12649337
1/50	2	350.00	122500	42875000
	1	700.00	490000	343000000

On the double microscope.

The double microscope is composed of two convex lenses placed at any distance not less than the sum of their focal lengths; and a lens with a large aperture and focal distance is generally fixed a little beyond the anterior focus of the eye-glass, for the purpose of enlarging the field of view. As the focal lengths of the lenses as well as their distances are altogether arbitrary, different opinions have been entertained respecting the most suitable values of these quantities. I have found, however, from experience, that a good compound microscope may be formed by making the object-glass $\frac{4}{16}$ of an inch in focal length, and the eye-glass 1 inch, their distance being about 8 inches. The amplifying lens or second eye-glass should generally be $1\frac{3}{4}$ inch in diameter, with $2\frac{1}{4}$ inches of focal length, and placed at $1\frac{1}{4}$ inch before the eye-glass; and the aperture of the object-glass should not exceed *one-tenth* of an inch. If, however, we increase the magnifying power of the microscope by augmenting the distance between the glass next the object and that next the eye, we must likewise enlarge the aperture of the object-glass; but if we increase the magnifying power by augmenting the curvature or diminishing the focal length of the object-glass, the aperture must be proportionably diminished. The distance of the eye from the eye-glass should be equal to the focal distance of the latter; and the hole through which the rays are finally transmitted should not exceed *one-seventh* of an inch.

The method of finding the magnifying power of double microscopes when only two lenses are employed has been shown in the first volume. But as an amplifying lens or second eye-glass is always used, we shall show the method of determining the magnifying power of these instruments when three lenses are employed. The following rule we believe is new. Divide the difference between the distance of the two first lenses, or those next the object, and the focal distance of the second or amplifying glass, by the focal distance of the second glass, and the quotient will be a first number. Square the distance between the two first lenses, and divide it by the difference between that distance and the focal distance of the second glass, and divide this quotient by the focal distance of the third glass, or that next the eye, and a second number will be had. Multiply together the first and second numbers, and the magnifying power of the object-glass, and the product will be the magnifying power of the compound microscope.

ON TELESCOPES.

On the refracting telescope.

Having already described the nature and operation of refracting telescopes, we have now only to lay before the reader a new table of the apertures and magnifying powers of these instruments, more accurate, we trust, than any which has yet been published. It is a remarkable circumstance, that the only table of this kind which has appeared, was copied by succeeding writers from Dr. Smith's optics, as the

production of the celebrated Huygens, while it was calculated only by the editors of his dioptrics. An excellent telescope of Huygens, indeed, was the standard upon which the table was constructed; but this philosopher informs us in his *Astroscopia Compendiaria*, that he had a refracting telescope with an object-glass 34 feet in focal length, which, in astronomical observations, bore an eye-glass of $2\frac{1}{2}$ inches focal distance, and therefore magnified 163 times, which is considerably greater, in proportion, than the magnifying power of the standard telescope upon which the old table was founded. Since the lenses of these instruments therefore may now be wrought with the same accuracy as in the time of Huygens, we have computed the following table according to this new standard, the apertures, magnifying powers, and the focal length of the eye-glass being severally as the square roots of their focal lengths. As the dimensions of the standard-telescope of Huygens were taken in Rhinland measure, the following table is suited to the same measure, but the second and third columns may be converted into English measure by dividing them by 7, the focal lengths of the object-glasses being supposed English feet.

A NEW TABLE,

Of the Apertures, Focal Lengths, and Magnifying Power of Refracting Telescopes.

Focal length of the object-glass.	Linear aperture of the object-glass.	Focal distance of the eye-glass.	Magnifying power.
Feet.	Inch. Dec.	Inch. Dec.	Times.
1	0.65	0.50	28
2	1.03	0.62	39
3	1.30	0.75	48
4	1.45	0.87	55
5	1.61	1.00	60
6	1.79	1.07	67
7	1.96	1.15	73
8	2.14	1.21	77
9	2.20	1.30	83
10	2.32	1.38	87
13	2.63	1.58	99
15	2.81	1.70	106
20	3.31	1.95	123
25	3.73	2.15	139
30	4.01	2.40	150
35	4.34	2.58	163
40	4.64	2.76	174
45	4.92	2.93	184
50	5.20	3.08	195
55	5.48	3.22	205
60	5.71	3.36	214
70	6.16	3.64	231
80	6.58	3.90	246
90	7.02	4.12	262
100	7.39	4.35	276
200	10.41	6.17	389
300	12.89	7.52	479
400	14.72	8.71	551
500	16.52	9.71	618

On the Gregorian Telescope.

To the observations already made upon this instrument, we have only to add a few practical remarks. In order to remove the tremors from reflecting telescopes, the springs and screws should be taken away from the back of the speculum, and three small screws employed, which pass through the tube perpendicular to its axis, and touch the back of the mirror merely with their sides. As the speculum is apt to bend when it is supported wholly upon its lower extremity, it should be made to rest upon two points 45 degrees distant from its lowest part, and on each side of it; and if the metal be wedged in at these points with bits of card, it will be prevented from falling backward or resting upon its lowest point. Some reflecting telescopes may be much improved, as Dr. Maskelyne has shown, by inclining the great mirror about $2\frac{1}{2}$ degrees* to the axis of the tube, so that the pencils of rays might fall obliquely on its surface.†

The diameter of the small eye-hole may be found by dividing the aperture of the telescope in inches by its magnifying power; but is generally about $\frac{1}{50}$ of an inch.

The following table, formed upon the computations of Dr. Smith, contains all the dimensions of Gregorian telescopes, and is more comprehensive and accurate than that which Mr. Edwards published.

* This degree of inclination greatly improved the six feet Newtonian reflector in the Observatory of Greenwich; but different specula will require different degrees of obliquity, and some may rather be injured by such an inclination.

† These observations are also applicable to the metals of Cassegrainian and Newtonian telescopes.

TABLE,

Of the Dimensions, Focal Lengths and Apertures, of Gregorian Telescopes, as constructed by Mr. Short, according to the computations of Dr. Smith.

See Plate LIV, Fig. 7.

Focal length of the great speculum.	Breadth of the great speculum.	Breadth of the small speculum, and of the hole in the large one.	Focal length of the small speculum.	Distance between the two specula.	Distance between the large speculum and the plane surface of the first eye-glass.	Focal distance of the first eye-glass, or that next the metals.	Focal distance of the second eye-glass, or that next the eye.	Distance between the plane sides of the two lenses.	Distance between the second eye-glass and the small eye-hole.	Diameter of the diaphragm placed in the anterior focus of the lens *S*.	Magnifying power.
Pm	*DF*.	*UV=hg*	*Ln*	*PL*	*PR*	*R*	*S*	*RS*	*Se*	*ab*	
Inch. Dec.	Inch. Dec.	Inch. Dec.	Inch. Dec.	Inch. Dec.	Inch. Dec.	Inch. Dec.	Inch. Dec.	Inch. Dec.	Inch. Dec.	Inch. Dec.	Times.
5.65	1.55	0.31	1.11	6.78	1.76	2.45	0.81	1.63	0.41	0.27	40
9.60	2.30	0.40	1.50	11.25	3.36	3.13	1.04	2.09	0.52	0.35	60
15.50	3.30	0.50	2.15	17.84	3.95	5.97	1.31	2.63	0.66	0.44	86
36.00	6.26	0.65	3.43	39.72	1.44	5.12	1.71	3.41	0.85	0.57	165
60.00	9.21	0.83	5.01	65.39	2.78	6.43	2.14	4.29	1.07	0.72	243
1	2	3	4	5	6	7	8	9	10	11	12

On the Cassegrainian Telescope.

From the following table of the dimensions of Cassegrainian telescopes, founded on Dr. Smith's calculations, it appears, that though they are shorter by twice the focal distance of the small speculum than those of the Gregorian form with the same focal length, yet they have a greater magnifying power. A Cassegrainian telescope, $15\frac{1}{2}$ inches in focal length, will magnify, according to the table, 93 times; while a Gregorian one, with a similar speculum, magnifies only 86 times. This great difference between the performauce of these instruments, does not subsist merely in theory; for Mr. Short constructed a telescope of the Cassegrain form, of 24 inches focus, which, with an aperture of 6 inches, magnified 355 times. With this power, indeed, it was rather indistinct; but it bore a power of 231 times with sufficient distinctness. In the observatory at Greenwich, there is a Gregorian telescope of Short's construction, which magnifies 250 times when the smallest mirror is employed, which is considerably less than the power of the Cassegrainian one of the same size.

TABLE,

Of the Dimensions, Focal Lengths and Apertures, of Cassegrainian Telescopes. See Plate XVIII, Fig. 7. in which the small convex speculum is supposed to be placed at GH.

Focal length of the great speculum.	Breadth of the great speculum.	Breadth of the small speculum, and of the hole in the large one.	Focal length of the small speculum.	Distance between the two specula.	Distance between the large speculum and the plane surface of the first eye-glass.	Focal distance of the first eye-glass, or that next the metals.	Focal distance of the second eye-glass, or that next the eye.	Distance between the plane sides of the two lenses.	Distance between the second eye-glass and the small eye-hole.	Diameter of the diaphragm, placed in the anterior focus of the lens *S*.	Magnifying power.
Pm	*DF*	*UV*			*PR*	*R*	*S*	*RS*	*Se*	*ab*	
Inch. Dec	Inch. Dec.	Inch. Dec	Inch. Dec	Inch. Dec	Inch. Dec.	Inch. Dec.	Inch. Dec	Inch. Dec.	Inch. Dec.	Inch. Dec.	Times.
15.50	3.52	0.40	1.97	13.51	1.41	3.17	1.06	2.11	0.53	0.18	93
36.00	6.57	0.59	3.57	32.75	1.65	4.69	1.56	3.03	0.78	0.26	173
60.00	9.61	0.77	8.17	55.21	2.97	6.06	2.02	4.04	1.01	0.34	253
1	2	3	4	5	6	7	8	9	10	11	12

On the Newtonian Telescope.

As the Newtonian telescope was powerfully recommended to the world by the simplicity of its construction, as well as by the name of its illustrious inventor, it is a matter of surprise that its merits should have been so long overlooked. During the last century Gregorian telescopes seem universally to have been preferred to those of the Newtonian form, till the celebrated Dr. Herschel introduced the latter into notice, by the splendour and extent of the discoveries which they enabled him to make. This philosopher, equally distinguished by his virtues and his talents, has constructed Newtonian telescopes from 7 to 40 feet* in focal length, by which he has greatly enlarged our knowledge of the solar system, and disclosed many new and important facts respecting the structure of the heavens.

Plate XVIII. Fig. 7. In the Newtonian telescope, the large parabolic speculum is not perforated with a hole *UV*. A small elliptical plane mirror, inclined 45° to the axis of the tube, is placed at *GH*, about as much nearer the speculum than its focus, as the centre of the small mirror is distant from the tube; that is, the distance *Gm* of the small speculum from the focus of the great one, should be nearly equal to *PT*, half the diameter of the tube. The rays which form the image *IK* of the object *AB*, instead of proceeding to

* A description of this noble instrument may be seen in the Phil. Trans. 1795, pt. 2. The diameter of the speculum is 4 feet, its thickness about 3½ inches, and its magnifying power 6000.

form it at m, are intercepted by the plane speculum at GH, and refracted upward through an aperture in the side of the tube TT, where the image is formed and magnified by a double convex lens of a short focal distance.

As the small plane mirror has an oblique position to the eye, it must be of an elliptical form. In order to find its conjugate or shortest diameter, say as the focal length of the great speculum is to its aperture, so is the distance of the small speculum from the focus of the great one to the conjugate diameter of the small mirror, that is, the conjugate diameter of the small mirror is $= \frac{Gm \times DF}{Pm}$. Its transverse or longest diameter will be $= \frac{Gm \times DF}{Pm} \times 1.4142$, that is, equal to the conjugate diameter multiplied by 1.41242, or which is the same thing, its transverse will be to its conjugate diameter as 7 to 5,* which is nearly the ratio of the diagonal of a square to one of its sides. If a rectangular prism be substituted in place of the small mirror, having its sides perpendicular to the incident and emergent rays, a less quantity of light will be lost, than when the reflection is made from a mirror of the common kind.

In most of Dr. Herschel's telescopes the plane mirror is thrown away, and the focal image IK is viewed directly with a small eye-glass, placed at TE, the lower side of the tube. When the aperture of the speculum is very large, the loss of light occasioned by the interposition of part of the observer's head is trivial;

* Mr. Adams in his *Introduction to Natural Philosophy*, vol. ii, p. 534, erroneously observes, that the length of the small speculum should be to its breadth as 2 to 1.

but when the aperture is small, the speculum must be inclined a little to the incident rays. I have frequently taken a Newtonian speculum, $3\frac{4}{10}$ inches in diameter, and 30 inches in focal length, out of its tube, and viewed the moon in this manner with great satisfaction. The superior performance of Newtonian telescopes, without the plane mirror, can be conceived only by those who have made the experiment.

As it is more difficult to find any of the heavenly bodies with a Newtonian than with a Gregorian telescope, it has been customary to fix a small astronomical telescope on the tube of the former, so that the axes of the two instruments may be parallel. The aperture of its object-glass is large, and cross-hairs are fixed in the focus of the eye-glass. The object is then found by this small telescope, which is called the *finder;* and if the axes of the instruments are rightly adjusted, it will be seen also in the field of the large telescope. When the Newtonian telescope, however, is large, and placed upon its lower end to view bodies at great altitudes, the finder can be of no use, from the difficulty of getting the eye to the eye-piece.— On this account I would propose to bend the tube of the finder to a right angle, and place a plane mirror at the angular point, so as to throw the image to one side, or rather above the upper part of the tube, that the eye-piece of the finder may be as near as possible to the eye-piece of the telescope. If the latter of these plans be adopted, the angular point, where the plane mirror is fixed, should be placed as near as possible to the focal image, in order that only a small part of the finder may stand above the tube; for in this way the eye can be transferred with the

greatest facility from the one eye-piece to the other.

The only table, containing the apertures, magnifying power, &c. of Newtonian telescopes, which has hitherto been published, was calculated by Dr. Smith,* from the middle aperture and power of Hadley's excellent Newtonian telescope, as a standard, the focal length of the great speculum being 5 feet $2\frac{1}{2}$ inches, its aperture 5 inches, and power 208. A speculum, however, 3 feet and 3 inches in focal length, was wrought, by Mr. Hauksbee, to so great perfection, as to magnify 226 times.† It showed the minute parts of the new moon very distinctly, as well as the belts of Jupiter, and the black list or division of Saturn's ring. For these objects, it bore an aperture of $3\frac{1}{2}$ or 4 inches; but in cloudy weather it showed land objects most distinct, when the whole surface of the metal was exposed, which was $4\frac{1}{2}$ inches in diameter. Since the method of grinding specula, and giving them a true parabolic figure, is much better understood at present than it was at the time of Mr. Hauksbee, Newtonian telescopes may be made as perfect as this instrument of his construction. Upon it, as a standard, therefore, we have computed the following new table, upon the supposition, that reflecting telescopes, of different lengths, show objects equally bright and distinct, when their linear apertures, and their linear amplifications, or magnifying powers, are as the *square-square roots*, or *biquadratic roots*, of their focal lengths,

* Optics, vol. I, p. 148. Dr. Smith's table was continued from 17 to 24 feet, by Mr. Edwards.

† Smith's Optics, vol. ii. *Remarks*, p. 79, col. ^.

and, consequently, when the focal distances of their eye-glasses are as the square-roots of their lengths.

The first column contains the focal length of the great speculum in feet, and the second its linear aperture in inches, and 100ths of an inch. The third and fourth columns contain Sir Isaac Newton's numbers, by means of which the apertures of any kind of reflecting telescopes may be readily computed.* The fifth column exhibits the focal length of the eye-glasses in 1000ths of an inch; and the sixth contains the magnifying power of the instrument.

* See Gregory's Optics, Appendix, p. 229, and the Philosophical Transactions, No. 81, p. 4004.

A NEW TABLE,

Of the Apertures and Magnifying Power of Newtonian Telescopes.

Focal length of the concave speculum.	Aperture of the concave speculum.	Sir Isaac Newton's numbers.		Focal length of the eye-glass.	Magnifying power.
Feet.	Inch. Dec.	Aperture of the speculum.	Focal length of the eye-glass.	Inch. Dec.	Times.
½	1.34	100	100	0.107	56
1	2.23	168	119	0.129	93
2	3.79	283	141	0.152	158
3	5.14	383	157	0.168	214
4	6.36	476	168	0.181	265
5	7.51	562	178	0.192	313
6	8.64	645	186	0.200	360
7	9.67			0.209	403
8	10.44	800	200	0.218	445
9	11.69			0.222	487
10	12.65	946	212	0.228	527
11	13.58			0.233	566
12	14.50	1084	221	0.238	604
13	15.41			0.243	642
14	16.25			0.248	677
15	17.11			0.252	713
16	17.98	1345	238	0.256	749
17	18.82			0.260	784
18	19.63			0.264	818
19	20.45			0.268	852
20	21.24	1591	251	0·271	885
21	22.06			0.274	919
22	22.85			0.277	952
23	23.62			0.280	984
24	24.41	1824		0.283	1017

The telescope which Dr. Herschel generally uses, and with which he has made many of his best discoveries, is a Newtonian reflector, with a speculum 7 feet in focal length, having an aperture of $6\frac{1}{3}$ inches, and powers of 227 and 460, though he sometimes employs a power of 6450 for the fixed stars. Dr. Herschel informs me, that he obtains such high powers, merely by using small double convex lenses for eye-glasses, and that he has some in his possession less than *one fiftieth of an inch* in focal length.

OPTICS.

Description of a New Fluid Microscope, invented by the E. Editor.

FOR the first idea of fluid microscopes we are indebted to the ingenious Mr. Stephen Grey, who published an account of his discovery in the Transactions of the Royal Society.* They consisted merely of a drop of water, taken up on the point of a pin, and placed in a small hole at *D*, $\frac{1}{30}$ of an inch in diameter, in the piece of brass *DE*, about $\frac{1}{18}$ of an inch thick. The hole *D* is in the middle of a spherical cavity, about $\frac{1}{8}$ of an inch in diameter, and a little deeper than half the thickness of the brass. On the opposite side of the brass is another spherical cavity, half as broad as the former, and so deep as to reduce the circumference of the small hole to a sharp edge. The water being placed in these cavities, will form a double convex lens, with unequal convexities. The object, if it be solid, is fixed upon the point *C* of the supporter *AB*, and placed at its proper distance from the water-lens, by the screw *FG*. When the object is fluid, it is placed in the hole *A*, but in such a manner as not to be spherical; and this hole is brought opposite the fluid lenses, by moving the extremity *G* of the screw into the slit *GH*.

PLATE XLVIII. Fig. 1.

* Phil. Trans. No. 221, 223. See also Smith's Optics, vol. ii, p. 394.

From this microscope of Mr. Grey's, the one which we are now to describe is totally different. It is represented, as fitted up, in Plate XLVII, Fig. 2. and some of its parts, on a larger scale, in Fig. 3 and 4. A drop of very pure and viscid turpentine-varnish is taken up by the point of a piece of wood, and dropped, at *a*, upon the piece of thin and well polished glass *abcdI;* and different quantities being taken up, and dropped, in a similar manner, at *b, c, d,* will form four or more plano-convex lenses of turpentine-varnish, which may be made of any focal length, by taking up a greater or a less quantity of the fluid. The lower surface of the glass *abcdI,* having been first smoked with a candle, the black pigment, immediately below the lenses *a, b, c, d,* is then to be removed, so that no light may pass by their circumferences. The piece of glass, *aIc,* is next to be perforated at *I,* and surrounded with a toothed wheel *CD,* which can be moved round *I* as a centre, by the endless screw *AB.* The apparatus *CDBA* is placed in a circular case, which is represented by *BH* in Fig. 2. and part of it, on a larger scale, by *CD* in Fig. 4. and to its sides the screw *AB* is fastened, by means of the two arms *m, n.* This circular case is fixed to the horizontal arm *R,* by means of a brass pin, which passes through its upper and under surfaces, and through the hole *I,* (Fig. 3.) which does not embrace the pin very tightly, in order that *CD* may revolve with facility. On the upper surface of *BH* is an aperture *K,* directly above the line described by the centres of the fluid lenses, when moving round *I;* and in this aperture is inserted a small cap, with a little hole at its top, to which

PLATE XLVII. Fig. 2, 3, 4.

the eye is applied. *EMN* is the moveable stage, that carries the slider *OP*, on which microscopic objects are laid, and is brought nearer, or removed from, the lenses, by the vertical screw *DE*. *RS* is the perpendicular arm to which the microscope is attached. *FG* is the pedestal; and *C* is a plane mirror, which has both a vertical and horizontal motion, in order to illuminate the objects on the slider.

When the microscope is thus constructed, the object to be viewed is placed upon *OP*, and the screw *AB* is turned, till one of the lenses be directly below the aperture *K*. The slider is then raised or depressed, by the screw *DE*, till the object be brought into the focus of the lens. In this manner, by turning the screw *AB*, and bringing all the lenses, one after another, directly below *K*, the object may be successively examined with a variety of magnifying powers.

The focal lengths of these fluid lenses will increase a little after they are formed; but if they are preserved from dust, they will last for a long time. The turpentine varnish should be as pure and viscid as possible; the glass on which it is dropped should be very thin;* and the microscope should stand on a horizontal surface.

I have even employed these fluid-lenses as the object-glasses of compound microscopes; and I once constructed a compound microscope, in which both the object-glass and eye-glass were made of turpentine varnish. It performed much better than I had expected, but gave rather a yellowish tinge to the objects which were presented to it.

* Instead of glass a thin lamina of fine transparent *talc* may be used with great advantage.—A. Ed.

DIALLING.

Description of an Analemmatic Dial, which sets itself.

PLATE XLVIII. Fig. 2. THE analemmatic dial is represented by *CD* in Fig. 2. of Plate XLVIII, and is generally described upon the same surface with a horizontal dial *AB*, for the purpose of ascertaining its proper position without the assistance of a meridian-line or compass. It is always of an elliptical form, approaching to that of a circle, as the place for which it is made recedes from the equator. Its stile is perpendicular, and has different positions in the line ♋ ♑, changing with the declination of the sun, and indicated by the names of the months marked upon its surface. From the obliquity of the stile of the one dial, and the rectangular position of the other, the motion of their shadows is so different, that the dial may be reckoned properly placed when the shadows of both stiles indicate the same hour.

Fig. 4. In order to understand the theory and construction of this dial, let *BE* be its length, perpendicular to the direction of the meridian.—Having bisected *BE* in *A*, make *AO* equal to the sine of the latitude of the place; and with the cosine of the latitude as radius, set off *AD* and *AC* equal to the tangent of 23° 28′, the

sun's greatest declination. The points *D* and *C* are the places of the stile in the time of the solstices, on the 21st of June and December, and if the tangent of the sun's declination for the first day of every month be set off in a similar manner between *A* and *D*, and *A* and *C*, the points thus found will be the place of the stile on those days, and the radius *BC* drawn from all these points to *B* will be the hour-line of six at these different times. PLATE XLVIII.

In order to prove this, let *ZÆNH* (Fig. 5.) be the meridian, *Pp* the six o'clock hour-circle, and *PH* the height of the pole, then *AZS* is the azimuth of the sun, and *PZS* its complement, *AS* the sun's declination, and *PS* its complement. Now, in the spherical triangle *PZS* right angled at *P*, we have by spherical trigonometry, (Simson's Euclid, prop. 17) Radius : Sin. *PZ*=Tang. *PZS* : Tang. *PS*; that is, Radius : Sin. *PZ*=Co Tang. azimuth : Co Tang. declination, for *PZS* is the complement of the azimuth, and *PS* the co-declination; but as radius is a mean proportional between the tangent and cotangent, (Def. IX, Cor. 1. plane trigonom.) the tangents will be in the reciprocal ratio of the cotangents, and consequently cotang. azimuth : cotang. declin.=Tang. declin. : Tang. azimuth. Therefore, Rad. : Sin. *PZ*=Tang. declin. : Tang. : azimuth; and the sine of *PZ* the colatitude, is the same as the cosine of the latitude. Fig. 5.

Now, if *AC* represents the six o'clock hour-line when the sun is in the equator. and *AC* the tangent of the sun's declination, for a radius equal to the cosine of the latitude, or *AC*=Tang. declin. × cosine. latitude, the angle *ABC* will be equal to the sun's azimuth, for

PLATE XLVIII. from the last analogy, Tang. declin. × cos. latitude = Rad. × Tang. azimuth, therefore *AC*= Rad. × Tang. azimuth, that is, *AC* is equal to the tangent of the sun's azimuth when *AB* is radius; and consequently *ABC* is the sun's azimuth since *AC* is its tangent. If the sun were in the equator and the stile at *A*, his azimuth from the south would be at *OAB*, whereas when the stile is at *C*, his azimuth is *OCB*, which is equal to *OAB*—*ABC*, therefore *ABC* is the sun's azimuth from the east or west at 6 o'clock, and *BC* the 6 o'clock hour line. In the same way it might be shown, when the stile is placed in any point between *C* and *D*, that a line drawn from it to the point *B* will be the six o'clock hour-line for that declination, and that the angle at *B*, comprehended between this line and *AB*, will be equal to the azimuth of the sun.

In order to determine the horary points and the circumference of the dial, we must consider, that if the equator be projected upon the horizon of any place, it will form an ellipse whose conjugate or shortest diameter is equal to the sine of the latitude of that place. Let *BMF*,
Fig. 6. therefore, (Fig. 6.) be the equator projected on the horizon of a given place, so that *AM* half the conjugate axis is to *AB*, half the transverse axis, as the sine of the latitude of that place is to radius. Then having described the semicircle *BXIIF*, divide the quadrants *BXII*, and *XIIF* into 6 equal parts for the hours, into 12 for the half-hours, and into 24 for the quarters, each hour being 15° in the daily motion of the sun, each half-hour 7° 30′, and each quarter 3° 45′, and from these points, from the point *III*, for example, draw *IIICE* parallel to *AXII*, or perpendicular to *AB*, the point *C*,

where this line cuts the ellipse, will be the horary point, and *DC* will be the three o'clock hour-line when the stile is at *D*. PLATE XLVIII.

As there is some difficulty, however, in describing an ellipse with accuracy, we shall show how to find the horary points without describing this conic section. Take *BC* (Fig. 3.) equal the breadth of the dial, and having bisected it in *A*, draw *A* 12 perpendicular to *BC*, and equal to the sine of the latitude, *AC* being radius. Then upon the centre *A*, with the distance *A* 12, describe the semicircle *D* 12 *E*, and with the distance *AB* the semicircle *CHB*. Divide the quadrant *HB* into six equal parts for hours in the points *m*, *n*, *o*, *p*, *q*, and the quadrant 12 *E* into the same number of equal parts in the points *a*, *b*, *c*, *d*, *e*; and through *a*, *b*, *c*, &c. draw *a* 11, *b* 10, *c* 9, &c. parallel to *CB*; and through *m*, *n*, *o*, &c. draw *m* 1, *n* 2, *o* 3, parallel to *HA*; the points of intersection 1, 2, 3, 4, 5, will be the horary points, and will be in the circumference of an ellipse. The horary points being thus known, it is not necessary to trace the ellipse, otherwise it might be easily done with the hand. If the divisions *Hm*, *mn*, &c. be subdivided into half hours and quarters, or even lower, the corresponding points in the ellipse 12 *B* may be determined in a similar manner. Fig. 3.

In order to demonstrate that *C* is the horary point of three o'clock, and *DC* the hour-line when the sun is at his greatest north declination, we must find from the construction the angle *CDM*, or the sun's azimuth, reckoned from the south, and see if the triangle *PZS* (Fig. 7.) furnishes us with a similar expression of the angle *Z*, or sun's azimuth. In Fig. 6. *CH*, or its equal *AE*, is evidently the sine of the horary Fig. 6.

PLATE XLVIII. angle AB being radius; and since CE or AH is the cosine of the horary angle, in a circle whose radius is AM, or the sine of the latitude, we will have CE or AH=Cos. horary angle × Sin. lat. But according to the first part of the construction AD = Tan. declin. × Cos. lat.; therefore DH, the difference between AD and AH, will be = Cos. hor. angle × Sin. lat.—Tang. declin. × Cos. lat.; and the tangent of the angle CDH or $\frac{CH}{DH}$ will then be equal to

$$\frac{\text{Sin. Hor. Angle}}{\text{Cos. Hor. Angle} \times \text{Sin Latit.} - \text{Tang. Decl.} \times \text{Cos. Lat.}}$$

Fig. 7. Now, in order to find a similar expression for the angle PZS, (Fig. 7.) let SO be a perpendicular upon PZ; and the sines of the segments PO, ZO, will be in the inverse ratio of the tangents of the angles at the base P and Z, (Simson's Spher. Trig. Prop. XXVI.); that is, Sin. ZO : Sin. PO=Tang. P : Tang. Z; and therefore, Tang. $Z=\frac{\text{Sin } PO \times \text{Tang. } P}{\text{Sin. } ZO}$. But, Sin. ZO= Sin. $\overline{PZ-PO}$*=Sin. PO×Cos. PZ—Sin. PZ ×Cos. PO.—Now, since Rad. : Tang.=Sin. : Cosine, and since Cos. : Sin.=Rad. : Tang. we have, by the rule of proportion, Sin. PO= Cos. PO×Tang. PO; and Tang. $PO=\frac{\text{Sin. } PO}{\text{Cos. } PO}$. Therefore, $\frac{\text{Sin. } PO}{\text{Sin. } ZO}$=

$$\frac{\text{Cos. } PO \times \text{Tang. } PO}{\text{Sin. } PO \times \text{Cos. } PZ - \text{Sin. } PZ \times \text{Cos. } PO}.$$ Dividing by Cos. PO we have $\frac{\text{Sin } PO}{\text{Sin. } ZO}$=

$$\frac{\text{Tang } PO}{\frac{\text{Sin. } PO \times \text{Cos. } PZ - \text{Sin. } PZ}{\text{Cos. } PO}};$$ and since

* See Trail's Algebra, Appendix, No. VI, *on the Arithmetic of Sines*, Theorem II.

Tang. $PO = \frac{\text{Sin. } PO}{\text{Cos. } PO}$, we will have, by substitution, Plate XLVIII.

$$\frac{\text{Sin. } PO}{\text{Sin. } ZO} = \frac{\text{Tang. } PO}{\text{Tang. } PO \times \text{Cos } PZ - \text{Sin } PZ}.$$

Again, by Simson's Spher. Trigon. Prop. XX. Cos. P : Rad.=Tang. PO : Tang. PS, consequently Tang. PO = Tang. $PS \times$ Cos. P. Substituting, therefore, this new value of Tang. PO in its room, in the last equation, multiplying the whole by Tang. P, and dividing by Tang. PS,* we will have,

$$\frac{\text{Tang. } P \times \text{Sin. } PO}{\text{Sin. } ZO} = \frac{\text{Cos. } P \times \text{Tang. } P}{\text{Cos. } PZ \times \text{Cos. } P - \text{Sin. } PZ \times \text{Cot } PS}.$$

But since Tang. : Rad.=Cos. : Sin., Sin. P= Cos. $P \times$Tang. P. By substituting Sin. P in place of its value, we will have Tang. Z, or its equal

$$\frac{\text{Tang. } P \times \text{Sin. } PO}{\text{Sin. } ZO} = \frac{\text{Sin. } P}{\text{Cos. } P \times \text{Cos } PZ - \text{Sin } PZ \times \text{Cot. } PS};$$

that is, by substituting the names of the symbols, Tang. Z=

$$\frac{\text{Sin. Hor. Angle}}{\text{Cos Hor. Ang.} \times \text{Sin. Lat.} - \text{Tang. Dec.} \times \text{Cos. Lat.}}$$

which is the same expression of the tangent of the sun's azimuth, or angle Z, as was deduced from the former construction.

The analemmatic dial being thus demonstrated, its construction will be better understood by taking an example. Let it be required, therefore, to construct one of these dials for latitude 56° north, which nearly answers to Edinburgh. Let *AC* (Fig. 3.) be taken for half the breadth or radius of the dial, and let it be divided into 1000 parts, then *A* 12, which must be equal to the sine of the latitude or 56°, will be 829, which are the three first figures of the Fig. 3.

* Since the tangents are in the inverse ratio of the cotangents, multiplying any number by the cotangent, is the same as dividing it by the tangent.

PLATE XLVIII. Fig. 4.

natural sine of 56° in a table of sines. In order to find the points *D*, *C*, (Fig. 4.) where the stile is to be placed at the solstices on the 21st of June and December, take the tangent of 23° 28′, the sun's declination at that time, and it will be 434, if the radius were *AC* or 1000; but as the radius is the cosine of the latitude, which is 559, we must say, as 1000 : 559 = 434 : 243, the length of *AD* and *AC*. On the 21st of February, April, August, and October, the sun's declination is nearly 11° 19′, the tangent of which for a radius of 1000 is 200; but for a radius of 559, the cosine of the latitude, it will be 112, which is the distance of the stile from *A* on both sides on the 21st of the months already mentioned. On the 21st of January, May, July, and November, the sun's declination is nearly 20° 8′, the tangent of which, for the radius 1000, is 367; but for the radius 559, it will be 205, which is the distance of the stile from *A*, on both sides, on the 21st of these months, the names of the months being inserted beside the points, as in Fig. 2. The horary points are now to be determined in the manner already mentioned,* and the dial will be finished. In order to place the dial, we have only to turn it round till the stile of the analemmatic dial indicates the same hour with that of the horizontal one, and it will then be properly placed.

* See page 467.

DIALLING.

*Description of a new Dial, in which the hours are at equal distances in the circumference of a circle.**

WITH any radius describe the circle *FXIIB*: draw *AXII* for the meridian, and divide the quadrants *FXII*, *BXII*, each into six equal parts for hours. To the latitude of the place add the half of its complement, or the height of the equator, and the sum will be the inclination of the stile, or the angle *DAC*. Thus, at Edinburgh, the latitude is 55° 58′, the complement of which, or the altitude of the equator, is 34° 2′; the half of which 17° 1′, being added to 55° 58′, gives 72° 59′ for the inclination of the stile or the angle *DAC*. The position of the stile in the figure is that which it must have on the 21st of March and September, when the sun crosses the equator; but when the sun has north declination, the point *A* must move towards *D*, and when he is south of the equator, it must move in the opposite direction. In order to find the position of the point *A* for any declination of the sun, multiply together the radius of the dial, the tangent of half the height of the equator at the place for which the dial is constructed, and the tangent Plate XLVIII. Fig. 6.

* This dial was invented by M. Lambert, and is described and demonstrated in the *Ephemerides de Berlin*, 1777, p. 200, written in German.

of the sun's declination, and the product of these three quantities divided by the square of the radius of the tables, will give the distance of the moveable point *A* from the centre of the circle *FXIIB*.

Let it be required, for example, to find the position of the point *A* on the 21st of December and June, when the declination of the sun is a maximum, or 23° 28′ the radius *AB* of the dial being divided into 100 equal parts.

$$\begin{aligned} \text{Log. } 100 &= 2.0000000 \\ \text{Log. Tang. } 17°\ 1' &= 9.4857907 \\ \text{Log. Tang. } 23°\ 28' &= 9.6376106 \\ \hline \end{aligned}$$

Sum 21.1234013=Log. of product. From this logarithm substract 20, the logarithm of the square of the radius, and the remainder will be 1.1234013 = Log. 13.29. Take $13\frac{1}{4}$ parts, therefore, in your compasses, and having set them both ways from *A*, the limits of the moveable stile will be marked out.

For any other declination, the position of the point *A* may be found in a similar manner. It will be sufficient in general to determine it for the declination of the sun when he enters each sign, and place these positions on the dial, as represented in Fig. 2.

The length of the stile *AC*, or its perpendicular height *HC*, must always be of such a size that its shadow may reach the hours in the circle *FXIIB*. For any declination of the sun, its length *AC* may be determined by plane trigonometry. *AXII* is always given, the inclination of the stile *DAC* is also known, the angle *AXIIC* is equal to the sun's meridian altitude, and therefore the whole triangle may be easily found

in the common way, or by the following trigonometrical formula: *AC* the length of the stile $= \frac{A\,XII \times \text{Sin. Merid. Alt.}}{\text{Sin. } (180^\circ - \text{Angle of Stile} + \text{Merid. Alt.})}$

Notwithstanding the simplicity in the construction of this dial, the motion of the stile is troublesome, and should, if possible, be avoided. For this purpose, the idea first suggested by the celebrated La Grange will be of essential utility. He allows the stile to be fixed in the centre *A*, and describes with the radius *AB* circles upon the different points where the stile is to be placed between *A* and *D*, and on the other side of *A*, which is not marked in the figure. All these circles must be divided equally into hours like the circle *FXIIB*, and when the sun is in the summer solstice, the divisions on the circle nearest the stile are to be used; when he is in the winter solstice, the circle farthest from *A* must be employed, and the intermediate circles must be used when the sun is in the intermediate points. This advice of La Grange may be adopted also in analemmatic dials.

ASTRONOMY.

On the cause of the Tides on the side of the Earth opposite to the Moon.

IT has always been reckoned difficult for those unacquainted with physical astronomy, to understand why the sea ebbs and flows on the side of the globe opposite to the moon. This fact, indeed, has frequently been regarded, and sometimes adduced, by the ignorant, as an insurmountable objection to the Newtonian theory of the Tides, in which the rise of the waters is referred to the attraction of the sun and moon. From an anxiety to give a popular explanation of this subject, Mr. Ferguson has been led into an important error, in so far as he ascribes the tides on the side of the earth opposite the moon, to the excess of the centrifugal force above the earth's attraction.* It cannot be questioned, indeed, that the earth revolves round the common centre of gravity of the earth and moon, at the distance of nearly 6000 miles from that centre; and that the side of the earth opposite the moon has a greater velocity, and consequently a greater centri-

* See Vol. I.

fugal force than the side next the moon; but as the side of the earth farthest from the moon, is only 10,000 miles from the centre of gravity, it will describe an orbit of 31,415 miles in the space of 27 days 8 hours, or 656 hours; this will give only a velocity of 47 miles an hour, which is too small to create a centrifugal force, capable of raising the waters of the ocean.

The true cause of the rise of the sea may be understood from Plate XLII, Fig. 4. where *ABC* is the earth, *O* the common centre of gravity of the earth and moon, round which the earth will revolve in the sume manner as if it were acted upon by another body placed in that centre. Let *AM*, *BN*, *CP*, be the directions in which the points *A*, *B*, *C*, would move, if not acted upon by the central body; snd let *Bbn* be the orbit into which the centre *B* of the earth is deflected from its tangential direction *BN*. Then, since the waters at *A* are acted upon by a force, as much less than that which influences the centre of the earth, as the square of *OB* is less than the square of *OA*, they cannot possibly be deflected as much from their tangential direction *AM*, as the centre *B* of the earth; that is, instead of describing the orbit *Am*, they will describe the orbit *ea*. In the same manner the waters at *c* being acted upon by a force as much greater than that which influences the centre *B* of the earth, as the square of *OB* exceeds the square of *OC*, will be deflected farther from their tangential direction than the centre of the earth, and instead of describing the orbit *cp*, will describe the orbit *hci*.

PLATE XLII. Fig. 4.

As the earth, therefore, when revolving round the centre of gravity *O*, will be acted upon by

the moon, in the same way as by another body placed in that centre, it will assume an oblate spheroidal form *abc;* so that the waters at *c* will rise towards the moon, and the waters at *a* will be *left behind*, or will be *less deflected* than the other parts of the earth, by the lunar attraction, from that rectilineal direction in which all revolving bodies, if influenced only by a projectile force, would naturally move.

MECHANICS.

The following article on Wheel-carriages, taken from Marrat's Mechanics, controverts some of the opinions advanced by Dr. Brewster, on the same subject, in the Appendix of this Work.—A. Ed.

ON WHEEL-CARRIAGES.

ON the subject of wheel-carriages much has been written by philosophers, but to very little purpose; for so contradictory to matter of fact are the arguments adduced by some writers, that a practitioner, in this branch of mechanics, would be apt to suspect that those persons had never seen the carriages, the construction of which they are attempting to explain.

A greater number of mistaken notions were never condensed into one article, than are to be found in Dr. Brewster's remarks on wheel-carriages, in his Appendix to "Ferguson's Lectures on Select Subjects:" as Dr. Brewster is undoubtedly a man of learning and talents, it is much to be lamented that he should have spent so much time on a subject with which he is so *totally unacquainted;* for almost every observation which he has made, is inconsistent with the mode now adopted by wheel-wrights and coach-makers, in the construction of carriages: indeed, no such carriages as the Doctor describes are any where to be met with in the southern parts of Britain.

Experimental knowledge on any subject is always allowed to be preferable to theory alone, but a judicious combination of both, always conducts us the nearest way to obtain our ends.

What most writers have said on this subject, has been principally collected from experiments *made on a very small scale;* what I have to offer, either in conjunction with, or opposition to other authors, is chiefly collected from experiments made on the carriages themselves, and from the accounts I have been able to obtain from consulting good workmen.

The principle advantage obtained by the use of wheels, arises from their turning on their axles; for, when a wheel turns on an axle, the force to overcome the friction is diminished in the ratio of the radius of the wheel to the radius of the axle; and no advantage is gained if they do not turn.

A very small power will move a wheel along a firm level road, where no obstacles intervene; but, in this case, a large wheel will evidently move with a less power than a small one; because the lever which overcomes the friction of the axle, is longer, as the diameter of the wheel increases.

PLATE XLIV. Fig. 6. Let us suppose that the wheel meets with an obstacle, as *O*, (Fig. 6.) then, the spoke *OC* will represent a lever whose centre of motion is *O*; *CD* being the direction of the weight with which the wheel is charged; and *CE*, which is parallel to the horizon, the direction of the power applied to move it.

Now, by the property of the bended lever, *GO* will denote the energy of the power acting in the direction *CE*, and *OH* will represent the energy by which the weight opposes the power; and, in case of an equilibrium, $P : W :: OH : OG$; therefore, $P = \frac{W \times OH}{OG}$; from whence it is plain that the power required to move the wheel

decreases as *OG* increases, or as *OC* increases; for the obstacle remaining the same, *OH* decreases as *OG* or *OC* increases. Here, also, great wheels have the advantage of small ones: but all this is on the supposition that the power rises as the diameter of the wheel increases, so that the direction of the power may always be parallel to the horizon.

Let the direction of the power be *inclined* to the horizon, and first let us suppose it to be *below* the horizontal line *CE*, as in the direction *Ce*; then, it is evident that the power decreases as the angle *ECe* increases; or, when *E* continues at the same height, as *CD* increases; and a great part of the power is lost by forcing the wheel into the ground.

Secondly, let us suppose the direction of the power to be *above* the horizontal line *CE*; then, the power manifestly increases as the line of direction rises, till it comes into the situation *CF*, which is perpendicular to *CO*, and then the power is represented by *CO*, and is a maximum.

Hence, it is plain, that the wheels must not be so large as to cause the line of direction of the power to fall below the horizontal line *CE*, nor so small as to cause it to rise above the direction *CF*.

Consequently, the power requisite to move the wheel whose radius is *GO*=*CH*, when it acts in the direction *CF*, is the same as would be required to move the wheel whose radius is *CO*, when the power acts in the direction *CE*; both being charged with the same weight. Therefore, if the height of the obstacle *Oo* be 4 inches, or if the wheel sink perpendicularly to the depth of 4 inches; then, if *CF* be the direction of the power, the wheel whose radius is *CO* will be

drawn with as much ease, as one of which the radius is 4 inches more, when the power acts in the horizontal direction *CE*.

In order then to know the height of fore wheels, it is necessary to determine the height above the ground, of that point from whence a *middle sized horse* generally draws.

This altitude is about 4 feet, as any person may easily prove by *actually measuring it:* now the height of the fore wheels is commonly about $4\frac{1}{2}$ feet, the radius, therefore, is 2 feet 3 inches; to which adding 3 inches for half the thickness of the axle-tree, and 4 inches more for the thickness of the shaft, we have 2 feet 10 inches for the height of the upper side of the shafts from the ground; and this taken from 4 feet, leaves 1 foot 2 inches for the difference of the heights of the point from whence the horse draws, and the upper surface of the shafts, at the axle-tree. If we take 3 inches from this, which is generally lost by the obliquity of the chains by which the shaft horse draws, we shall still have a surplus of 11 inches; hence, it is manifest, that the fore wheels might be at least a foot higher, or $5\frac{1}{2}$ feet in diameter, and then the line of direction in which the horse draws would be 5 inches above a horizontal direction.

But this is on the hypothesis that the roads are perfectly hard and level, which is very frequently not the case; we will, therefore, suppose the wheels to move in soft or silt roads, where they sink to the depth of 4 inches, and often more; here it is evident, from what has been said in the preceding page of this article, that there is a great advantage gained, and the power is actually increased, by the line of direc-

tion in which it acts being considerably inclined above the horizontal direction, and especially in moving up a hill, where the power is most required. From whence it follows, even from the theory, that the height of fore wheels, where the roads are soft, should not much exceed about $4\frac{1}{2}$ feet; but it has been *experimentally* proved, that when the fore wheels of waggons are more than $4\frac{1}{2}$ feet in diameter, that a greater power is required to draw them along soft roads (the weight being the same) than when the wheels are about that height.

It is, therefore, not probable, that small fore wheels will be abandoned for larger ones, since they are not only stronger, and of course will bear more weight, but they last longer, and are far more commodious and manageable.

We may observe also, that, in soft roads, the horses generally sink to the depth of 3 or 4 inches, which lowers the point of application of the power, and is another reason for adopting low fore wheels.

The wheels of carriages are generally made dishing (as the workmen call it) or hollow, that is, the spokes are not placed perpendicular to the nave, but inclining forward; and wheels of this form are known, from experience, to be stronger than if they were cylindrical, or than if the spokes were placed perpendicular to the nave. This is obvious when we consider the violent strains to which wheels are liable from the push which the load frequently makes against the hind end of the nave, especially when the wheel suddenly falls into a rut; and it is further confirmed by observing that the spokes become more and more upright as the wheels grow older. From whence it is plain,

that if wheels were made cylindrical at first, they would soon become round, or the ends of the spokes at the ground would incline inwards, which would render them extremely defective. It must be observed, however, that wheels are sometimes dished much more than what appears to be necessary, but no general rule for a proper quantity of dish has yet been found out; it is, therefore, regulated according to the fancy of the workman: wheels that are $5\frac{1}{2}$ feet high are generally dished about 3 inches, and those that are $4\frac{1}{2}$ feet high are dished about $2\frac{1}{2}$ inches.

But though the spokes are inclined to the nave, yet the rims are *always* made *cylindrical*, and not conical, as Dr. Brewster asserts; and when the carriage is on level ground, the axletree is so formed that every spoke in the wheel, as they successively come to the ground, may be in a *vertical position*, or *perpendicular to the horizon:* (the workmen commonly let the foot of the spoke at the rim point outward about a quarter of an inch, because, as was observed above, the wheels always wear straighter) in which situation it is evidently the strongest.

Who ever saw a carriage running with the rims of the wheels as wide at the bottom as at the top?—Philosophers have long been mistaken in this point, for the wheels have been so posited, that the lowest spoke, when at the ground, might be perpendicular to the horizontal plane on which it moved, for 50 years at at least, and perhaps a great deal longer.

If the axle-tree were so formed that the whole rim of the wheel might be perpendicular to the horizon, the spokes would, of course, be inclined to the horizon; and the oblique pressure of the load upon the spokes, would cause the hin-

der bush to rub very hard against the otter (an iron plate fixed at the extremity of the thick end of the arm of the axle-tree), and, consequently, the wheel would not move freely; but this is remedied by bending, as it were, the small end of the axle-tree downwards, so that the spokes may always receive the load, or pressure, when in a perpendicular position.

The end of the axle-tree is also bent a little forwards, (this workmen call the *gather*) viz. about $\frac{2}{10}$ of an inch, for the purpose of making the carriage turn to the right or left with more ease, and also to prevent the fore sides of the wheels from gathering outwards; as they naturally would otherwise do, on account of the conical figure of the axle-tree. It is found, by experience, that the carriage runs much easier when the axle-tree is so bent, than when it is not.

When the dish of the wheels is 3 inches, the wheels at top, measuring to the outside of the rims, ought to be a foot wider than they are at the ground: in all cases they should be 4 times the dish wider at the top than bottom.

The distance of the outer edges of the wheels at the ground, (or what workmen call the road) is commonly about 5 feet 1 inch, in what are called narrow wheels, and 5 feet 4 inches in broad or 6 inch wheels.

We shall just add, to what has been said concerning concave or dishing wheels, that they are a great deal more *convenient* than cylindrical ones would be;—first, because they allow more room for the body of the carriage and the load; and, secondly, as they make the carriage more easy to turn in a small compass; lastly, they

are more suitable for large towns, where a great number of carriages are almost continually passing and repassing; as they prevent the wheels of those carriages from getting entangled, and so prevent also much confusion, and render travelling much less liable to accidents.

FINIS.

GENERAL INDEX.

A

	VOL. PAGE
ABSOLUTE fall of water - - -	ii. 146
Achromatic telescope - - -	ii. 403
——— object glass, double - -	ii. 407
tables of their radii	ii. 416
——— ——— triple - -	ii. 408
tables of their radii	ii. 411
telescopes with five lenses -	ii. 418
formulæ for - -	ii. 420
——— eye-pieces - - -	ii. 425
——— ——— formulæ for - -	ii. 426, 427
Adulteration of metals, how to detect - -	i. 179
Air, properties of - - - -	i. 189
—— component parts of - - -	i. 198
—— pump, experiments on it - -	i. 206
—— pump - - - -	i. 202
—— —— Haas's - - - -	i. 224
—— —— Smeaton's - - -	i. 224
—— —— Prince's - - - -	i. 224
—— —— Cuthbertson's - - -	i. 224
—— —— Edelcrantz's - - -	i. 224
—— —— Mendelsshon's - - -	i. 224
—— gun - - - - -	i. 226
Aldersea, Mr. his engine for raising water -	i. 142
Altitude of the sun, to find it by trigonometry -	ii. 55
Amontons on friction - - -	ii. 284
Analemmatic dial - - - -	ii. 464
Anemometer, Leslie's - - -	ii. 267
Angle of incidence - - - -	i. 232
——— refraction - - -	i. 232
Antipodes - - - - -	i. 304
Antœci - - - - -	i. 306

VOL. PAGE

Archimedes, his proposition for finding the deceit in king Hiero's crown - - - i. 171
——— his proposition for finding the area of a circle, and the solidity of a cylinder raised upon that circle i. 156
Archimedes's screw engine - - - ii. 119
Arithmetic, mechanical - - - - i. 54
Armillary sphere - - - - i. 351
Atmosphere, its whole weight upon a man - i. 192
——— its whole weight upon the earth - i. 193
Attraction of cohesion - - - i. 6
——— of aggregation - - - i. 6
——— of composition - - - i. 6
——— of gravitation - - - i. 9
——— of magnetism - - - i. 19
——— of electricity - - - i. 20
Atwood's machine - - - - i. 32
Azimuth - - - - - i. 321
Azimuth of the sun, to find it by trigonometry - ii. 55, 60

B

Balance - - - - - i. 55
——— Helsham's property of - - i. 55
——— hydrostatical - - - i. 173
Barometer - - - - i. 193
Bellows, hydrostatical - - - i. 121
Barker's water-mill, without wheel or trundle - ii. 103
——— various improvements upon it - ii. 105
Bevelled wheels, on the formation of their teeth ii. 232
Blakey's hydraulic engine - - - ii. 114
Blowing-machine - - - - ii. 400
Bodies moving in orbits have a tendency to fly off from them i. 33
——— move faster in small orbits than in large ones i. 34
——— their centrifugal force - - i. 34
Breast-mills - - - - ii. 184
Bulfinger on friction - - - ii. 284
Burning-glasses, the force of their heat - i. 235
Butterfly, valve - - - - i. 132
Button, valve - - - - i. 132

C

Camera obscura - - - - i. 271
Capstane, description of a simple and powerful one ii. 369

	VOL. PAGE
Carriage-wheels - - - - -	ii. 310
———— conical rims disadvantageous in	ii. 323
———— inclination of spokes disadvantageous	ii. 317
———— Mr. Marrat's directions for making	ii. 477
Cartesian vortices - - - -	i. 38
Cassegrainian telescope - - -	ii. 451
———— table for - -	ii. 452
Centre of gravity - - - -	i. 14 & 40
———— the curves described by bodies moving round it	i. 27
———— mechanical method of finding it	ii. 374
Central forces - - - -	i. 22
Centrifugal forces - - - -	i. 34 & 36
Circles of the sphere - - -	i. 287
Clack, valve - - - -	i. 132
Climates - - - - -	i. 326
Clocks and watches, how to regulate them -	ii. 67
Colours, theory of - - - -	i. 277
——— prismatic, make a white when blended together	i. 280
——— transparent, become opaque if put together	i. 280
Colours - - - - -	i. 335
Combined forces, their effect - - -	i. 23
Compressibility of water - - -	i. 114
Conjugate foci - - - -	i. 237
Constellations - - - -	i. 336
Coulomb on wind-mills - - -	ii. 246
——— on friction - - -	ii. 288
Course of discharge - - -	ii. 144
——— of impulsion - - -	ii. 144
Cranes - - - - -	i. 98 & 62
Crane, description of a new and safe one, invented by the author	ii. 95
Cylindrical sun-dial for showing the time of the day, the sun's place, and altitude -	ii. 127
———— method of using it - -	ii. 130

D

Damps - - - - - -	i. 199
Danger of people's rising hastily in a coach or boat, when it is likely to be overset - - -	i. 15
Days lengthened by the refraction of the sun's rays	i. 231
Declining dials - - - -	ii. 4
Descending velocity gives a power of equal ascent	i. 14
Dial, horizontal - - - -	ii. 3, 6, 16
—— vertical - - - -	ii. 4, 6, 9
—— inclining - - - -	ii. 4

	VOL.	PAGE
Dial, reclining	ii.	4
—— declining	ii.	4
—— erect, direct, south	ii.	6, 9
—— erect declining	ii.	10
—— on a card	ii.	19
—— Hardie's universal	ii.	22
—— double horizontal	ii.	63
—— Babylonian	ii.	63
—— Italian	ii.	63
—— Analemmatic	ii.	464
—— Lambert's description of	ii.	471
Dials, method of placing them	ii.	67
—— to make three on three different planes with one gnomon	ii.	131
—— universal, on a plain cross	ii.	132
———— by a terrestrial globe, and the shadows of several gnomons at the same time	ii.	135
Dialling lines, how constructed	ii.	17
—— by trigonometry	ii.	43
—— principles and art of	ii.	1
—— by the terrestrial globe	ii.	5
—— cylinder, universal	ii.	123
Direction, line of	i.	14
Divisibility	i.	3
———— example of	i.	4
———— infinite	i.	5
Dolland's achromatic eye-piece	ii.	431
Double microscopes	ii.	454
—— mills	ii.	131
Double projectile force, a balance to a quadruple power of gravity,	i.	36

E

Earth, proof of its being globular	i.	283
—— motion of the, demonstrated	i.	49
Earthquakes	i.	202
Eclipses	ii.	79, 81
—— limits of	ii.	85
—— period of	ii.	85
Ecliptic	i.	283
Electricity	i.	20
Engine (any mechanical) how to compute its power	i.	53
—— for working pumps by water	i.	141
—— for raising water by horses	i.	142
—— Newsham's, for extinguishing fire	i.	140

	VOL.	PAGE
Engine, steam	ii.	376
Epicycloids, exterior, method of forming them mechanically	ii.	221
——— ——— method of forming them geometrically	ii.	223
——— interior	ii.	221
Equation of time	i.	349
Equation of time	ii.	35
Equinoctial	i.	285
Erect, direct, south dial	ii.	96
—— declining dial	ii.	10
Euler on wind-mills	ii.	261
—— on friction	ii.	284
Expansion of metals	i.	17
Eye described	i.	239
Eye, accommodation of, to different distances	i.	240
—— dimensions of	i.	241

F

	VOL.	PAGE
Face of the heavens and earth, how represented in a machine	i.	284
Fermentation	i.	200
Finder, a new one for Newtonian telescopes	ii.	456
Fire damps	i.	201
—— engine	i.	162 & 140
Float-boards, number of	ii.	149
——— size of	ii.	149
——— position of	ii.	155
Flour mills, improvement upon	ii.	200
Fluid, definition of	i.	114
Fluid microscopes	ii.	461
——— Grey's	ii.	461
——— single, invented by the editor	ii.	462
——— double, invented by the editor	ii.	463
Fluids press as much upwards as downwards	i.	115
Fly-wheels, on the nature and operation of	ii.	302
——— how they become regulators of machinery	ii.	302
——— how they become accumulators of power	ii.	308
Forces, central	i.	22
—— combined	i.	23
Foundation principle of all mechanics	i.	52
Fountain at command	i.	131
Friction, how to find the *momentum* of, in wind-mills	ii.	266
—— nature and effects of	ii.	283
—— Amonton's observations upon	ii.	284

	VOL. PAGE
Friction, Bulfinger's observations on -	ii. 284
——— Parent's ——— -	ii. 285
——— Euler's ——— -	ii. 285
——— Ferguson's ——— -	ii. 286
——— Vince's ——— -	ii. 287
——— Coulomb's ——— -	ii. 288
——— Leslie's ——— -	ii. 293
——— method of diminishing the effects of	ii. 296
——— wheels - -	ii. 297
Frigid zone - - -	i. 310

G

Glass, the shapes into which it is generally ground for optical uses - - -	i. 284
Glass-grinding - - -	ii. 433
Globe, terrestrial use of - -	i. 300
——— celestial use of - -	i. 339
Gold, how much heavier than its bulk of water	i. 173
Gold-beaters, to what a prodigious extent they can hammer out gold - - -	i. 4
Gravity - - -	i. 2
——— decreases as the square of the distance increases	i. 2
Gregorian telescopes - -	ii. 450
——————— table for - -	ii. 451
Grey's water microscopes - -	ii. 461
Grinding lenses, method of - -	ii. 433
Grinding mirrors, method of - -	ii. 438
Gudgeons, form and size of - -	ii. 159

H

Halley's, Dr. period of eclipses -	ii. 85
Hand-mill - - -	i. 92
Harvest-moon - - -	i. 346
Heavens, their apparent motion -	i. 286
Herschel's telescope - -	ii. 454, 460
Horizon, sensible and rational -	i. 287
Horizontal dial, how to construct it -	ii. 6, 16
——————— description of -	ii. 3
——— mills - - -	ii. 176
Horse-mill - - -	i. 94
Horses, way in which they draw -	ii. 329
——— power of - -	ii. 397
Hour of the day, to find it by trigonometry -	ii. 57

	VOL. PAGE
Hydraulics - - -	ii. 115
Hydraulics - - -	i. 114
Hydraulic engine, Blakey's - -	ii. 115
Hydrodynamics - -	i. 114
Hydrometer - - -	i. 183
Hydrostatics - - -	i. 114
Hydrostatics - - -	ii. 106
Hydrostatical balance - - -	i. 173
———— bellows - -	i. 121
———— paradox - -	i. 117
———— paradox - -	ii. 106

I

Inactivity of matter - -	i. 2 & 32
Inclined plane - - -	i. 66
Inclining dials - - -	ii. 4
Infinite divisibility of matter - -	i. 5
Intermitting springs - -	i. 132

K

Kepler's problem - -	i. 36

L

Lambert's dial, description of -	ii. 471
Latitude of a place, rules for finding it -	ii. 37
Laws of the planet's motions - -	i. 26
Laywell spring - - -	ii. 113
Lead, how it may be made to swim in water	i. 123
Lenses - - -	i. 234
Leslie's anemometer - -	ii. 267
—— on friction - -	ii. 293, 366
—— on the construction and effect of machines	ii. 254
Leewenhoek, his account of the number and size of the small animals in the milt of a cod-fish -	i. 5
Lever, its use - - -	i. 54
Light, the amazing smallness of its particles	i. 228
—— argument for the materiality of -	i. 228
—— refracted - - -	i. 229
—— reflected - - -	i. 229
—— colours of - - -	i. 277
Lightning - - -	i. 200

	VOL. PAGE
Limits of eclipses	ii. 83
Line of direction	i. 14
Lloyd's hand-mill	i. 23
Loadstone, its properties	i. 19
Long (Rev. Dr.), his curious experiment with a concave mirror	i. 258
—— his glass sphere	i. 351
Looking-glass	i. 274
———— need be only half the length and half the breadth of a man to show him his whole image	i. 275
Lunations, table of mean	ii. 87

M

Machine, water-blowing	ii. 400
———— thrashing	ii. 336
———— in place of the common hydrostatical bellows	ii. 111
Machines, on the construction and effect of	ii. 354
Maclaurin on wind-mills	ii. 258
Magic lanthorn	i. 252
Magnetism, law of action in	i. 20
———— according to Helsham	i. 20
———— Lambert	i. 20
Man, how he may raise himself up by his breath	i. 123
Man, estimate of his force	ii. 271
Marrat on wheel-carriages	ii. 477
Matter, its properties	i. 1
Mechanical powers	i. 52
———— —— all combined in one engine	i. 53
———— arithmetic	i. 153
Mechanics	ii. 95, 145 & 477
Meridian line, how to draw one	ii. 67
Metals expand by heat	i. 117
——— their specific gravities	i. 177
———— fluid, Grey's	ii. 461
———— —— single, invented by the editor	ii. 462
———— —— double	ii. 463
Microscope, single	i. 247
———— method of making it	i. 248
———— double	i. 249
———— solar,	i. 250
———— —— improvement upon	i. 251
———— reflecting, Sir I. Newton's	i. 252
———— —— Dr. Smith's	i. 252

	VOL. PAGE
Microscopes, single,	ii. 442
——— table for	ii. 445
double	ii. 446
Mill course, on the construction of	ii. 142
Mills, on the performance of undershot	ii. 165
—— horizontal	ii. 176
—— double	ii. 181
—— breast	ii. 184
—— overshot	ii. 189
—— improvement on flour	ii. 209
—— water	i. 80
—— breast	i. 80
—— undershot	i. 80
—— overshot	i. 80
—— wind	i. 94
—— driven by hand	i. 92
—— driven by horses	i. 94
—— double	i. 94
—— corn	i. 30
Millstone, on the formation, size, and velocity, of it	ii. 160
Mill-wright's table	i. 90
——————— description of one on new principles	ii. 169
Mirrors, how they reflect the light	i. 252
Mobility	i. 3
Momentum	i. 52
Monsoons	i. 197
Moon, the law of her motions	i. 27, 28, 29
Motion, all naturally rectilineal	i. 22
Multiplying glass	i. 271

N

New and full moons, how to calculate the mean time of	ii. 72
Newsham's water-engine	i. 140
Newtonian telescope	ii. 454
———— telescopes, table for their apertures, &c.	ii. 459
———— ———— a new finder for	ii. 456

O

Objects, how their images are formed by means of glasses	i. 237
——— why they appear erect, notwithstanding their images are inverted in the eye	i. 243

	VOL. PAGE
Objects, why they appear coloured when seen through some telescopes - - -	i. 264
Opera glass - - -	i. 273
Optics - - -	i. 227
Optic nerve, why that part of the image which falls upon it is lost - - -	i. 245
Overshot-wheels, formation of their buckets	ii. 192
——— velocity of - -	ii. 192
——— table for - -	ii. 197
——— Borda's observations upon -	ii. 198
Overshot-mills, - - -	ii. 188
——— power of - -	ii. 188
——— performance of - -	ii. 190

P

Padmore, Mr. his improvement of cranes -	i. 101
Paradox, hydrostatical - -	ii. 117
——— hydrostatical - -	ii. 106
Parent on windmills - -	ii. 257
——— on friction - -	ii. 285
Periœci - - -	i. 304
Period of eclipses - -	ii. 85
Peritrochio, axis in - -	i. 60
Persian wheel - - -	i. 170
Pile-engine - - -	i. 111
Pipes, bursting of, method of preventing -	i. 128
——— easy method of measuring their contents	i. 151
Pivots, form and size of - -	ii. 159
Planetary motions, laws of - -	i. 26
Planets, three lately discovered - -	i. 337
Pneumatics - - -	i. 189
Poles of the earth and heavens -	i. 288
Polar circles - - -	i. 292
Porosity of bodies - -	i. 16
Precepts for calculating the mean times of new and full moons	ii. 72
Prism - - -	i. 277
Prismatic spectrum, important discoveries concerning it by Herschel and Ritter - -	i. 277
Projectile force - -	i. 26, 27
Pulley - - -	i. 63
——— White's improvement on -	i. 63
——— Smeaton's ——— - -	i. 65

	VOL. PAGE
Pulley, Garnett's improvement on -	i. 65
Pump, common - - -	i. 132
—— forcing - - -	i. 138
—— engine to work by water -	i. 141
—— by horses -	i. 142
Pump-makers, table for - -	i. 138
Pump-mill, quadruple - -	ii. 190
Pyrometer - - -	i. 17
—— Ramsden's, &c. &c. -	i. 19
—— description of an accurate one -	ii. 100

Q

Quadruple pump-mill - -	ii. 190
Quantity of matter in bodies, is in exact proportion to their weight - - -	i. 16
Quicksilver, its weight - -	i. 175

R

Rackwork, method of forming its teeth -	ii. 231
Rain-bow, solar - -	i. 281
lunar - -	i. 282
Ramsden's achromatic eye-piece -	ii. 430
Rays of light - -	i. 227
invisible - -	i. 277
Reciprocating springs - -	ii. 113
Reclining dials - -	ii. 4
Reflecting telescopes - -	ii. 450
table for - -	ii. 451, 453, 459
Refracting telescopes - -	ii. 447
table for -	ii. 449
Relative fall of water - -	ii. 146
Repulsion - - -	i. 9
Right ascension - -	i. 339
Running water, its weight - -	i. 159

S

Sails of a wind-mill, their proper form and angle	i. 94
—— their incredible velocity	i. 95
Saw-mill - - -	ii. 140
Screw - - -	i. 74
—— Hunter's improvement on -	i. 75
—— its power - - -	i. 75
—— shown by a machine -	i. 76

	VOL. PAGE
Screw-engine, Archimedes's	ii. 119
Seasons, how they may be shown by a small globe	i. 293
Ship, how to estimate its weight	i. 183
Signs of the zodiac	i. 290
Silver, how much heavier than its bulk of water	i. 179
Single microscopes	ii. 442
Siroc wind	i. 197
Slare, Dr. his dangerous experiment	i. 225
Smeaton's maxims on undershot-mills	ii. 168
Smeaton on windmills	ii. 274
Solanos	i. 197
Solidity	i. 1
Specific gravities of bodies	i. 171
Spectacles, why some eyes require them	i. 246
——— for divers	i. 248
Specula, method of casting, grinding, and polishing	ii. 438
Spirituous liquors, how to try	i. 180
Spouting fluids, velocity of	i. 125
Springs, reciprocating	ii. 113
——— Laywell	ii. 113
Spur-wheel, formation of	ii. 156
Statera, Roman	i. 56
Steam-engine, Breghton's	i. 162
—— engine	ii. 376
—— ——— Watt's	ii. 378
—— ——— Woolff's improvements on	ii. 391
—— ——— on the power of, and the method of computing it	ii. 395
Steelyard, Roman	i. 55
Strength of materials, table of	i. 79
Sun	i. 285
Sun appears above the horizon when he is really below it	i. 231
Syphon	i. 128

T

Table of the strength of materials	i. 79
—— for mill-wrights	i. 90
—— of Sines for the elevation of water-pipes	i. 149
—— of the quantity and weight of water in a pipe of a given length, with 4 feet diameter of bore	i. 153
—— of the power of the steam-engine	i. 169
—— of specific gravities	i. 177
—— of troy weight reduced to avoirdupois	i. 186

	VOL. PAGE
Table of avoirdupois weight reduced to troy	i. 187
—— of the rarity and expansion of air -	i. 191
—— of the force and velocity of winds -	i. 196
—— of the miles in a degree of longitude in all latitudes	i. 307
—— for single microscopes - -	ii. 445
—— for refracting telescopes -	ii. 449
—— for Gregorian telescopes -	ii. 451
—— for Cassegrainian telescopes -	ii. 453
—— for Newtonian telescopes -	ii. 459
Tables of the sun's place and declination -	ii. 26
—— for calculating new and full moons and eclipses	ii. 87
—— of the equation of time - -	ii. 32
—— of mean lunations - -	ii. 87
—— of the number of floatboards in undershot water wheels, according to Pitot - -	ii. 151
—— for mill-wrights, constructed on new principles	ii. 175
—— for breast-mills - -	ii. 186
—— for overshot-mills - -	ii. 197
—— of the velocity of thrashing machines -	ii. 341
—— of the power of - -	ii. 348
—— for achromatic telescopes -	ii. 416
Tantalus's cup - -	i. 130
Teeth of wheels, method of forming them -	ii. 202
method of disposing them -	ii. 221
Teeth of rackwork, method of forming them	ii. 231
Telescope, refracting - -	i. 259
—— Galilean - -	i. 259
—— astronomical - -	i. 259
—— reflecting -	i. 265
—— Gregorian - -	i. 265
—— Cassegrainian - -	i. 265
—— Newtonian - -	i. 268
—— La Maire's - -	i. 268
—— Herschel's - -	i. 268
—— achromatic - -	ii. 408
—— refracting - -	ii. 447
—— —— table for - -	ii. 449
—— Gregorian - -	ii. 450
—— table for - -	ii. 451
—— Cassegrainian - -	ii. 452
—— table for - -	ii. 453
—— Newtonian - -	ii. 454
—— —— table for - -	ii. 459

		VOL. PAGE
Telescope, Herschel's	- -	ii. 454, 460
——— Dolland's	- -	ii. 415
——— with fluid object glasses	-	ii. 424
Temperate zones	- -	i. 310
Thales's eclipse	- -	ii. 81
Thermometer	- -	i. 180
——— Fahrenheit's	- -	i. 181
——— Reaumur's	- -	i. 181
——— Celsius's	- -	i. 181
——— De la Hire's	- -	i. 181
——— Amonton's	- -	i. 181
——— Crucquius's	- -	i. 181
Newton's	- -	i. 181
Thrashing-machine	- -	ii. 336
driven by water	-	ii. 338
driven by horses	-	ii. 342
on the power of	-	ii. 148
Thunder	- - -	i. 200
Tides	- - -	i. 45
——— St. Pierre's theory of	- -	i. 50
——— on the cause of	- -	ii. 474
Toricellian experiment	- -	i. 192
Tornadoes	- - -	i. 197
Torrid zone	- - -	i. 310
Traction, on the line of	- -	ii. 328
Tropics	- - -	i. 292
Trundle, formation of	- -	ii. 156
Turpentine varnish microscopes	-	ii. 461

U

Universal dialling cylinder	- -	ii. 123
Undershot water-wheels, construction of	-	ii. 141
——— table of the number of floatboards in, according to Pitot		ii. 151
Undershot-mills, on the performance of them		ii. 165
Up and down, only relative terms	-	i. 284

V

Valve, clack.	- - -	i. 132
——— butterfly	- - -	i. 132
——— button or tail	- -	i. 132
Vauloue's pile engine	- -	i. 111

	VOL. PAGE
Velocity of spouting water	i. 125
—— experiment upon, by Brisson	i. 126
Verreir's windmill, description of	ii. 248
Vertical dials	ii. 4
Vince on friction	ii. 287
Vision, how caused	i. 240

W

Water-blowing machine	ii. 400
Water, compressibility of	i. 114
—— how conveyed over hills and valleys	i. 127
—— microscopes	ii. 461
—— mills	i. 89
—— wheel, size of	ii. 149
—— —— number of floatboards in	ii. 149
—— —— position of floatboards	ii. 149
Wedge	i. 69
Westgarth's hydraulic machine	i. 171
Wheel and axle	i. 60
Wheels, bevelled, on the formation of their teeth	ii. 228
—— on the teeth of	ii. 202
Wheel-carriages	i. 102
Wheels, friction	ii. 297
—— fly	ii. 302
—— of carriages, on the advantages of high ones	ii. 315
—— on the position of	ii 327
Wheel-carriages	ii. 310
—— on the formation of	ii. 311
—— how to place the centre of gravity	ii. 333
—— Marrat's observations on	ii. 477
Whirling-table	i. 30
Wilson, Professor, his hydrometer	i. 176
Winds, cause of	i. 194
—— trade	i. 195
Wind, method of finding its velocity	ii. 267
—— by Coulomb	ii. 268
Wind-mills	i. 94
—— description of Verrier's	ii. 248
—— those in Holland	ii. 254
—— sails, on the form and position of	ii. 256
—— —— angle of their inclination, according to Parent	ii. 257
—— —— Maclaurin's theorem concerning	ii. 258
—— —— Euler's observations upon	ii. 261

	VOL. PAGE
Wind-mills, table of their power, &c.	ii. 264
——— horizontal	ii. 277
——— comparison between vertical and horizontal ones	ii. 280
——— Smeaton's observations on	ii. 274
——— Coulomb's ———	ii. 254
Wipers of stampers, &c. mode of forming them	ii. 231
Wood, though light, may be made to lie at the bottom of water	i. 126
World, whether has a tendency of itself to come to an end	i. 44

Z

Zodiac	i. 337
Zones	i. 310
Zurich machine	i. 171

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